Center for X-ray Optics

The Physics of Polycapillary Optics

MAXWELL'S EQUATIONS FOR A NON-MAGNETIC INSULATOR:

$$\left.\begin{aligned} &\nabla \cdot \vec{E} = 0 \\ &\nabla \cdot \vec{B} = 0 \\ &\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &\nabla \times \vec{B} = \mu_o \in \frac{\partial \vec{E}}{\partial t} \end{aligned}\right\} \Rightarrow$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_o \in} \nabla^2 \vec{E}$$

$$v = \frac{1}{\sqrt{\mu_o \in}} = \frac{1}{\sqrt{\mu_o \in_o}} \sqrt{\frac{\in_o}{\in}} = c \cdot \frac{1}{n}$$

$$n, \text{index of refraction} = \sqrt{\frac{\in}{\in_o}}$$

So, given an incident plane wave of the form:

$$\vec{E} = \vec{E}_o e^{i\omega t}$$

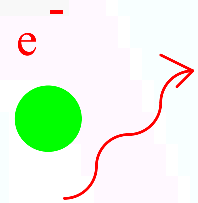
we expect a response

$$\vec{D} = \in \vec{E} = \in_o \vec{E} + \vec{P}$$

$$D = \in E = \in_o E + P \Rightarrow \in = \in_o + \frac{P}{E}$$

$$P = Nqx$$

Free Electrons:



$$E = E_o \cos\omega t$$

$$F = m\ddot{x} = qE = qE_o \cos\omega t$$

$$x = x_o \cos\omega t$$

$$\Rightarrow -m\omega^2 x_o = qE_o \Rightarrow x_o = \frac{-qE_o}{m\omega^2}$$

$$P_o = Nqx_o = \frac{-Nq^2}{m\omega^2} E_o$$

$$\Rightarrow \in = \in_o + \frac{-Nq^2}{m\omega^2} = \in_o \left(1 - \frac{\frac{Nq^2}{m \in_o}}{\omega^2}\right)$$

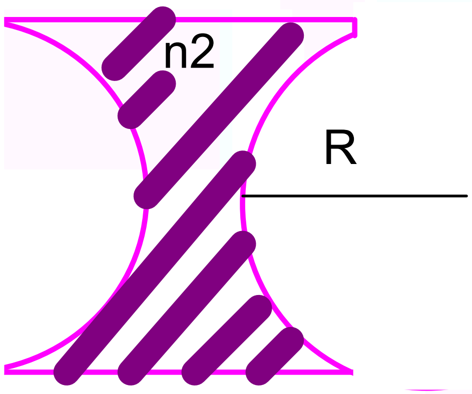
$$\in = \in_o \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_o}} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \approx 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}$$

$$= 1 - \delta$$

$$\delta = \frac{1}{2}\left(\frac{30\text{eV}}{10\text{KeV}}\right)^2 \approx 5 \times 10^{-6}$$

Consequences for refractive optics:



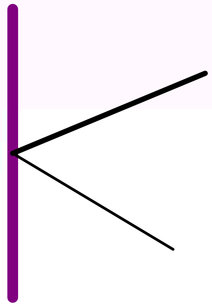
Lens-maker's rule:

$$f = \frac{1}{1-n2}\frac{R}{2} \approx \frac{1}{5\times 10^{-6}}\frac{2\times 10^{-2}}{2}$$

$$\approx 2\ km$$

☹

Consequences for normal incidence mirror:



$$R(\theta = 0) = \left(\frac{n1 - n2}{n1 + n2} \right) = \frac{\delta^2}{2} \approx 10^{-11}$$

