Spring 1999 Microeconomic Theory Comprehensive Exam Answers

1.a) In common language, goods are complements if they are used together. The effects of ration price changes cannot tell us whether meat and butter are complements in that sense. The same price effects could occur with goods that are clearly never used together. Butter is a gross complement of meat if a decrease in the price of meat increases the quantity of butter demanded when other prices and consumer incomes are fixed. If the increase in butter consumption occurred for a consumer who was only constrained by the ration points then butter was a gross complement of meat for that consumer. The effect of the ration price reduction for meat (m) could be the move from A' to C' in Figure 2, below. If both the ration and the money budget constraints are binding then we cannot conclude that the goods are gross complements. In Figure 1, meat becomes cheaper in ration points and the quantity of butter demanded increases (the move from A to B). But the demand for butter would decrease (A to C) if the money budget constraint were not binding. In any case, we cannot conclude that the goods are Hicksian complements. (Two goods are *Hicksian complements* [resp. *substitutes*] if the Hicksian compensated demand for one good falls [resp. rises] when the price of the other rises.) In the two good case in Figures 1 and 2, the goods are Hicksian substitutes yet the fall in the ration price of meat increases the demand for butter.

b) Some people do eat meat with butter (on average richer people). Meat and butter are complements in the common usage for those people. But even for those people meat and butter might not be gross or Hicksian complements. For the majority of Americans in the 1940's, and meat and butter were both luxuries, and were not used together. Spending more on one took away money for the other. This suggests that they were more likely gross and Hicksian substitutes. c) Suppose that a consumer has a fixed budget for the two goods, as in both Figures 1 and 2. If both constraints are binding and the consumer consumes both goods as in Figure 1, then the demands are determined by the budget sets, not by preferences, so income and substitution effects are irrelevant. If only the ration constraint is binding, as in Figure 2, the substitution effect is the move from A' to B', reducing the quantity of butter. The only way for butter demand to increase is for its income effect (in the move from B' to C') to be large. The problem suggests that both the demand for butter and for meat rose. This can happen only if the total amount of money spent on the two goods rises, which rules out the situation in Figure 1 for at least some consumers.

2. a) [i] Each consumer provides labor L and maximizes utility $ln(4L^{(1/2)})+ln(1-L)$. The solution is $L = 1/3$, so leisure is $l = 2/3$, consumption is $c = 4/\sqrt{3}$ and utility is $ln(8/(3\sqrt{3}))$. [ii] Let the wage be 1 and let p be the price of consumption. The firm chooses input L to max $4p\sqrt{L} - L$. The solution is $L = 4p^2$. Profit is $4p^2$. Each consumer gets half of the profit and has wealth $1 + 2p^2$. The demand for leisure by the Cobb-Douglas consumer is $l = (1 + 2p^2)/2$. Total leisure demand is $1 + 2p^2$. Total labor supply is $1 - 2p^2$. In equilibrium it equals labor demand $4p^2$, so the equilibrium price is $p = 1/\sqrt{6}$. Each consumer demands 2/3 units of leisure, consumes $c = (1 + 2p^2)/2p = 2\sqrt{6}/3$ and gets utility $ln(4\sqrt{6}/9)$, less than in [i].

[iii] When the consumption price is p, The firm makes profit $4p^2$ as above and consumer i receives the fraction $(1/2) + (1/2)(l_j - l_i)$ of it. So the wealth of consumer i is $w_i = 1 + 2p^2(1 + l_j - l_i)$, and the consumer maximizes $ln\, c_i + ln\, l_i$ subject to $pc_i + l_i \leq w_i$, taking account of the change in w_i due to changing l_i. The solution is the same as if the consumer maximized $ln(pc_i) + ln\, l_i$ subject to the budget constraint, since the consumer treats p as fixed. The budget constraint holds with equality, so we can use it to solve for pc_i. Then the consumer chooses l_i to maximize $ln[1 + 2p^2(1 + l_j - l_i) - l_i] + ln\, l_i$. The first order conditions imply $1+2p^2(1-l_j-l_i) = (1+2p^2)l_i$, so for each i, the optimal l_i is a linear function of l_j with a negative derivative less than 1 in magnitude. It follows that $l_1 = l_2$. This can be seen by plotting l_1 as a function of l_2 and l_2 as a function of l_1. Therefore we have a single equation in l_i, which is solved by $l_i = (1+2p^2)/(2+2p^2)$. As in [ii], labor demand is $L = 4p^2$. In equilibrium, it equals labor supply $2 - l_1 - l_2 = 2 - 2[(1 + 2p^2)/(2 + 2p^2)]$. This yields a second degree polynomial in p^2, which can be solved for $p^2 = (-4 + 4\sqrt{2})/8 > 1/6$. So the price is higher than in [ii]. This implies that the labor supply and consumption by each consumer is higher than in [ii]. The consumers' utilities are lower than in [ii]. This can be shown by computing them or as follows. Suppose

