

STATE UNIVERSITY OF NEW YORK AT ALBANY
Department of Economics

Ph. D. Preliminary Examination: Macroeconomics
Fall, 2002

Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 1 Hour) *For 4 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. A government budget deficit brought about by a tax cut will increase current consumption.
2. A decline in the number of hours worked during a recession indicates the presence of inefficiencies in the labor market.
3. If a consumer has separable preferences and a flow utility function ($u(c_t)$) with a positive third derivative, he/she will engage in precautionary saving.
4. In a real business cycle model, an increase in current government spending generates an increase in current output.
5. In the baseline consumption-smoothing model, a 1-unit increase in current income will generate less than a 1-unit increase in current consumption.
6. Economic growth occurs due to the presence of increasing returns to scale.

Section 2. (Suggested Time: 2 Hours) Answer any 3 of the following 4 questions.

7. Consider the following model of consumer behavior. The consumer derives utility from her expected consumption and leisure streams:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j (\ln(c_{t+j}) + \ln(1 - \ell_{t+j})) \right\},$$

where ℓ_{t+j} denotes the fraction of the consumer's time endowment she supplies to the labor market. The consumer's real wage, w_t , follows an i.i.d. stochastic process. In particular, each period the consumer's wage takes on the value $w_L > 0$ with probability $\pi \in (0, 1)$ and the value $w_H > w_L$ with probability $1 - \pi$. The consumer can insure against this income risk by buying insurance, i_t . At time $t+1$, this financial instrument will pay the consumer i_t units of the consumption if $w_{t+1} = w_L$ and zero units if $w_{t+1} = w_H$. The consumer can also invest in a risk-free asset, a_t . The consumer's financial resources thus follow

$$\begin{aligned} w_t \ell_t + i_{t-1} \times 1 \{w_t = w_L\} &= c_t + s_t + \frac{\pi}{R} i_t, \\ a_{t+1} &= R a_t + s_t, \end{aligned}$$

where: s_t denotes saving in the risk-free asset; $R > 1$ is the risk-free rate of return, with $\beta R \leq 1$; π/R is the unit price of insurance; and $1 \{w_t = w_L\}$ is an indicator that returns 1 if $w_t = w_L$ and 0 otherwise. The consumer also faces the following constraints

$$\lim_{T \rightarrow \infty} R^{-T} a_{T+1} = 0,$$

$$c_t \geq 0; \quad \ell_t \in [0, 1]; \quad a_0, i_0 \text{ given.}$$

- Assuming interiority, find the first order conditions for utility maximization. Be sure to differentiate between high- and low-wage quantities: you should contrast c_t^H and ℓ_t^H with c_t^L and ℓ_t^L .
- Does the consumer engage in precautionary saving? Why or why not?
- Solve for i_t and interpret your answer.
- Suppose that the market for insurance vanishes, so that i_t must equal 0. Using intuitive arguments, briefly describe how c_t^H , ℓ_t^H , c_t^L and ℓ_t^L will change for any given value of a_t .

8. Consider the following infinite-horizon exchange economy populated by two agents that behave competitively. (Alternatively, there is a large and equal number of each type of agent.) Agent-1 has preferences represented by the following utility function:

$$\begin{aligned} u^1(c_1^1, c_2^1, \dots, c_T^1) &= \min \{c_t^1\}_{t=1}^{\infty} \\ &= \min \{c_1^1, c_2^1, \dots\} \end{aligned}$$

where c_t^1 is the period- t consumption of Agent-1. Agent-2's preferences are given by

$$\sum_{t=1}^{\infty} \beta^{t-1} \ln c_t, \quad 0 < \beta < 1,$$

where c_t^2 is the period- t consumption of Agent-2. Each agent is endowed with y units of a single, perishable consumption good in each period: $y_t^1 = y_t^2 = y$, for all $t = 1, 2, \dots$. There is also government that finances a stream of exogenously determined purchases $\{g_t\}_{t=1}^{\infty}$ by levying lump-sum taxes $\{\tau_t^1, \tau_t^2\}_{t=1}^{\infty}$ on each agent. The government is allowed to issue debt, but must abide by its intertemporal budget constraint. In each period, the agents trade a one-period bond that is in zero net supply. Each bond purchased in period- t delivers 1 unit of consumption in period- $(t+1)$. Letting b_t denote the quantity of bonds purchased (or issued) in period- t and p_t be the period- t price of bonds (in units of period- t consumption), each agent's period- t budget constraint is written

$$c_t^j + p_t b_t^j = y_t^j - \tau_t^j + b_{t-1}^j, \quad j = 1, 2.$$

Assume that debt is initially zero for each consumer ($b_0^1 = b_0^2 = 0$) and that neither can engage in Ponzi-schemes. Note that the real interest rate r_t solves $1 + r_t = \frac{1}{p_t}$ given p_t .

- (a) As precisely as you can, define the competitive equilibrium for this economy. Is the competitive equilibrium Pareto optimal? Explain.
- (b) Suppose that, the government taxes Agent-1 τ units of the good in odd periods and 0 units in even periods, and taxes Agent-2 τ units of the good in even periods and 0 units in odd periods. (You may assume that after-tax income is positive in each period.) Determine the period- t competitive equilibrium interest rate and consumptions.
- (c) Now suppose that the government changes the constellation of taxes in part (b) by eliminating all taxes for Agent-1 and instead, requires that Agent 2 pays the τ units in odd periods (in addition to his previous tax burden). Note that this change leaves the present value of the government's tax revenue unchanged. What are the effects, if any, on the equilibrium path for the real interest rate and individuals consumptions?