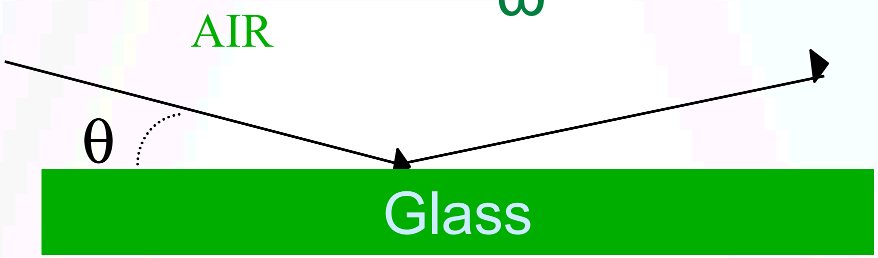
☹

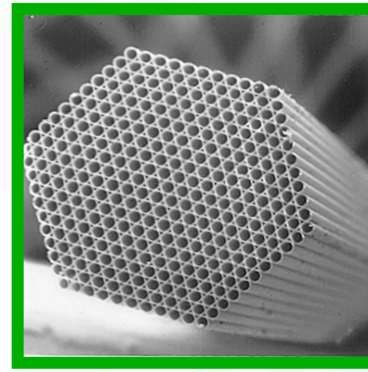
POLYCAPILLARY OPTICS

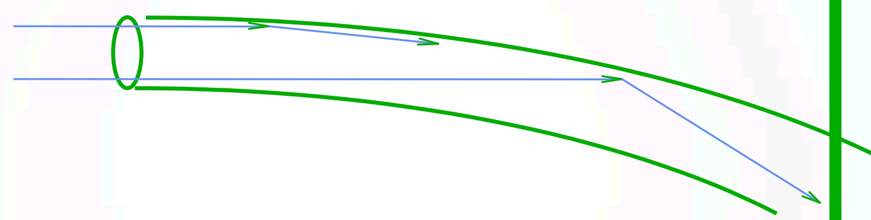
$$n = 1 - \frac{\omega_p^2}{\omega^2}$$

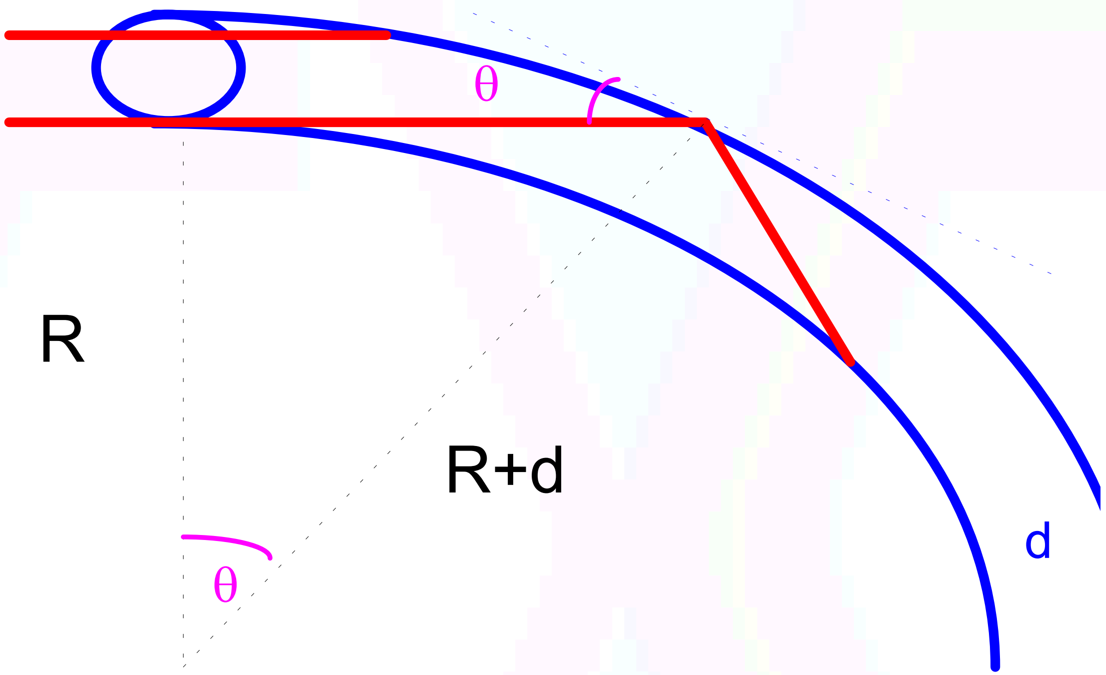
Total External Reflection

$$\theta < \theta_c$$





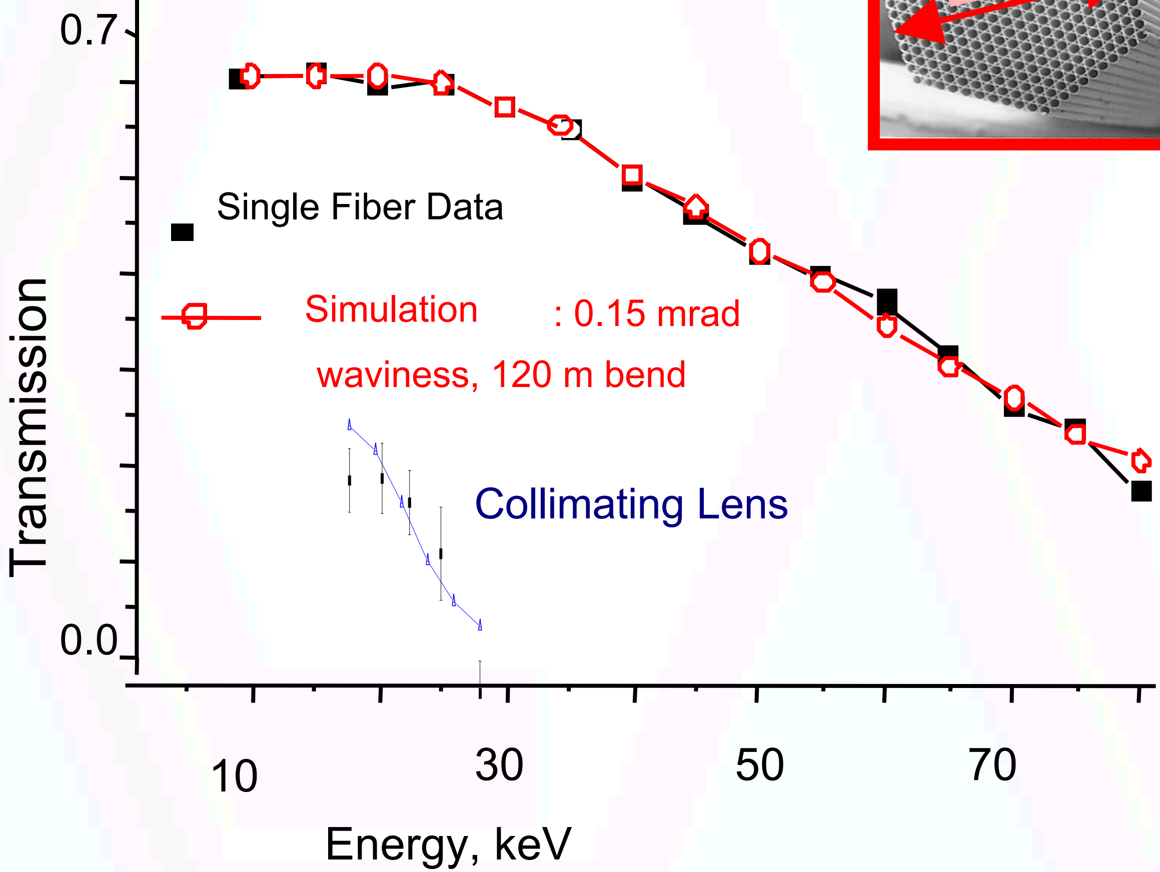


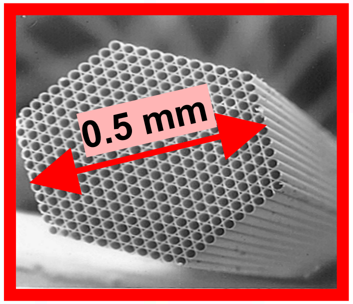


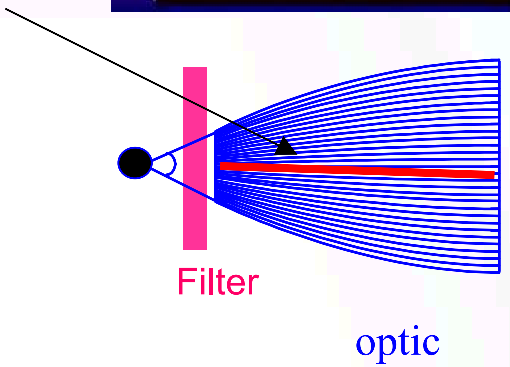
$$\cos(\theta) = \frac{R}{R+d} \Rightarrow 1 - \frac{\theta^2}{2} \approx \frac{1}{1+\frac{d}{R}} \approx 1 - \frac{d}{R}$$

$$\theta^2 \approx \frac{2d}{R} < \theta_c^2 \Rightarrow \gamma = \frac{R\theta_c^2}{2d} > 1$$

