Let $a$, $b$, and $c$ be the lengths of the sides of a triangle. Do there exist real numbers $x$ such that
\[ b^2x^2 + (b^2 + c^2 - a^2) x + c^2 = 0? \] (1)

Solution:
This is a quadratic equation in $x$. Its discriminant is given by
\[
D = (b^2 + c^2 - a^2)^2 - 4b^2c^2 \\
= (b^2 + c^2 - a^2 - 2bc)(b^2 + c^2 - a^2 + 2bc) \\
= ((b - c)^2 - a^2)((b + c)^2 - a^2) \\
= (b - c - a)(b - c + a)(b + c - a)(b + c + a).
\]

Now, $a$, $b$, and $c$ are the sides of a triangle and therefore they are positive real numbers that satisfy the triangle inequalities
\[
a + b > c, \\
b + c > a, \\
a + c > b.
\]

Therefore $b - c - a < 0$, $b - c + a > 0$, $b + c - a > 0$, and $b + c + a > 0$ and thus $D < 0$, which implies that (1) has no real solutions.

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