

Digest: A New Tool for Creating Insightful System Stories

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Abstract

“System stories” are systematic verbal accounts of how key parts of a model’s structure influence important patterns of behavior in the model. System stories are how managers make sense out of model output and understand a system’s dynamics. This paper identifies three ways of thinking all of which support the creation of system stories—operational thinking, feedback loop thinking, and dynamic thinking. Dynamic thinking requires that the modeler be prepared to describe precisely what structure influences most strongly a given pattern of behavior. The paper briefly defines and describes the Pathway Participation Metric (PPM), a mathematical calculation (that can be related to eigenvalue analysis) as a useful algorithm for identifying influential structure. Next, the *Digest* software is presented. *Digest* is not a simulation language. Rather *Digest* accepts a text version of a simulation model from any commercial system dynamics package and performs post formulation analysis of the model. *Digest* is an experimental piece of software that automatically detects and then displays which feedback loops are most influential in explaining a selected pattern of behavior in a model. Output from a sample *Digest* run is presented and described.

System Stories: Understanding How and Why Patterns of System Behavior Arise from Most Influential System Structure

An important purpose of many system dynamics modeling efforts, indeed of most such modeling projects, is to help teams of non-technical managers better understand the systems which they manage and in which they live. One key task in this search for

insightful, system level understanding is the telling of “system stories.” By system stories we mean coherent and dynamically correct explanations of how influential pieces of system structure give rise to important patterns of system behavior.

A key task in creating system stories is accurately detecting exactly what system structure gives rise to (or contributes most importantly to) some pattern of behavior identified in one or more simulation runs. Richardson (1997) has identified this task as one of the key research problems presently facing the field of system dynamics.

Most skilled practitioners approach this problem with some combination of intuition and analysis coupled to a program of repeated simulations, testing hypotheses about what structure controls what behavior in a controlled way with some of experimental logic.¹ For linear dynamic systems, some mathematical tools exist to make this trial-and-error process more tractable. Indeed, patterns of overall system behavior have a clearly defined meaning in the concept of modes of overall system behavior for linear systems. System behavior is understood to arise from a linear combination of the dynamics associated with the eigenvalues of the linear matrix of system structure. Hence, the calculation of a system’s eigenvalues can go a long way toward describing overall behavior modes of a system.

Closely coupled to eigenvalue analysis of dominant modes of system behavior is the notion of “dominant loops.” Dominant loops are seen as a reduced set of closed feedback paths that contribute most to the overall behavior mode of a system. Indeed, for linear systems, one can work out mathematical relationships between a set of such dominant loops and the system’s eigenvalues (Forrester 1982).

The work presented in this paper continues in the line of eigenvalue and dominant loop analysis in that it continues the search for formal analytic approaches to support the detection of which pieces of system structure contribute most to selected patterns of system behavior. However, as opposed to previous attempts to solve this problem, our approach does not focus on eigenvalues or on dominant loops as the key building blocks of influential system structure. Rather pathways, links of causal structure between two system stocks, are envisioned as the primary building blocks of influential structure.² Of course, one or more pathways can define closed feedback loops. Under this new approach, some combinations of pathways (some of which form a closed feedback loop) define most influential system structure. This most influential system structure, explicitly linked to a pattern of behavior identified by the modeler, forms the basis for creating insightful system stories.

Below, we begin by exploring how it is that stocks, flows, feedback loops, and pathways contribute to a manager’s understanding of how system structure determines system

¹ For example standard text in the field such as Richardson (1984, 1986) describe an approach to model analysis that relies on repeated simulations, as does Sterman (2000)

² Actually, while most pathways are from one system state variable to another, some pathways can connect a state variable to an ordinary auxiliary variable found in the system “between” two or more state variables. See Mojtahedzadeh (1996) for a more formal definition of a pathway.

behavior (the creation of system stories). Next we briefly define a new measure called the pathway participation metric (PPM) and how it can be used to help detect most influential system structure. A PPM is a mathematical algorithm first developed by Mojtahedzadeh in 1996 that can automatically detect which pathway leading into a given variable contributes most to the behavior of that variable at any given point in time. For the purposes of PPM calculations, “behavior” is defined in terms of the slope and curvature—the first and second time derivatives—of any variable selected by the modeler to be explained.