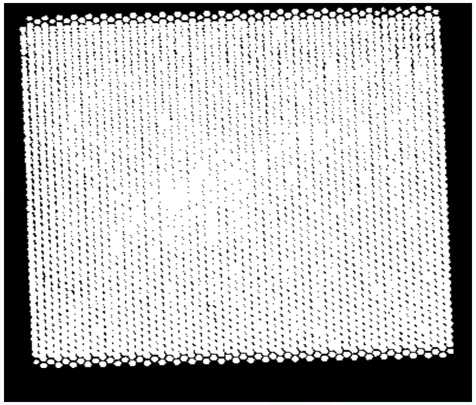
Optic Simulation



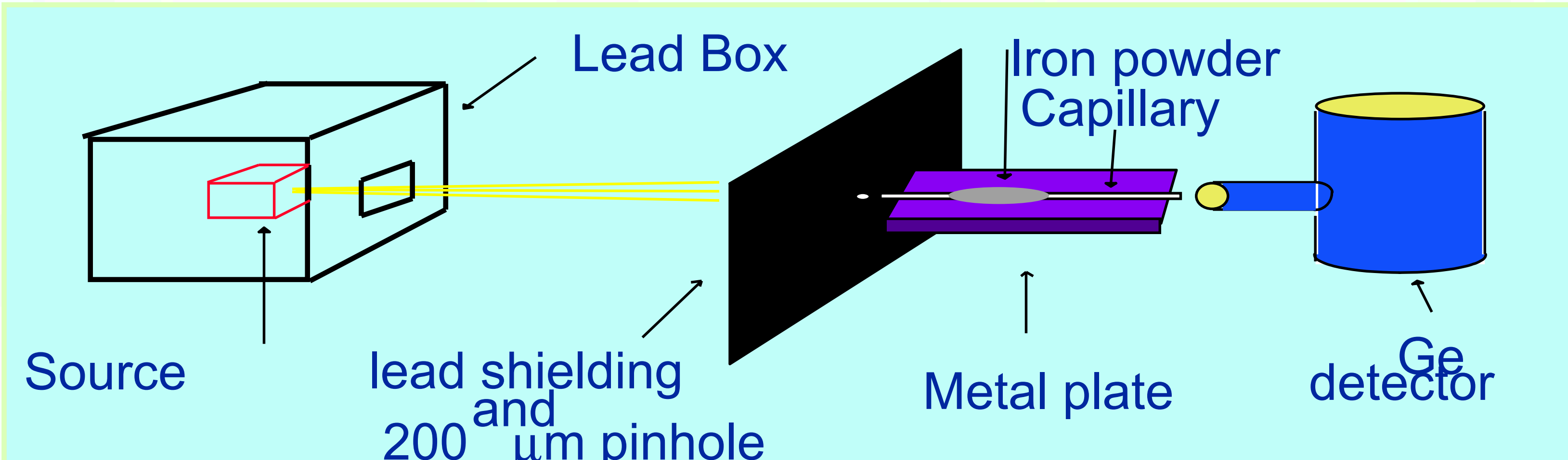


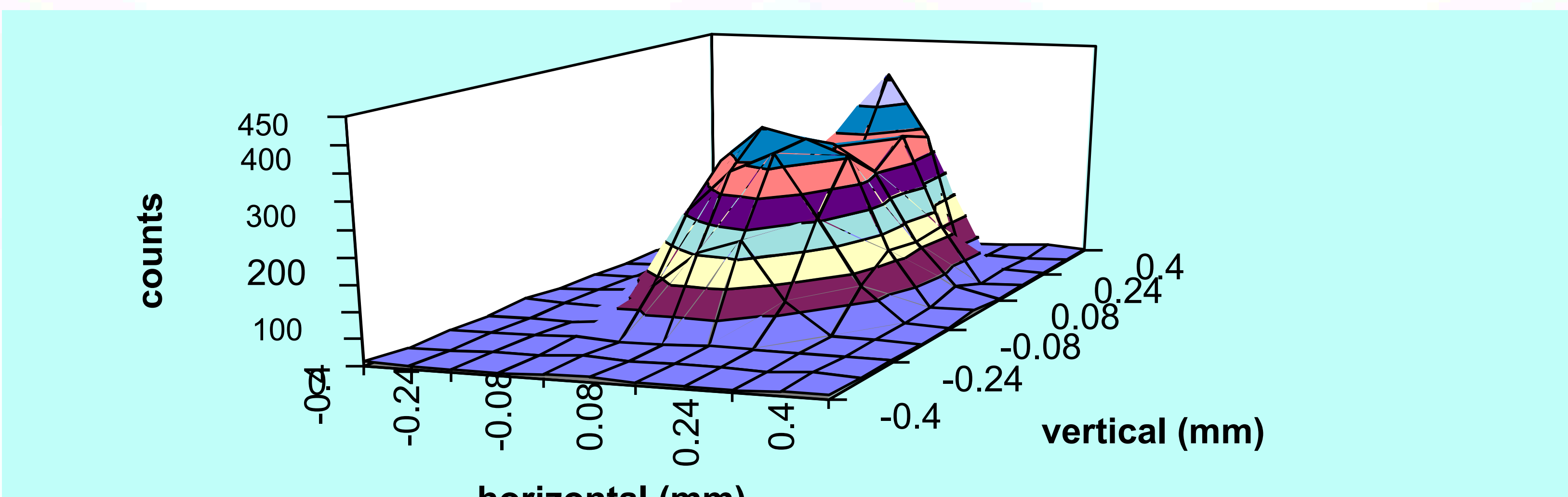


1 cm x 1 cm output
25 cm input focal length
12 μm channel



EXPERIMENTAL SETUP





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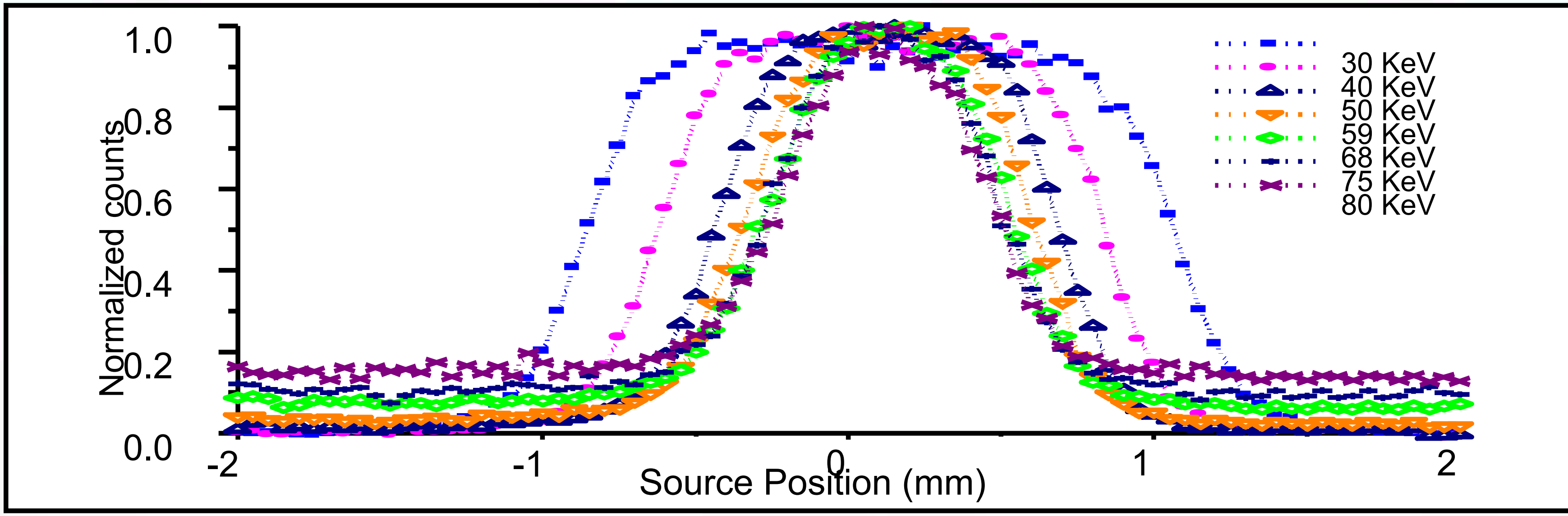
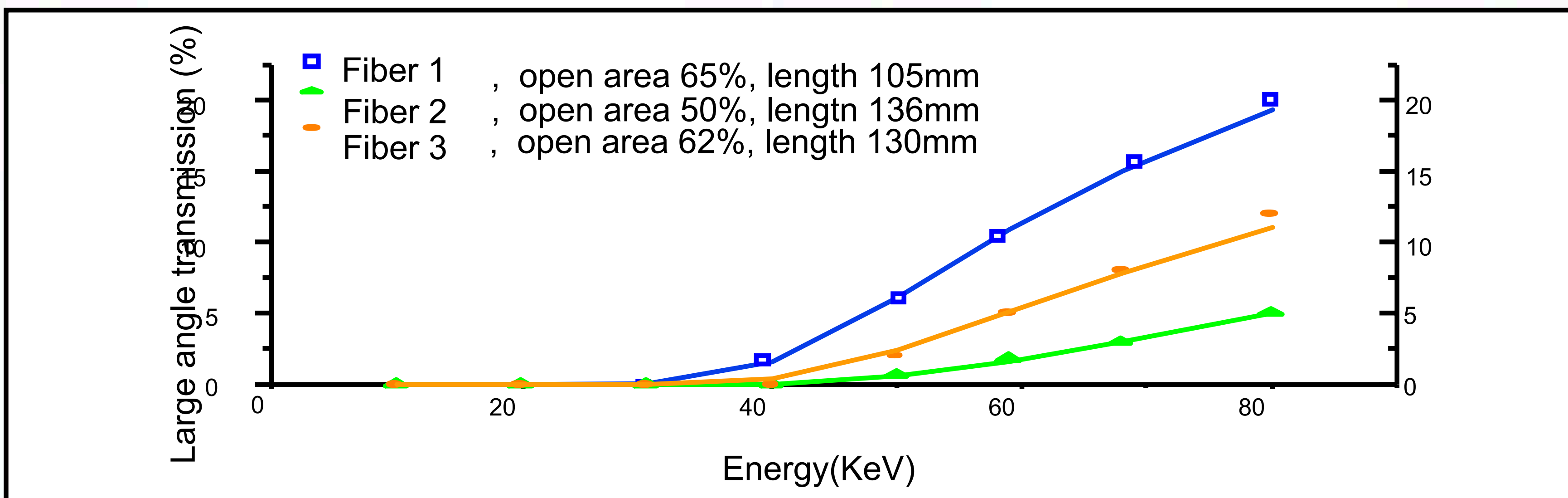
SOURCE MOTION





