High School Math Problems
2017
Week 14
Problem and Solution

If $x, y \in \mathbb{R}$, what is the minimum value of

$$B(x,y) = x^2 + 2xy + 2y^2 + 4y + 2x - 2017$$

and for what values of x and y is it achieved?

Solution:

We observe that

$$\begin{aligned} B(x,y) &= x^2 + 2xy + 2y^2 + 4y + 2x - 2017 \\ &= \left(x^2 + y^2 + 1 + 2xy + 2y + 2x\right) + \left(y^2 + 2y + 1\right) - 2019 \\ &= (x+y+1)^2 + (y+1)^2 - 2019 \\ &\geq -2019, \end{aligned}$$

where the equality is achieved when

$$\left| \begin{array}{l} x + y + 1 = 0 \\ y + 1 = 0 \end{array} \right. ,$$

which is achieved when $y = -1$ and $x = 0$.

Thus the minimum value of B is $B(0, -1) = -2019$.