the utilities are at least as high as in [ii]. The utility functions are strictly quasiconcave and increasing, so at the equilibrium prices of [ii], a consumer's bundle in [iii] costs more than the consumer's wealth in [ii]. This implies that the allocation in [iii] is not feasible (using the same argument as in the proof that a competitive equilibrium allocation is Pareto optimal). The contradiction proves that each consumer gets lower utility in [iii] than in [ii].
b) As shown above, the consumers get higher utility in [i] than in [ii]. The first fundamental welfare theorem implies that the allocation in [ii] is Pareto optimal, so it is impossible for both consumers to get higher utility than they do in [ii] given the technology in [ii]. But the technology in [i] is different. In [ii] there is only one producer, whereas in [i], both consumers can produce separately. Since the technology exhibits decreasing returns, the total production set (the sum of the sets of the two consumers) is larger than the production set of a single consumer and larger than the production set of the firm in [ii]. As a result both consumers obtain higher utility in [i].

3. a) Bertrand competitors set prices independently. Suppose that firm 1 discovers the innovation while firm 2 does not, and firm 2 charges the price p in period 2. If $p < c - \Delta$ then firm 1 makes a loss charging $p' \leq p$. So it charges more than p and firm 2 makes a loss. This is not optimal for firm 2, so $p \geq c - \Delta$. The best response for firm 1 is the price p. Then firm 1 has output A/p and profit $A(p-c+\Delta)/p$. Firm 2 gets 0 profit. If $p > c$ then firm 2 could get higher profit by reducing its price. Otherwise it cannot, so the joint strategy with both firms charging p is a Nash equilibrium if and only if $c - \Delta \leq p \leq c$. But for $p < c$ the equilibrium is not trembling hand perfect. If there is any chance that firm 1 would charge a price higher than p, firm 2 would make a loss. Thus if the equilibrium must be trembling hand perfect, then both firms charge $p = c$ and the profit of firm 1 is $A\Delta/c$.
b) For both firms, first period profit is 0, and second period profit is 0 if both firms innovate. If firm i invests k_i, $i = 1, 2$, then its profit is positive only when it has a marginal cost reduction and the other firm, j, does not. This happens with probability $\pi(k_i)(1 - \pi(k_j))$. The expected profit of firm i, discounted to period 1, is therefore $(\delta A\Delta/c)\pi(k_i)(1 - \pi(k_j)) - k_i$. If $k_j \geq 1$ then $\pi(k_j) = 1$ and the optimal k_i is 0. If $k_j < 1$, then the optimal k_i satisfies $\pi(k_i) = k_i^{1/2} = (\delta A\Delta/2c)(1 - \pi(k_j))$ when this last term is less than 1, and $\pi(k_i) = k_i = 1$ otherwise. Since the problem for firm j is the same, the same solution holds with the indices i and j reversed. Define $a \equiv \delta A\Delta/2c$. As long as $a \neq 1$, the equations $\pi(k_i) = a(1 - \pi(k_j))$ and $\pi(k_j) = a(1 - \pi(k_i))$ have a unique solution $\pi(k_i) = \pi(k_j) = a/(1 + a)$. If $a = 1$ then there are multiple Nash equilibria with $\pi(k_1) + \pi(k_2) = a = 1$. For $a \neq 1$, the firms invest more if a is higher, i.e., if δ, A or Δ are higher or if c is lower. Thus there is more investment when there is less discounting, higher demand, higher potential cost reduction and lower initial marginal cost. The last result may seem odd, but what matters is ratio Δ/c, the fraction of the marginal cost that can be eliminated. This fraction rises when c falls.
c) If firm 1 commits to investing k_1, then, from above, the optimal investment for 2 satisfies $\pi(k_2) \leq a(1 - \pi(k_1))$, with equality if the right side is no greater than 1. Define $\pi_2(k_1)$ to be $a(1 - \pi(k_1))$ or 1, whichever is smaller. Then $\pi_2(k_1)$ is the probability that firm 2 has a marginal cost reduction when it invests optimally given the investment k_1 by firm 1. From b), the discounted expected profit of firm 1 becomes $2a\pi(k_1)(1-\pi_2(k_1))-k_1$. This has a higher derivative with respect to k_1 than the profit function in b) if $\pi_2(k_1) < 1$ (since in that case $\pi_2'(\cdot) < 0$). Suppose that $\pi_2(k_1) < 1$ at the optimal k_1. We will show below that this has to be true. Then the derivative of the discounted expected profit of firm 1 is positive when k_1 is evaluated at the equilibrium of part b). Therefore the optimal k_1 with commitment is higher than in the equilibrium of part b). Firm 1 invests more and firm 2 invests less than in b). In the answer to part b) we saw that the discounted expected profit for firm 1 when it chooses any given k_1 is higher if firm 2 invests less, so firm 1 gains by being able to commit. Finally, note that $\pi_2(k_1) < 1$ at the optimal k_1 with commitment. If not, then $a(1 - \pi(k_1)) \geq 1$ and $\pi_2(k_1) = 1$, and firm 2 makes positive discounted expected profit ρ, whereas the discounted expected profit of firm 1 is $-k_1 \leq 0$. But this cannot be optimal for firm 1 since the firm can reverse the situation and invest $k = 1$. By symmetry, the optimal investment for firm 2 is then 0 and firm 1 makes profit $\rho > 0$. This