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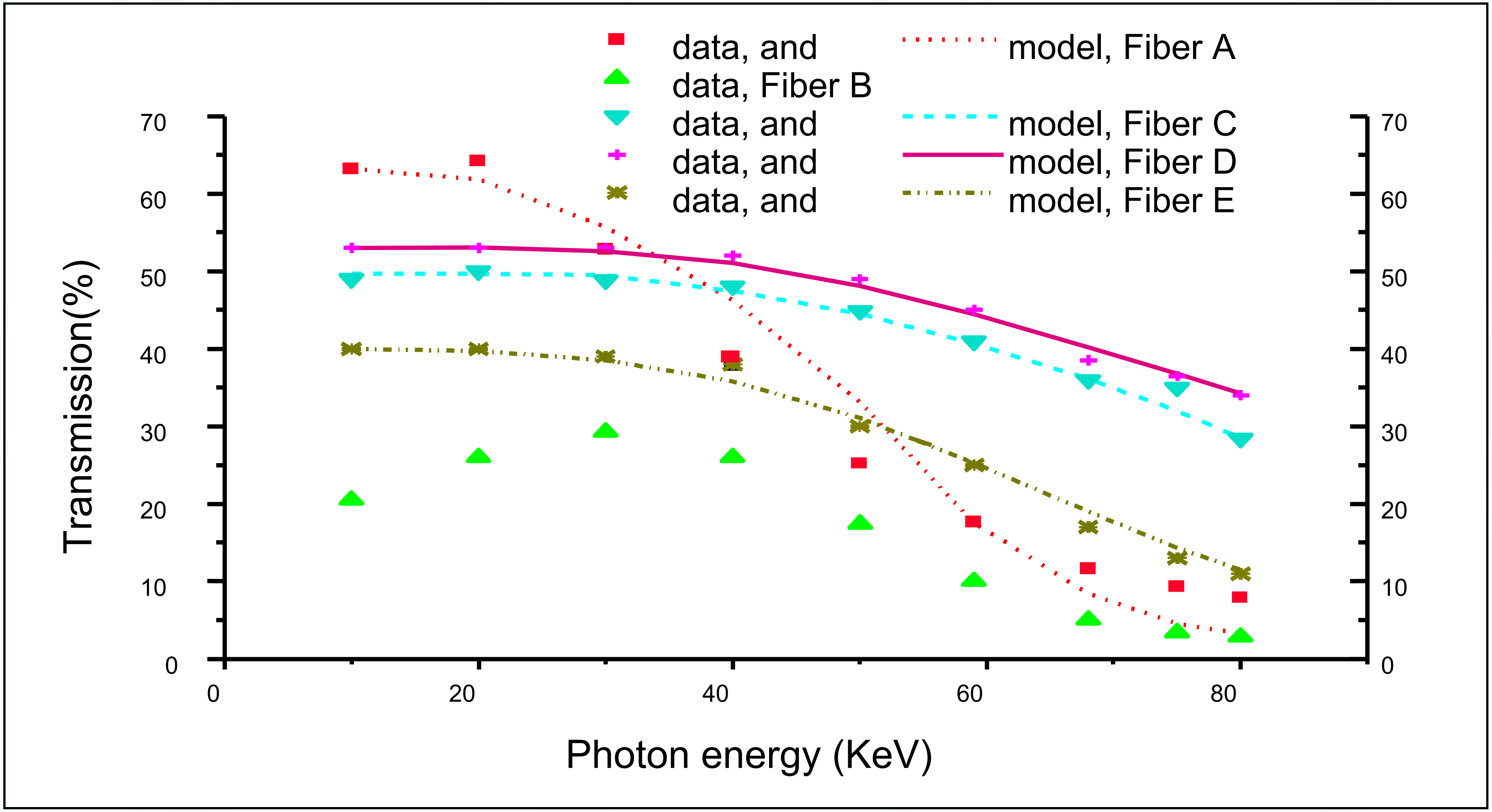

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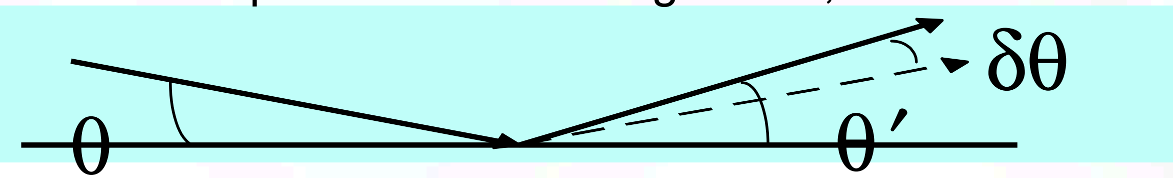
Defect Simulation

Bending: $\lambda_b > 10$ cm parameter: bending radius, R



Waviness:
$\lambda_r < \lambda < \lambda_b$
parameter: $\Delta\theta$

Modeled by random angle shifts of δθ after each bounce:

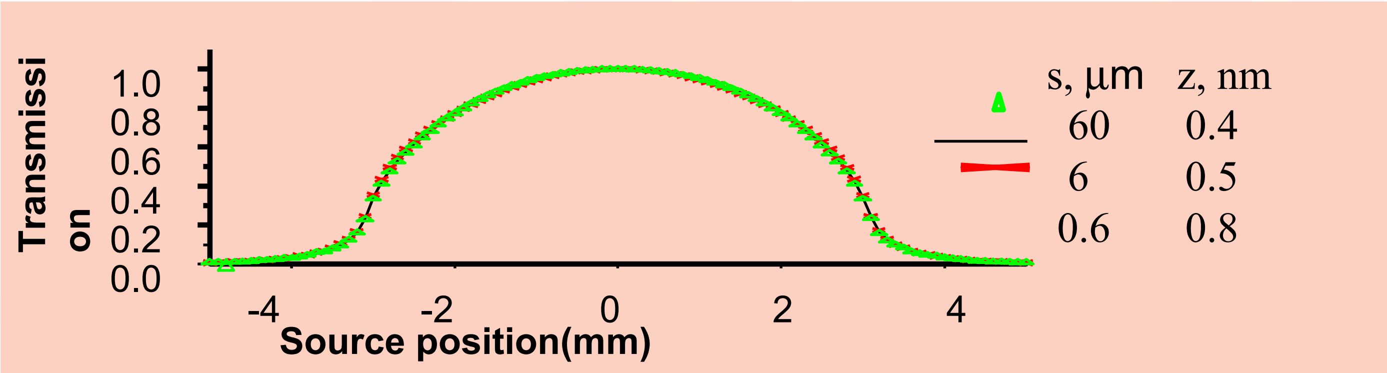
$$\theta' = \theta + \delta\theta, \quad -\Delta\theta < \delta\theta < \Delta\theta$$

$$(-\theta < \delta\theta < \Delta\theta \text{ if } \theta < \Delta\theta)$$

Roughness:
$\lambda_r < 1\ \mu m$ parameters: correlation length, s
rms height, $\bar{z}$

Roughness correlation assumed exponential:

$$g(\Delta x) = \frac{1}{L}\int_0^L Z(x)Z(x+\Delta x)\,dx = \overline{Z}^2 e^{-\frac{|\Delta x|}{s}}$$