One problem with PPM as an algorithm is that it is cumbersome and difficult to compute and no existing commercial simulation software packages support these calculations. *Digest* is a piece of experimental software that automatically computes the PPM and then uses information from the PPM to automatically detect and display influential pathways and feedback loops. The major purpose of this paper is not to derive and explain the mathematics that lies behind the calculations of the PPM—that task has been reported elsewhere (Mojtahedzadeh 1996). Rather, the major purpose of this paper is to illustrate how *Digest*, an experimental implementation of the PPM calculations, can be used to create system stories. The paper gives the “look and feel” of using *Digest* to automatically detect and display the most influential system structure driving a specific pattern of system behavior. The information obtained from PPM calculations, while closely related to eigenvalue and dominant loop analysis for linear systems, gives a qualitatively different understanding both of what modelers mean by “patterns of behavior” and by what we mean by “influential system structure.” We believe that the system structure automatically identified by *Digest* forms that basis for insightful system stories that can help non-technical managers better to understand the story of how the systems that they manage and live in evolve over time.

Operational Thinking, Feedback Loop Thinking as the Basic Components of System Stories

Using the PPM as implemented in *Digest* to detect and display influential structure is a necessary but not sufficient component of the process of creating insightful system stories. The classic literature in system dynamics returns to the basic problem of how it is that managers’ mental models grapple with and make sense out of the complexity of dynamic systems. Many tools come into play as human actors learn about the systems that they must manage.

The argument is that while complex systems carry enormous variables, connections and relationships within a system, the mental models of managers and analysts are singularly ill-prepared to comprehend the complexity of reoccurring processes. There seems to be a consensus that managers’ mental models have this information, however, they are not organized in a way to produce the knowledge necessary for managing their organization. Senge (1990) suggests that “... The increasing complexity of today’s world leads managers to assume that they lack information to act effectively.” Senge adds, “I would suggest that the fundamental ‘information problem’ faced by managers is not too little information but too much information”(p. 128).

What is needed to overcome the mass of information is a framework to organize and process the available information in a way that expands the ability to produce the results policy makers really want. Forrester (1968) writes, “Without an integrating structure, information remains a hodgepodge of fragments. Without an organizing structure, knowledge is a mere collection of observations, practices and conflicting incidence.” The contribution of the system dynamics approach is to provide a framework to organize and structure available knowledge about the complex systems, thus producing a better understanding of the system. For such purpose a few thinking skills are required, described in detail by Richmond (1997), Senge (1991), and Richardson (1981). To some extent, these skills are recognized as at least three fold: operational thinking; feedback loop thinking; and dynamic thinking.

Operational Thinking: The basic principle of organizing knowledge in system dynamics approach lies in differentiating stocks from flows and how flows are determined. Identifying the integration points facilitates understating a source of dynamic behavior in the system. An example can reveal the contribution of stock-and-flow exercises in learning about complex systems.

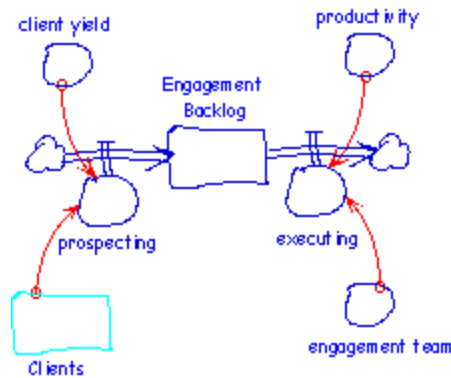


Figure 1: A simple stock-and-flow structure for Engagement Backlog

Figure 1 depicts a simple stock-and-flow structure for engagement backlog. This simple structure tells a story about the engagement backlog in a consulting firm: Engagement backlog increases by prospecting rate and decreases by executing rate. Prospecting is a product of the number of clients and client yield. Executing rate depends on engagement team efforts. It is determined as a product of the size of engagement team and their productivity.

