University at Albany, State University of New York, Department of Economics
Ph.D. Preliminary Examination in Microeconomics

Instructions: Answer 3 of the following 4 questions. Show all of your work. Write your answer to each question in a separate bluebook. On the cover of the bluebook, write the number of the question under "Section." DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 3 hours.

[1] The owner of a competitive firm must choose an output level before knowing the output price. The cost of producing y units of output is $c(y)$, payable at the time the output is sold, where c is convex, with $c' > 0$. The owner knows that when the output is sold, its price will be either p_1 or p_2, with equal probability, where $0 < p_1 < p_2$. The owner chooses the output level to maximize expected utility of the profit. The owner's utility from a sure profit x is $u(x)$.

a. Formulate the owner's optimization problem.

b. Explain what it means for the owner to be risk neutral.

c. Characterize the optimal output level assuming that the owner is risk neutral. What would be the effect on the optimal output level if $p_1 - p_2$ increased but the expected output price remained the same?

d. In the remaining parts of problem 1, assume that $u(x) = -e^{-kx}$, where $k > 0$. What can be said about the owner's risk preference or risk aversion and the dependence of this risk preference or aversion on the level of expected profit?

e. How would the optimal output level be affected if k increased?

f. How would the optimal output level be affected if $p_1 - p_2$ increased but the expected output price remained the same?

g. How would the owner's expected utility be affected if $p_1 - p_2$ increased but the expected output price remained the same? Would the owner prefer to have price uncertainty rather than having the output price fixed at its expected value?

[2] The entire output in an industry is produced by two plants. The cost of producing y_i units of output at plant i is $c_i y_i$, for $i = 1, 2$. The inverse demand function in the industry is $p = a - by$, where y is the total industry output and p is the output price.

a. Suppose that the two plants are owned by a single monopoly firm. Find the industry output.

b. Suppose that the two plants are owned by separate firms that act as Cournot duopolists. Find the output of each firm. Assume for the remaining parts of problem [2] that both duopolists produce in Cournot equilibrium. Compare the industry output level to the level in part a.

c. Is the total industry output in part b produced in the cheapest possible way?

d. Suppose that the owner of plant 1 can reduce the marginal cost c_1 by paying a fixed cost. Show that if the fixed cost is sufficiently small and if the industry is a duopoly then the owner will pay for the marginal cost reduction. Is the same necessarily true if the industry is a monopoly?

e. Suppose that $c_1 \leq c_2$. Show that if the marginal cost reduction were made, the industry output would expand by a larger amount if the industry were a monopoly than if it were a duopoly.

f. Suppose that a monopoly owner would choose to make the marginal cost reduction in part d. Is it possible that the marginal cost reduction would not be made if the industry were a duopoly? Hint: Use the answer to part e.

g. Suppose that in a duopoly the owner of plant 1 would choose to make the marginal cost reduction in part d. Is it possible that the marginal cost reduction would not be made if the industry were a monopoly?

h. Give an intuitive explanation for the answers to parts f and g.

[3] Consider an exchange economy with two traders 1 and 2. There are two goods, apples and bananas. We denote a typical allocation by $((a_1, b_1), (a_2, b_2))$, where a_1 (resp. b_1) denotes of the amount of apple (resp. banana) that goes to trader 1 and similarly for trader 2. Trader 1 has the endowment of 1 apple and no banana. Trader 2 has the endowment of one banana and one apple. The utility of trader 1 is given by $u^1(a_1, b_1) = \min(a_1, b_1)$. The utility of trader 2 is given by $u^2(a_2, b_2) = a_2 + b_2$. We normalize by setting the price of a banana, p^b, equal to 1. Thus, the price of apple, p^a, is in terms of bananas.

a) What are the excess demand functions ($z_1(p^a)$ and $z_2(p^a)$) of trader 1 and trader 2?

b) What are the aggregate excess demand functions $(z^a(p^a), z^b(p^a))$ of apple and banana (as functions of p^a)?

c) What are competitive equilibrium prices and allocations?

e) Suppose that p^a adjusts by $dp^a/dt = z^a(p^a)$. The price of bananas remain fixed at 1. What can you say of the stability of the competitive equilibrium prices?

f) What are the strictly positive Pareto optimal allocations?

g) What are the strictly positive core allocations?

[4]. Ms. Chen has finished her degree and has to depart Albany in a hurry for China. She is leaving a car to a friend to sell for her. She knows that the amount her friend will get for the car will depend partly on the effort her friend puts into selling it. She offers to pay her friend y, for the service. The friend chooses an effort level e from the set $\{0, 1, 2\}$. In the following matrix, the first column represents effort levels and the corresponding rows show probabilities of getting the top row prices.

$$\begin{pmatrix} \text{efforts} \setminus \text{prices} & \$3000 & \$3900 & \$4800 \\ 0 & 2/3 & 1/3 & 0 \\ 1 & 1/3 & 1/3 & 1/3 \\ 2 & 0 & 1/3 & 2/3 \end{pmatrix}$$

Her friend maximizes the expected value of $y - 200e$. Her friend will refuse to help if the maximum expected value of $y - 200e$ is less than zero. Chen maximizes expected net income from the sale. For simplicity, we ignore the time cost of selling the car.

a) Suppose that Chen cannot observe the effort level e. Chen offers a simple contract of $y = ax,\ a \geq 0$, where x is the selling price. Formulate the procedure for finding an optimal a for Chen.

b) What is the optimal a for Chen? What are the corresponding expected incomes for Chen and her friend respectively?

c) Now, suppose instead that Chen could find out her friend's effort level (through the watchful eyes of another friend) and pay accordingly. What would be the optimal solution of (y, e) for Chen? Is Chen better off with perfect information? How about her friend?