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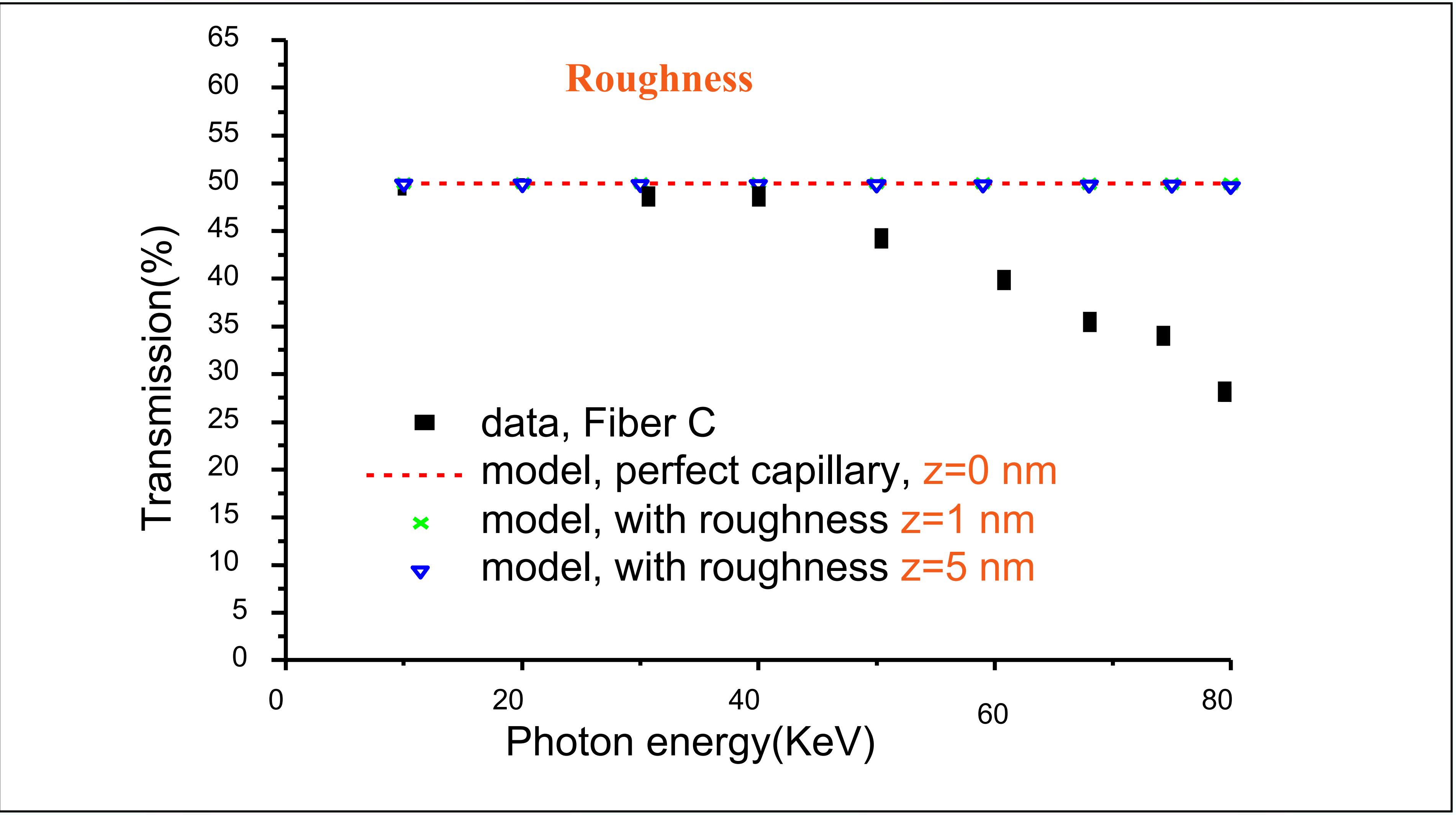

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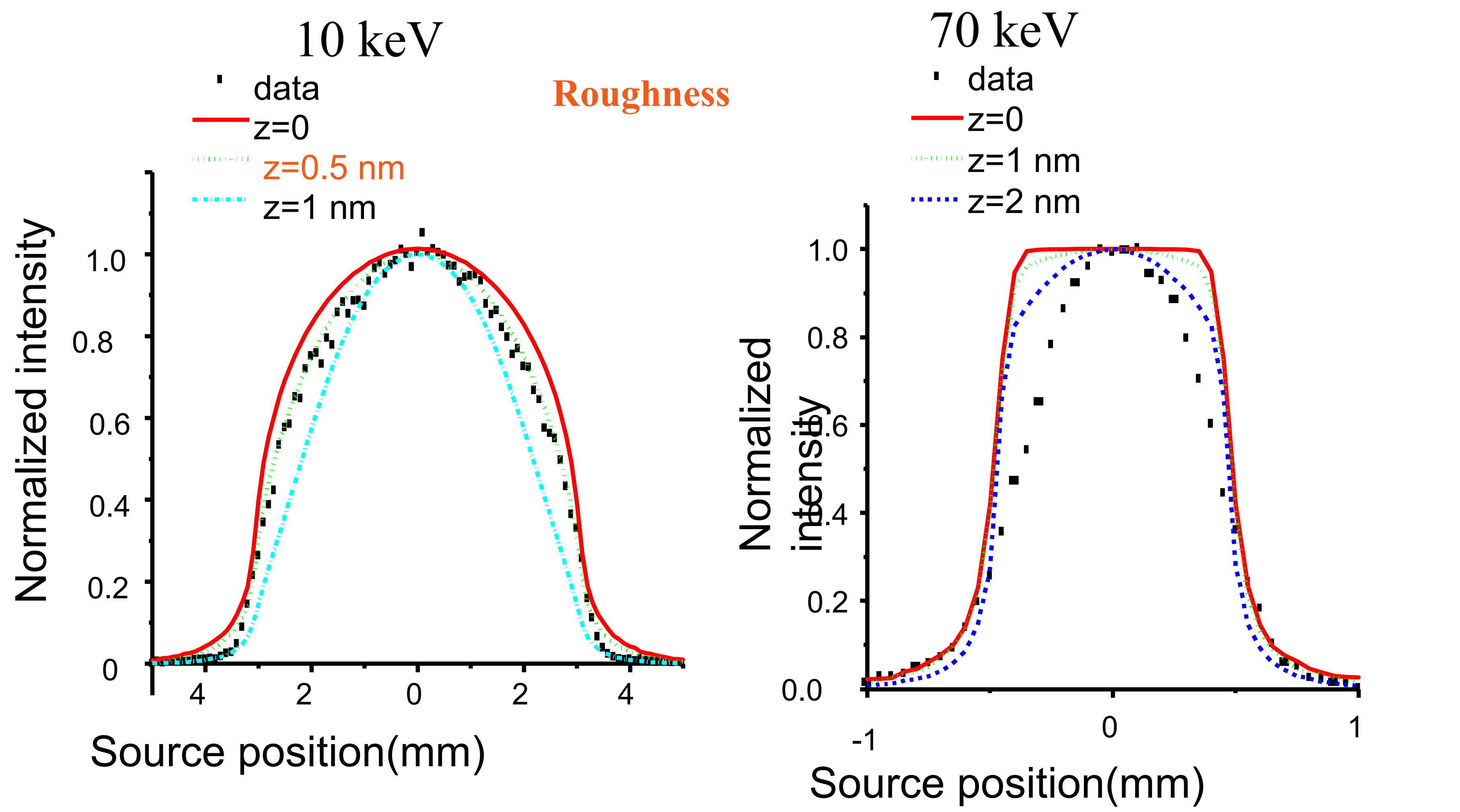
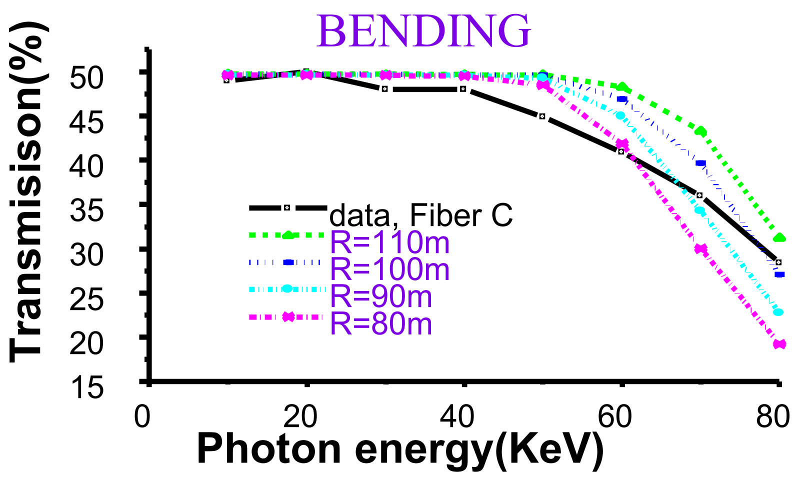
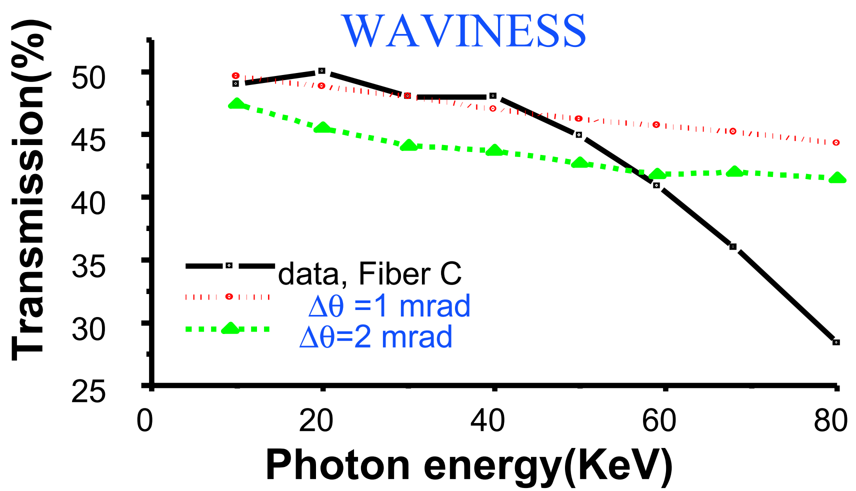
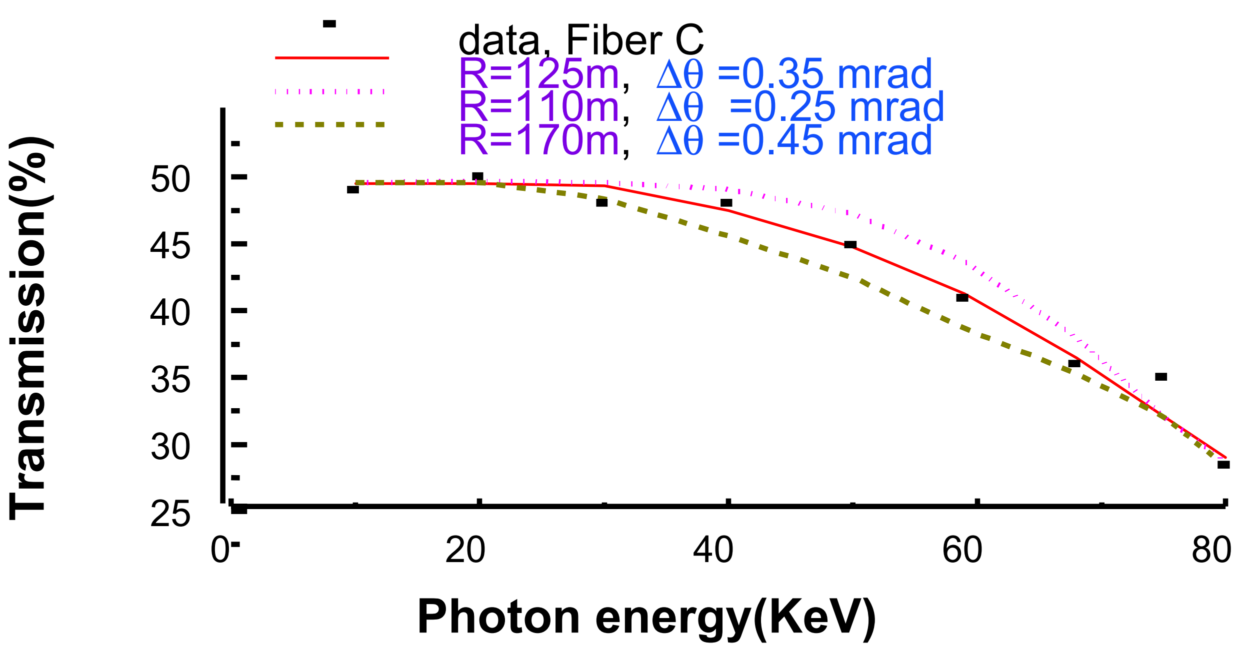
Roughness