A stock-and-flow diagram shows flows and accumulations, which are essential in creating the dynamic behavior of a system. It also gives a brief description of how flows of the system are determined. Stock-and-flow diagrams accelerate what Richmond (1994) calls operational thinking. In operational thinking one not only identifies the integration points, but focuses on how the flows in the system, like prospecting in the above example, are determined. Stock-and-flow diagrams divulge and visualize the scope and breadth of our thinking about a system as we attempt to investigate a problem.

Feedback Loop Thinking: Although stock-and-flow diagrams indicate the integration points, they may not reveal reinforcing and balancing feedback loops that are present in the system. Feedback loop diagrams, in turn, convey information about feedback loops and circular causality in a system. Feedback loop thinking is used to explore how the operating system at the present time is affected by its past. In feedback loop thinking, dynamic thinking is implicit; each feedback loop has its behavioral implications, however, the overall dynamics of the whole structure remain unclear. Feedback loop thinking helps us to understand how a variable can influence other variables and how, in turn, they change the original variable.

Figure 2 shows a feedback loop concerning employee layoffs in an organization. The feedback loop states that increasing layoffs leads to a decrease in employee confidence. Declining employee confidence causes an increase in anxiety. As anxiety rises, the employee performance declines, which in turn can lead even to more layoffs.

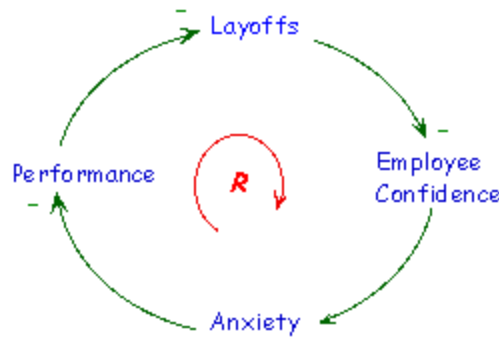


Figure 2: Employee-layoffs feedback loop

The story told in Figure 2 is based on how layoffs and anxiety interact and reinforce both layoffs and anxiety. The nature of information conveyed by Figure 1 is different than that of Figure 2. Whereas a stock-and-flow diagram focuses on how variables like engagement backlog, prospecting new engagement and executing are determined, the focal point of a feedback loop is how changes in a variable like layoffs can lead to a bigger change in layoffs. Both types of information are constructive in learning about complex systems. Both diagrams organize available knowledge about a system in a way that is mutually complementary. Both forms of information contain details on how different parts of the system are connected. Both types of information are captured in a full simulation model.

Dynamic Thinking: Neither stock-and-flow diagrams nor feedback loops illuminate the dynamic consequences of inter-relationships as a whole. In a complete system dynamics model, various pieces of the structure are defined with enormous details. While each piece is associated with an over time behavior, the dynamics of the whole system are the result of the interactions between all the feedback loops and stock-and-flows in the system. Once details about the structure of a system are captured and formulated, the dynamics of the system can be determined. Computer simulation allows us to reveal the over time behavior of various variables. At this stage, we have much

information about the structure and we know the overall dynamic of the whole structure. When formulating system structure we are aware of how individual pieces of structure operate. However, when examining the overall behavior, we have little information as to which pieces of structure are most influential. Getting at which pieces of the whole system are mainly responsible for creating the behavior is at the heart of understanding dynamic complexity.

The fundamental principle for creating insightful system stories is to integrate learning from operational thinking, feedback loop thinking, and dynamic thinking. In stock-and-flow thinking, we tell stories about how market share is operationally determined. What do we exactly mean by profitability? What influences quality and how can it be changed? Those are the questions for which operational thinking can help in finding good answers. Although stock-and-flows have dynamic implications, they are weakly connected to the overall behavior of the whole system.

In feedback loop thinking, on the other hand, the story centers on how low profitability could influence other variables like insufficient investment and ultimately lead to a lower profitability. Again, feedback loops are dynamic concepts; they are associated with some sort of behavior, but they alone have only indirect connections with the behavior of the system as a whole. The focus in both stocks-and-flow and feedback loops is in detail complexity, if they are not linked explicitly to the overall behavior of the whole structure.

