

High School Math Problems  
2017  
Week 2  
Problem and Solution

Let  $(a_k)_{k=1}^{\infty}$  be an arithmetic progression. Let  $m > n > 0$  be integers such that  $a_{m+n} = b$  and  $a_{m-n} = c$ , where  $b$  and  $c$  be given real numbers.

Find  $a_m$  and  $a_n$ .

**Solution:**

Let  $d$  be the common difference of the given arithmetic progression.

Then

$$a_{m+n} = a_m + nd = b$$

$$a_{m-n} = a_m - nd = c.$$

Therefore

$$a_m = \frac{b+c}{2}.$$

On the other hand,

$$2nd = b - c$$

and therefore

$$d = \frac{b-c}{2n}.$$

From this we obtain that

$$a_n = a_m + (n-m)d = \frac{b+c}{2} + (n-m) \cdot \frac{b-c}{2n} = \frac{(2n-m)b + mc}{2n}.$$