10 keV
data
z=0
z=0.5 nm
z=1 nm
Normalized intensity
1.0
0.8
0.6
0.4
0.2
0
4
2
0
2
4
Source position(mm)
70 keV
data
z=0
z=1 nm
z=2 nm
Normalized intensity
1.0
0.8
0.6
0.4
0.2
0.0
-1
0
1
Source position(mm)







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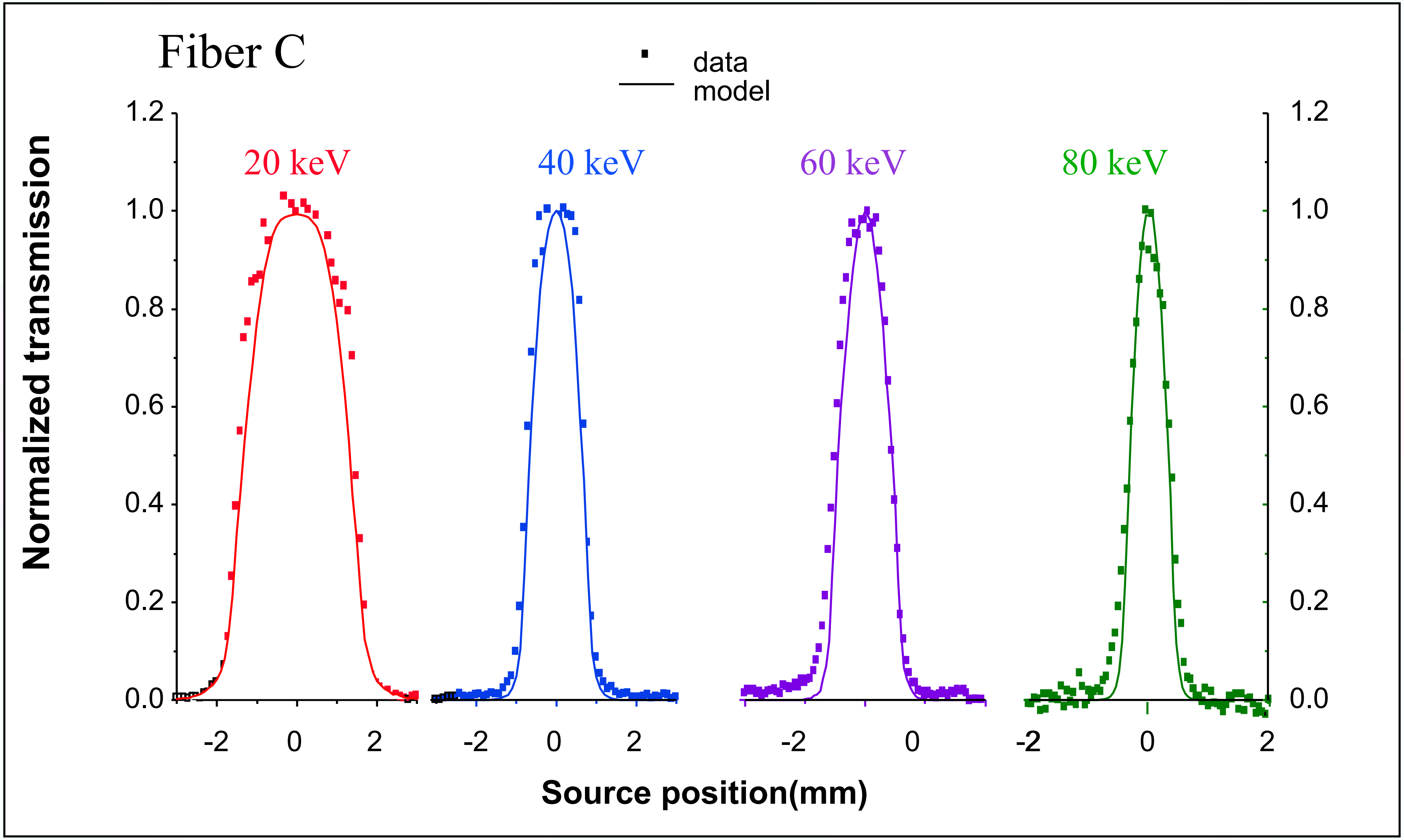

