Prove that the equation
\[ \sqrt{2 - x^2} + \sqrt[3]{3 - x^3} = 0 \]
has no real solutions.

**Solution:**

The domain of the equation is
\[ D = \{ x : x \in [-\sqrt{2}, \sqrt{2}] \} . \]  

(1)

On the other hand, since \( \sqrt{2 - x^2} \geq 0 \), we must have that \( \sqrt[3]{3 - x^3} \leq 0 \).
Therefore \( 3 \leq x^3 \), or, equivalently,
\[ x \geq \sqrt[3]{3} . \]  

(2)

Now, since \( \sqrt[3]{3} > \sqrt{2} \), conditions (1) and (2) cannot be satisfied simultaneously.
Thus the equation has no real solutions.