UNIVERSITY AT ALBANY, STATE UNIVERSITY OF NEW YORK
DEPARTMENT OF ECONOMICS
Ph.D. Preliminary Examination in Microeconomics
June 12, 2006

Instructions: Answer any three of the following four questions. Whenever possible, justify your answers. Write your answer to each question in a separate bluebook. Write the number of the question on the cover of the bluebook. DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

1. Consider an exchange economy with one good; two periods, $t = 1, 2$; and two consumers, A and B. The utility of A when he consumes x_{A1} at $t = 1$ and x_{A2} at $t = 2$ is $U_A(x_{A1}, x_{A2}) = \sqrt{x_{A1}} + \sqrt{x_{A2}}$. The utility function of B is $U_B(x_{B1}, x_{B2}) = \sqrt{x_{B1}} + \frac{1}{2}\sqrt{x_{B2}}$. Each of the consumers gets 1 unit of the good at $t = 1$, but only consumer B gets 1 unit of the good at $t = 2$. Initially assume the good is perishable, i.e., storage is not possible.

a. What are the Pareto efficient allocations? Sketch the loci of Pareto efficient allocations.

b. What are the core allocations?

c. Let p be the price of the good at $t = 1$ and let the price of the good at $t = 2$ be 1. What is the competitive equilibrium price and allocation in this model?

d. Now, suppose there is a firm that can transform period 1 good into period 2 good. Specifically, the firm's production set is $\{(y_1, y_2) | y_1 \leq 0, y_2 \leq -y_1\}$. Characterize the Pareto efficient allocations of this production economy. (Write down the values parametrically.)

e. What is the competitive equilibrium of this production economy?

f. Compare the welfare of individuals at the competitive equilibrium of the exchange economy with that of the production economy. Interpret the results.

2. Suppose there are two types of consumers, H and L. The total number (mass) of type H consumers is 1 and the total number of type L is 1 as well. The two types have different tastes. The H types have the utility function $U_H = 8\sqrt{x} + y$ and the L types have utility function $U_L = 6\sqrt{x} + y$, where x is a continuous variable measuring quality of a particular good, and y is money (representing all other goods). Commodity x is provided by a monopolist. It costs the monopolist \$1 to produce one unit of x. Initially, assume the reservation utility of both types is zero. The monopolist cannot identify the types directly (except as a consequence of a separating equilibrium).

a. Suppose the monopolist can offer (everyone) only linear pricing. What would be the profit maximizing price and the resulting profit?

b. Suppose the monopolist can offer a single two-part tariff pricing scheme consisting of an initial tariff (flat fee) T and a unit price p (that does not depend on quantity sold). What is the profit maximizing (T, p) for the monopolist? Do both types buy at the optimum two-part tariff?

c. Next, suppose the monopolist can offer menus to each type on a take-it-or-leave-it basis. A menu consists of a level of quality and a price (negative income). Derive $O_L = (x_L, y_L)$, the menu designed for the L types, and $O_H = (x_H, y_H)$, the menu designed for the H types, that maximize the monopolist's profit. Here, if an H type selects O_H, it has to pay y_H in exchange for a good of quality x_H. So, the utility H derives from the contract $O_H = (x_H, y_H)$ would be $8\sqrt{x_H} - y_H$, and similarly for the L type.

d. One can rank the profit of the monopolist in the above three cases without actually calculating them. Explain.

e. Now, suppose the reservation utility is 5 for both H and L. What are the optimal contracts for the monopolist in this case? Would the low type be served? Explain.

3. For parts (a) - (c), consider an N-player finite normal (strategic) form game in which player i has pure strategy set S_i and utility function u_i.

a. Define **strictly** and **weakly dominated (pure) strategies.**

b. Prove that a pure strategy played with positive probability in a mixed strategy Nash equilibrium survives the process of iterated deletion (elimination) of strictly dominated (pure) strategies (IDSD).

c. Without using Nash's existence theorem, prove that if IDSD produces a unique strategy profile, then it is the unique Nash equilibrium.

d. Consider the Cournot duopoly model with linear demand $p = a - bQ$ ($p = 0$ if $Q > \frac{a}{b}$), where $Q = q_1 + q_2$ and $a > 0, b > 0$. Both firms have zero fixed cost and constant marginal cost c ($0 < c < a$). Each firm i has a strategy set $S_i = [0, \infty)$. Solve the game by iterated deletion of strictly dominated strategies. (Only pure strategies need be considered.)

e. Again consider the Cournot model as in part (d) but suppose there are three firms. What is the result of iterated deletion of strictly dominated strategies.

4. Consider the issue of medical malpractice. A surgeon can exercise caution (c) to avoid committing errors during a procedure. Assume the marginal cost (in monetary units) of exercising caution is 1. The damage (also measured in monetary units) resulting from an error is given by $D(c)$, with $D'(c) < 0$. In the event of an error (hence after c is chosen), a patient can sue the physician. The likelihood of success in such a suit is given by $p(l)$, where l represents expenditure on legal fees. Assume $p(0) = 0$, $p' > 0$ and $p'' < 0$. If a suit is unsuccessful, the patient incurs legal fees and does not recover damages. If the suit is successful, the patient recovers damages but not legal fees. However, any compensation is subject to a tax at the rate t. Hence, the net-of-tax award in the event of a successful suit is $D(c)(1 - t)$. Assume (1) both the surgeon and the patient are risk neutral, and (2) there is complete information and c is observable.

a. In the event that damages occur, derive a first order characterization of the patient's optimal expenditure on legal fees as a function of c and t. Explain the meaning of the first order condition you obtain.

b. Are the above assumptions sufficient to ensure the existence of a solution to the first order condition? Are they sufficient to ensure that such a solution is optimal? Explain.

c. Determine the effects of changes in c and t on the optimal expenditure by the patient.

d. Assuming the surgeon can calculate the answer to part (a), characterize its optimal choice of c. Again explain any relevant first order condition.

e. Explain how to determine the effect of a change in t on the surgeon's optimal choice of c.

f. Are the above assumptions sufficient to determine the sign of the relationship in (e)? Explain.

g. Suppose the patient could also recover legal fees in the event of a successful suit. Would the above conditions be sufficient to determine the optimal expenditure on l? Explain.

h. Discuss what if any modifications would be necessary if c were unobservable by the patient. Intuitively, how do you think this would affect the results, i.e., the choices of l and c?