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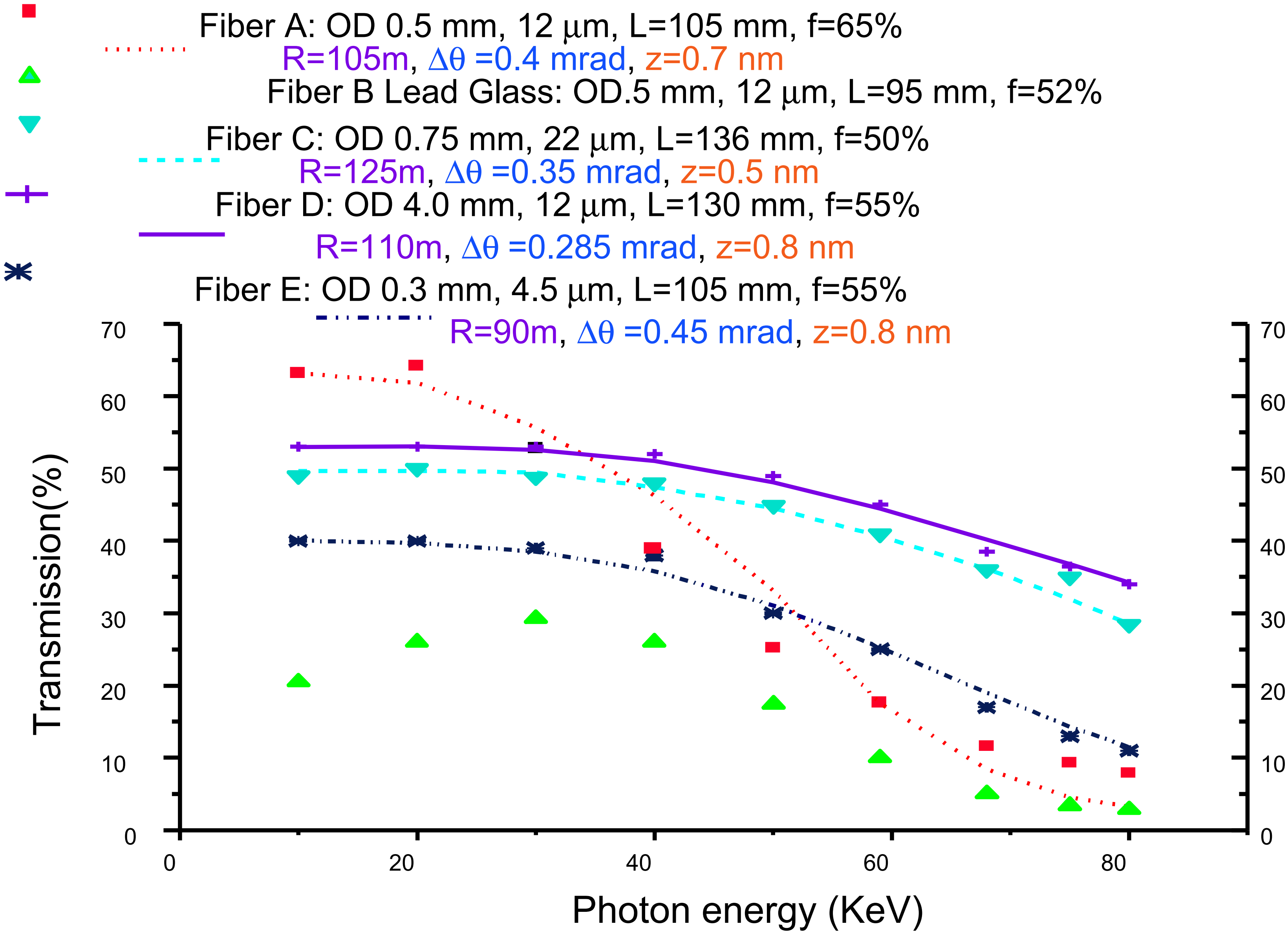

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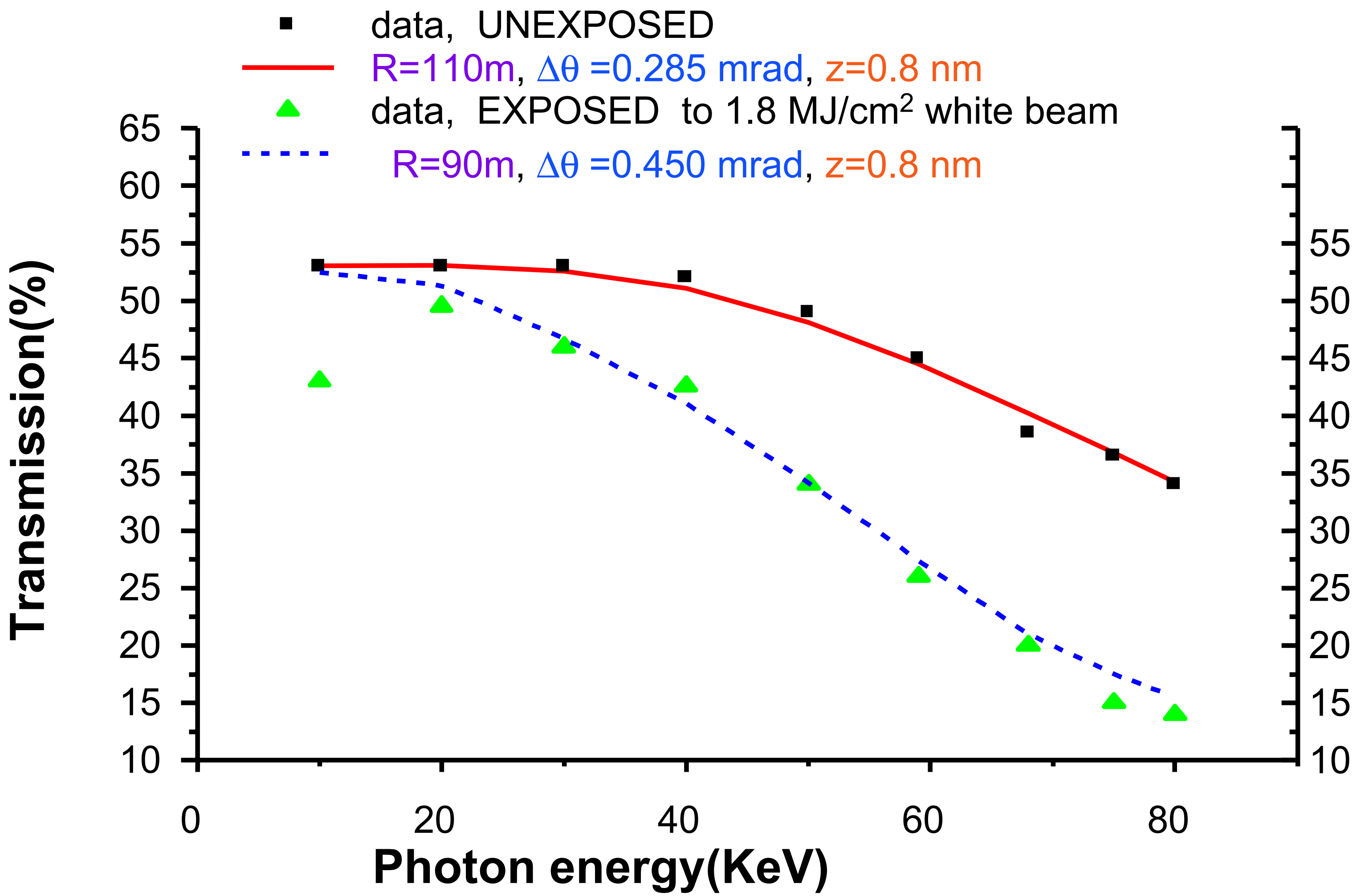
Application to Radiation Damage