9. Consider an economy with a large number of identical consumers indexed by $j = 1, 2, \dots, J$ and a single, representative firm. The economy has an infinite horizon and is dated in discrete time by the index $t = 1, 2, \dots$. Each consumer is endowed with one unit of time per period and the same initial amount of capital: $k_0^j = k_0$ for all j . In each period t , consumers sell labor time n_t^j (measured in time periods) and rents capital k_t^j to the firm, receiving the real wage w_t and real rental rate r_t , respectively. Preferences for each agent are given by the utility function

$$\sum_{t=0}^{\infty} \beta^t \ln c_t^j$$

where $0 < \beta < 1$ and c_t^i is agent i 's period- t consumption. Consumers receive the profits generated by the firm and have the ability to transform current output into future capital unit-for-unit, but the depreciation rate is 100% per period. The period- t budget constraint of consumer j is therefore

$$c^j + k_{t+1}^j = w_t n_t^j + r_t k_t^j + \pi_t^j$$

where π_t^j are consumer j 's share of period- t profits. The representative firm has access to a technology represented by the production function

$$y_t = A k_t^\alpha n_t^{1-\alpha} \bar{k}_t^\theta$$

where $A > 0$, $\theta > 0$, $0 < \alpha < 1$, k_t is firm's capital input at the beginning of period- t , n_t is the firm's period- t labor input, and \bar{k}_t is the average capital stock at the beginning of period- t : $\bar{k}_t = \frac{1}{J} \sum_{j=1}^J k_t^j$. Since any single agent is small relative to the total population J , he or she behaves competitively in the sense that they have no ability to influence aggregate outcomes.

- (a) Dropping the j superscripts to indicate per capita quantities, hypothesize the existence of a single representative consumer and carefully define the corresponding competitive equilibrium. Derive expressions for the equilibrium real wage and real rental rate in each period t .
- (b) Devise a (constrained) social planner's problem for the representative agent economy that, when solved, will yield the competitive equilibrium paths for labor effort, consumption, and capital accumulation. Write the social planner's problem in recursive form (i.e. present the Bellman equation and relevant constraints) and identify the period- t state variables. Assuming that the value function is differentiable, derive the corresponding first-order condition, envelope condition, and Euler equation.
- (c) Assuming that $\alpha + \theta \geq 1$, characterize the competitive equilibrium growth path of capital and consumption as completely as you can.
- (d) Is the competitive equilibrium growth rate faster or slower than the optimal growth rate? Explain the intuition behind your answer.

10. Consider the following simplified version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\begin{aligned} \max_{L_t \geq 0} \Pi_t &= Y_t - W_t L_t, \\ Y_t &= L_t^\eta, \quad 0 < \eta < 1, \end{aligned} \tag{PRF}$$

where: Π_t is profit; L_t is labor; and W_t is the real wage. There is no population or technology growth, and the population, number of firms and steady state technology level are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption (C_t) and labor (L_t) are given by:

$$\begin{aligned} E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \Gamma_t \frac{1}{1+\chi} L_t^{1+\chi} \right] \right), \\ 0 < \beta < 1, \quad \chi \geq 0, \quad \Gamma_t > 0. \end{aligned}$$

Households receive labor income and profits from firms. Households earn a gross return of $(1+r)K_t$ on their assets, K_t , with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption and investment in capital. Households also face the usual non-negativity and No-Ponzi-Game conditions.

Finally, the log of the preference parameter Γ_t follows an AR(1) process

$$\hat{\gamma}_t \equiv \ln(\Gamma_t/\Gamma) = \phi \hat{\gamma}_{t-1} + \varepsilon_t, \quad \Gamma > 0, \quad 0 \leq \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous stationary martingale difference sequence.

- (a) Write down the social planner's problem for this economy as a dynamic programming problem. Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.
- (b) Let lower-case letters with carats “ $\hat{\cdot}$ ” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\hat{\ell}_t = -\theta [\hat{\gamma}_t + \hat{c}_t]; \quad \hat{y}_t = -\alpha [\hat{\gamma}_t + \hat{c}_t].$$

Are θ and α increasing or decreasing in the parameter χ ? Why?

- (c) Suppose that the steady state consumption-to-capital ratio, C_{ss}/K_{ss} , is ψ , with $\psi > r$. It is then straightforward to show that the steady state output-to-capital ratio, Y_{ss}/K_{ss} , is $\psi - r$. Using these results, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1+r)\hat{k}_t - \omega_1 \hat{\gamma}_t - \omega_2 \hat{c}_t, \tag{CA'}$$

$$\omega_2 > \omega_1 > 0.$$

- (d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA')) to express consumption as a function of capital and preferences

$$\widehat{c}_t = \lambda \widehat{k}_t - \mu \widehat{\gamma}_t, \quad (\text{CF})$$

with

$$\lambda = \frac{r}{\omega_2} \in (0, 1); \quad \mu = \frac{r\omega_1}{\omega_2(1 - \phi + r)} \in (0, 1).$$

1. Using (CF) and your answer from part (b), express \widehat{y}_t and $\widehat{\ell}_t$ as functions of capital and preferences. Are the magnitude of the coefficients on $\widehat{\gamma}$ bigger or smaller in this equilibrium expression than in your answer to part (c)? Why?
2. Are the magnitudes of the coefficients on $\widehat{\gamma}$ increasing or decreasing in the parameter ϕ ? Why?