

High School Math Problems
2017
Week 8
Problem and Solution

Prove that the equation

$$\sqrt{2-x^2} + \sqrt[3]{3-x^3} = 0$$

has no real solutions.

Solution:

The domain of the equation is

$$D = \left\{ x : x \in \left[-\sqrt{2}, \sqrt{2} \right] \right\}. \tag{1}$$

On the other hand, since $\sqrt{2-x^2} \geq 0$, we must have that $\sqrt[3]{3-x^3} \leq 0$.
Therefore $3 \leq x^3$, or, equivalently,

$$x \geq \sqrt[3]{3}. \tag{2}$$

Now, since $\sqrt[3]{3} > \sqrt{2}$, conditions (1) and (2) cannot be satisfied simultaneously.
Thus the equation has no real solutions.