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Basic Research

- Scattering Theory
- Surface Effects
- Radiation Effects
- X-Ray Astronomy

Microelectronics Applications

- X-Ray Lithography
- Topography

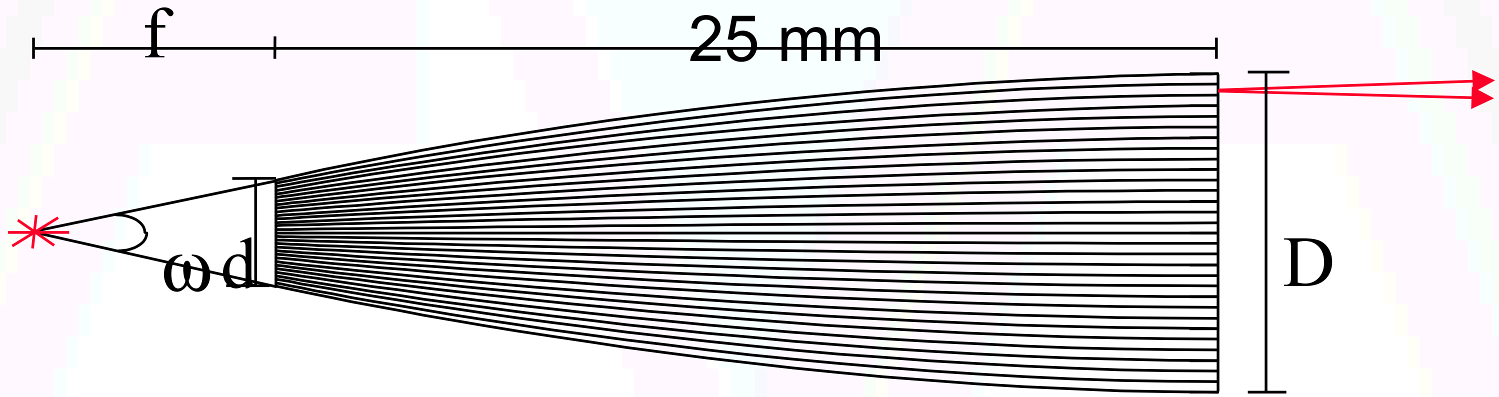
Materials Analysis Applications

- X-Ray Diffraction
- Micro-analysis
- X-Ray Fluorescence
- Neutron Analysis and Diffraction

Medical Applications

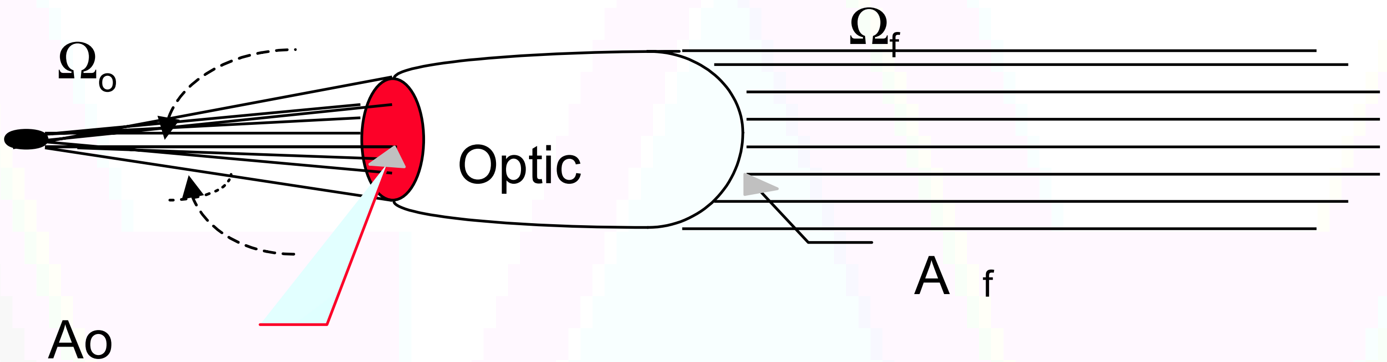
- Mammography
- Radiography
- Monochromatic Imaging
- X-Ray and Neutron Therapy

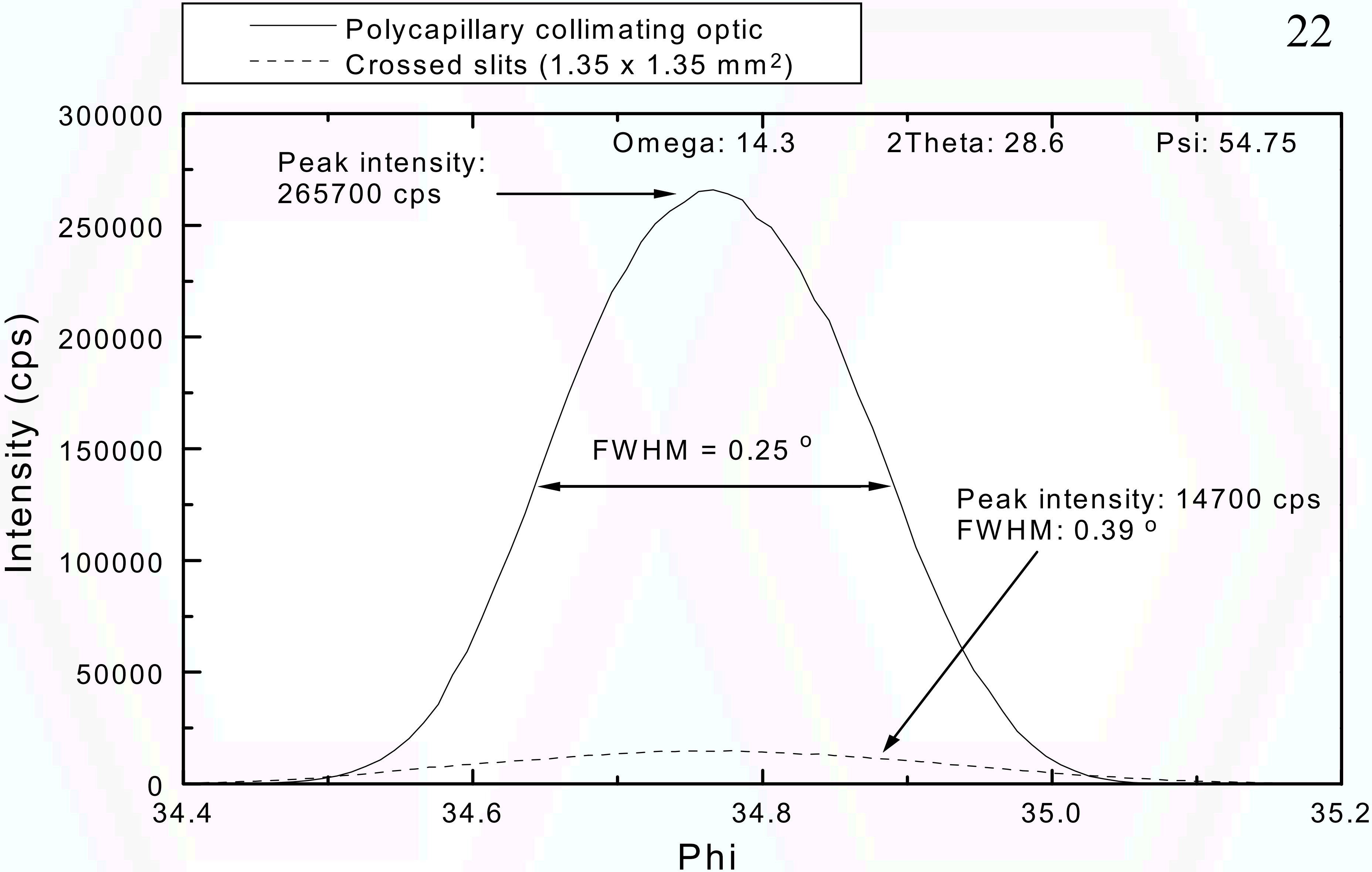
SMALL BEAM COLLIMATION



Liouville's Theorem:

$$A_f \Omega_f \geq A_o \Omega_o$$





Polycapillary collimating optic
Crossed slits (1.35 x 1.35 mm2)
Omega: 14.3
2Theta: 28.6
Psi: 54.75
Peak intensity:
265700 cps
FWHM = 0.25 o
Peak intensity: 14700 cps
FWHM: 0.39 o
Intensity (cps)
300000
250000
200000
150000
100000
50000
0
34.4
34.6
34.8
35.0
35.2
Phi