High School Math Problems
2017
Week 22
Problem and Solution

Solve the equation

$$\sqrt{2x^2+21x-11}-\sqrt{2x^2-9x+4}=\sqrt{18x-9}.$$

Solution:

The domain of the equation is $D=\left\{x : x\in\left\{\frac{1}{2}\right\}\cup[4,\infty)\right\}$.

We solve the equation as follows:

$$\sqrt{2x^2+21x-11}-\sqrt{2x^2-9x+4}=\sqrt{18x-9}$$
$$\sqrt{(2x-1)(x+11)}-\sqrt{(2x-1)(x-4)}=3\sqrt{2x-1}.$$

We easily check that $x=\dfrac{1}{2}$ is a solution.

For $x\neq\dfrac{1}{2}$ we can, therefore, divide both sides of the equation by $\sqrt{2x-1}$ and obtain

$$\sqrt{x+11}-\sqrt{x-4}=3$$
$$\sqrt{x+11}=3+\sqrt{x-4}.$$

Squaring both sides of the last equation, we now obtain

$$x+11=9+6\sqrt{x-4}+x-4$$
$$1=\sqrt{x-4}.$$

Again squaring both sides, we obtain

$$1=x-4$$
$$x=5\in D.$$

Therefore the solutions to the equation are

$$x_1=\frac{1}{2}\text{ and }x_2=5.$$

Thanks to Christos Irakleidis from Rhodes, Greece for suggesting a correction on an earlier version of this solution.

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