

High School Math Problems
2017
Week 19
Problem and Solution

For $n \in \mathbb{N}$ let S_n be the sum of the first n terms of a geometric progression with common ratio $q \neq 0, 1$ and first term a .

For $p \in \mathbb{N}$ find $\sum_{n=1}^p S_n$.

Solution:

For every $n \in \mathbb{N}$ we have that

$$S_n = a + aq + aq^2 + \cdots + aq^{n-1} = a \frac{1 - q^n}{1 - q}.$$

Therefore

$$\begin{aligned} \sum_{n=1}^p S_n &= \sum_{n=1}^p a \frac{1 - q^n}{1 - q} = a \sum_{n=1}^p \frac{1 - q^n}{1 - q} = a \left(\sum_{n=1}^p \frac{1}{1 - q} - \sum_{n=1}^p \frac{q^n}{1 - q} \right) \\ &= a \left(\frac{p}{1 - q} - \frac{1}{1 - q} \sum_{n=1}^p q^n \right) \\ &= a \left(\frac{p}{1 - q} - \frac{1}{1 - q} \cdot q \frac{1 - q^p}{1 - q} \right) \\ &= \frac{a}{1 - q} \left(p - q \frac{1 - q^p}{1 - q} \right). \end{aligned}$$