

High School Math Problems
2017
Week 20
Problem and Solution

Find the minimum and the maximum values of $y = \frac{x}{x^2 + 2}$, where $x \in \mathbb{R}$.

Solution:

From $y = \frac{x}{x^2 + 2}$ we have that

$$yx^2 - x + 2y = 0.$$

We now consider the following two cases:

Case 1: $y \neq 0$

Since x is now a real root to the above quadratic equation, we must have that the discriminant D of the equation is non-negative, i.e.,

$$D = 1 - 8y^2 \geq 0$$
$$y \in \left[-\frac{1}{\sqrt{8}}, 0\right) \cup \left(0, \frac{1}{\sqrt{8}}\right].$$

Case 2: $y = 0$

Then $x = 0$.

From Cases 1 and 2 we see that the maximum possible value for y is $\frac{1}{\sqrt{8}}$ and the minimum possible value for y is $-\frac{1}{\sqrt{8}}$.