

Instructions: Answer 3 of the following 5 questions. Show all of your work. Write your answer to each question in a separate bluebook. On the cover of the bluebook, write the number of the question under "Section." DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

1. Suppose individuals live for two periods. In the first period, they allocate their time between working and attending school, and in the second period they work. First period income is $y - s$, where s denotes the amount of time spent in school. Second period labor income is $w(\theta, s) = \theta s^\alpha$, where θ is a parameter representing the agent's ability and $0 < \alpha < 1$. Agents can borrow against their future earnings at the interest rate r .

a. Let c^t denote consumption in period t , and assume that agents have strictly quasi-concave utility functions of the form $U(c^1, c^2)$. Write the individual's wealth constraint, and derive and interpret the first order conditions for the individual choice of consumption and schooling.

b. Solve explicitly for the individual choice of schooling. How is the choice of schooling affected by consumer impatience and by the individual ability parameter?

c. Suppose θ can take on two values, θ_H or θ_L , where $\theta_H > \theta_L$. The government wishes to compensate those with low ability by imposing a redistributive tax on the income of the high ability agents. Evaluate the alternative consequences of imposing such a tax on first period income versus second period income. How do such taxes affect the level of schooling attained by ability type H ? Also, discuss the incentive effects of such a scheme on the level of schooling by ability type L .

2. A firm has a patent that gives it a monopoly over a new type of computer. Buyers of computers buy only one unit each. There are two types of buyers, low valuation buyers who are willing to pay at most V_L and high valuation buyers willing to pay up to V_H ($> V_L$). There are N_H high and N_L low valuation buyers, and the marginal cost of a computer is constant at c ($c < V_L$). The above information is common knowledge. Each consumer knows its own type, but the monopoly cannot distinguish among consumers.

a. Assuming there is only one time period, write down the profit function for the monopolist and describe the optimal pricing policy. Derive the condition under which the monopolist will sell only to high valuation buyers.

In the remaining parts of the problem suppose the monopolist can sell in period 1 or 2, with second period returns discounted by δ . If a buyer of type i waits until period 2 to purchase, the present value of the computer to the purchaser is δV_i . The monopolist must charge the same price to all consumers in any given period.

b. Treating the interaction between the monopoly and the consumers as a game, explain what a pure strategy is for the monopoly and for a typical consumer.

c. Under what, if any, conditions will the monopoly charge the price V_H in period 1 and V_L in period 2 in a Nash equilibrium?

d. Under what, if any, conditions will the monopoly charge the price V_H in both periods in a Nash equilibrium?

e. Find a sequential equilibrium for the two period game. Compare the monopoly profit to the profit in the single period case of part a. Explain.

3. Consider a production economy with two consumers/workers, 1 and 2, and two firms, A and B , that produce different outputs. Firm A produces $x = 4L_A$ units of its output when it uses L_A units of labor, and firm B produces $y = 8L_B$ units of its output when it uses L_B units of labor. Each firm can employ only one worker, and each consumer can work only one job. Firm A offers secure employment with a guaranteed wage of 4. In contrast, with probability θ , firm B will close, laying off its employee and producing no output. Otherwise, if it remains open, it will pay its employee a wage of 8. Each consumer is endowed with one unit of labor which it supplies inelastically to its employer. The consumers' have von Neumann-Morgenstern utility functions, where the utility of consumption is given by $u_1(x, y) = x^{1/2} + y^{1/2}$ and $u_2(x, y) = x^{1/4} + y^{1/4}$, respectively.

a. Identify the feasible allocations for this economy.

b. Prove that consumer 1 is less risk averse than consumer 2.

c. Evaluate the following claim: "Since consumer 1 is less averse to risk, efficiency requires that it works for firm B , thus bearing the risk associated with uncertain employment."

d. Explain how contingent markets for commodities x and y would allow the agents to share the risk associated with uncertain employment at firm B .

e. Assuming that agent 1 is employed by firm B , find a competitive equilibrium (in contingent commodities) for this economy. Evaluate whether the equilibrium allocation is Pareto efficient. Discuss the allocation of risk between the consumers.

4. There are two types of firms; high profitability ($\pi = H$ with probability $\frac{1}{2}$) and low profitability ($\pi = L$ with probability $\frac{1}{2}$). An entrepreneur knows the profitability of her firm but an outside investor does not. The entrepreneur wants to finance an investment project by offering a share $s \in \{s_l, s_h\}$ of the equity of the firm to the investor in return for the investment. Given an offer, the investor either accepts the offer or rejects it. The extensive form of the game is depicted in the figure below. The first number is the payoff to the entrepreneur and the second number the payoff to the investor.

a. Give all pure strategies of the entrepreneur.

b. Give all pure strategies of the investor.

c. Give all (pure strategy) pooling sequential equilibria, if they exist. Otherwise, show that there are none.

d. Give all (pure strategy) separating sequential equilibria, if they exist. Otherwise, show that there are none.

e. Interpret any equilibria you have found.

