

High School Math Problems  
2017  
Week 22  
Problem and Solution

Solve the equation

$$\sqrt{2x^2 + 21x - 11} - \sqrt{2x^2 - 9x + 4} = \sqrt{18x - 9}.$$

**Solution:**

The domain of the equation is  $D = \{x : x \in \{\frac{1}{2}\} \cup [4, \infty)\}$ .

We solve the equation as follows:

$$\begin{aligned}\sqrt{2x^2 + 21x - 11} - \sqrt{2x^2 - 9x + 4} &= \sqrt{18x - 9} \\ \sqrt{(2x - 1)(x + 11)} - \sqrt{(2x - 1)(x - 4)} &= 3\sqrt{2x - 1}.\end{aligned}$$

We easily check that  $x = \frac{1}{2}$  is a solution.

For  $x \neq \frac{1}{2}$  we can, therefore, divide both sides of the equation by  $\sqrt{2x - 1}$  and obtain

$$\begin{aligned}\sqrt{x + 11} - \sqrt{x - 4} &= 3 \\ \sqrt{x + 11} &= 3 + \sqrt{x - 4}.\end{aligned}$$

Squaring both sides of the last equation, we now obtain

$$\begin{aligned}x + 11 &= 9 + 2\sqrt{x - 4} + x - 4 \\ 3 &= \sqrt{x - 4}.\end{aligned}$$

Again squaring both sides, we obtain

$$\begin{aligned}9 &= x - 4 \\ x &= 13 \in D.\end{aligned}$$

Therefore the solutions to the equation are

$$x_1 = \frac{1}{2} \text{ and } x_2 = 13.$$