contradicts the assumption that k_1 was optimal and shows that $\pi_2(k_1) < 1$ at the optimal k_1.

4. This answer refers to the exam labeled "corrected 6/2000". Let i and n denote "investment" and "no investment" by the incumbent I. The entrant E enters (chooses e) or stays out (chooses o). If E enters, I chooses a (accommodate) or f (fight). b) A strategy for E is a list of moves, one in each of its two information sets. For example (no, ie) means that E stays out if I does not invest, and E enters if I does invest. E has four pure strategies: (no, io), (no, ie), (ne, io), (ne, ie). c) I has four information sets and two possible moves in each, so it has $2^4 = 16$ pure strategies. One strategy is $(n, nef, iuea, isef)$. In it, I chooses n, no investment, in the first node. Following that, if E enters (plays e), I chooses f. The strategy also lists moves for I in the information sets that follow investment. These moves are responses to E choosing e after investment (i) by I. After unsuccessful investment (iu) and entry by E, I chooses a: ($iuea$). After successful investment by I and entry by E, I chooses f: ($isef$).
d) In a pure strategy SPNE, I chooses the optimal response to e in each of three information sets: nea, $iuea$, and $isef$. If I plays n at the beginning, then E gets 2 by entering and 0 otherwise, so E enters and the payoffs are 2 for both players. Following i by I and e by E, E gets 2 or -4 with equal probabilities. So E chooses o after i and the expected payoffs are $(4.5, 0)$. This shows that I gets a higher expected payoff from i at the beginning. The unique SPNE is $(i, nea, iuea, isef)$ for I and (ne, io) for E. e) No, as shown in d).
f) Suppose that I chooses f whenever E chooses e. The best response for E is o no matter what. Then I gets expected payoff 4.5 from i and 5 from n. It is not possible for I to get a higher expected payoff than 5 when E plays (no, io). So $(n, nef, iuef, isef)$ is a best response for I, and that along with (no, io) for E forms a Nash equilibrium without investment. For this equilibrium to be played, I being able to commit to fighting even when it is not optimal for I (when the technology fails and E enters). In the equilibrium play of the game, the nonoptimal move f is not made. The information sets at which I chooses f are never reached. The payoff to I is higher than in the SPNE, and the payoff to E is 0 as in the SPNE. So the SPNE is less efficient for these two players than the NE without investment.
g) If E plays (ne, io) we showed above that I's best response is a strategy beginning with i, so E does not enter. If E plays (no, ie) then I's highest possible expected profit following i is 2.5, so I's best response begins with n, and again E does not enter. The only other possible strategy for E that contains the move e is (ne, ie). If I begins with n then the best possible payoff for I is 2. But I's expected payoff from $(i, nea, iuea, isef)$ is 2.5, and it is the best response to (ne, ie). But (ne, ie) is not the best response to $(i, nea, iuea, isef)$ since E does better with (ne, io). This shows that (ne, ie) is not part of a Nash equilibrium. Thus there is no Nash equilibrium play of the game in which the E enters. But the presence of E still has an effect on the outcome of the game. If E was sure to stay out, I would play n. But when commitment is impossible, we expect the outcome to be the SPNE in which I invests. So the threat of entry leads to investment. The incumbent can be interpreted as investing in part to prevent E from entering. h) If the firms are strictly risk averse then they prefer a sure profit to a random profit with the same expected value. Slight risk aversion does not change the ranking of expected utilities by much, so it does not change the answers. Strict risk aversion for E will not make E more likely to enter following i, so its preferences among strategies do not change. For I, strict risk aversion reduces the value of the random outcome that follows i when E plays ie. The best possible outcomes for I are $isef$ or $iuea$, with equal probability. The resulting profit for I is 5 or 0, with 2.5 expected profit. However, if I is sufficiently risk averse it prefers the sure profit 2 that it gets from nea. Then its optimal response to (ne, ie) begins with n, and in that case there is a Nash equilibrium play of the game in which E enters.