High School Math Problems
2017
Week 17
Problem

Let a_1, a_2, ..., a_n, ..., be a sequence of real numbers, which form an arithmetic progression with common difference d. For every $n \in \mathbb{N}$, let $S_n = a_1 + a_2 + \cdots + a_n$. Further, for $x \in \mathbb{R}$ let $f(x) = x^2 + (2p-1)x + 1$, where $p \in \mathbb{R}$, and assume that

$$2S_n + 1 = f(n) \tag{1}$$

for all $n \in \mathbb{N}$. Suppose also that the roots of the equation $f(x) = 0$ are real. If, lastly,

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 - a_1 a_2 a_3 a_4 = \frac{15}{16}, \tag{2}$$

find a_1, d, and p.