

High School Math Problems  
2017  
Week 7  
Problem

For  $x > 0$  let

$$f(x) = \left( x \sqrt[n]{1 + (a^n x^{-n})^{\frac{1}{n+1}}} + a \sqrt[n]{1 + (x^n a^{-n})^{\frac{1}{n+1}}} \right) \cdot \left( \frac{b}{5} \right)^{-1},$$

where  $b > a > 0$  and  $n \in \mathbb{N}$ .

Let  $p = f(x_0)$ , where  $x_0 = \left( b^{\frac{n}{n+1}} - a^{\frac{n}{n+1}} \right)^{\frac{n+1}{n}}$ .

Solve the equation

$$\sqrt{1 + \frac{20}{p}y} + \sqrt{\frac{20}{p} - y^2} + \sqrt{y^2 + z^2 - 8z - 3} = \sqrt[4]{y^4 - 16} + p - (z^6)^{\frac{1}{6}}.$$