In coupling behavior and structure, the focus shifts from detail structural complexity to dynamic complexity. Attention must focus on which part of the structure is mainly responsible for creating the overall dynamic behavior of the system. The story being narrated in this stage is based on why a variable such as profit is rising or falling. What is making the market share grow? Why does quality of products and services remain low despite efforts being made for its improvement? Perhaps, the key to improving managerial mental models is the ability to create insightful and precise stories that trace how the feedback structure of an organization or social system generates over time the dynamic behavior that characterizes such an organization or social system (Senge, 1990)

Despite the importance of understanding the linkages between the structure and dynamic behavior in simulation models, tools to accomplish this task are lacking. The only practical way to generate system stories about the linkage between system structure and system behavior is via repeated simulations guided by hypotheses generated by experienced modelers. Years of experience with system dynamics models is needed for launching artful hypotheses and testing them via repeated simulation, and no satisfying accounts exist in the published literature prescribing a precise set of steps for completing this key task. Even experienced modelers experience difficulty in the testing of their hunches about the connection between structure and behavior.

Pathway Participation Metrics (PPM): A Mathematical Algorithm for Detecting Most Influential Structure

Mojtahedzadeh (1996) has proposed the Pathway Participation Metrics (PPM) as a mathematical tool that could help support modeler intuition in dealing with the task of unraveling relationships between system structure and system behavior. Earlier methods have sought to characterize the overall behavior mode of a system in terms of a linear combination of the dynamics associated with each of the eigenvalues of the system. For linear systems, these overall modes of behavior persist for the whole time period of the simulation and hence identifying the eigenvalues of a linear system goes a long way toward defining its overall behavior mode.³ Another set of approaches to the analysis and understanding of structure-to-behavior linkages has focused on feedback loops as the primary unit of analysis. The question dominating this approach has been to identify what feedback loop or set of feedback loops dominates the overall behavior mode of the system. Some approaches (Forrester 1982) have combined dominant feedback loop analysis with eigenvalue analysis by mathematically computing relationships between linear system eigenvalues and the strength of feedback loops.

The PPM approach takes a more limited tact and in the first instance asks a smaller question. The basic structural building blocks of PPM analysis are pathways, not loops. A pathway is a causal link between two system states (or in some cases between one system state and some other non-state variable in the model). The basic behavioral building block of the PPM is a single phase of behavior for a single variable (as opposed to the overall mode of behavior for the whole system). A single phase of behavior for a selected variable is a time slice of the simulation where the selected variable has the same slope and curvature (first and second time derivatives). Hence, there are seven patterns of behavior that may exist within a single phase: (1) reinforcing growth, (2) linear growth, (3) balancing growth, (4) reinforcing decline, (5) linear decline, (6) balancing decline and finally, (7) equilibrium. Figure 3 depicts the 6 first pattern of behavior.

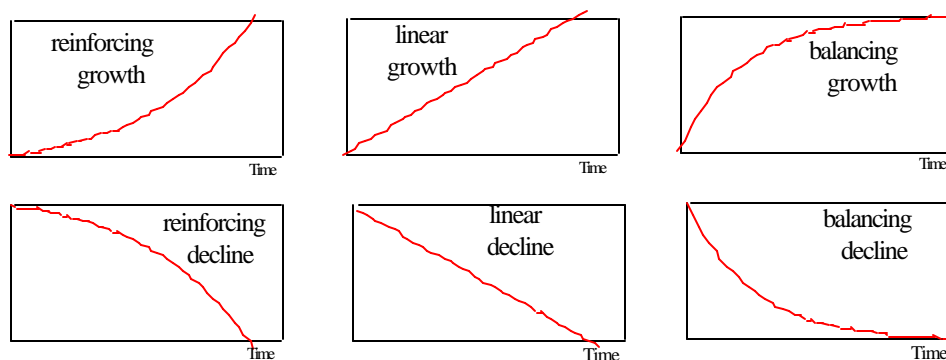


Figure 3: Six patterns of over time behavior

³ Indeed, the PPM method can be related to eigenvalue analysis. Appendix B demonstrates that for a second order linear system, the PPM method produces values that are mathematically related to the two eigenvalues of the system. The same result can be shown to hold for higher order linear systems.

With these two basic building blocks in mind the mathematics of the PPM sets out to address a single and apparently limited question. *Which pathway leading into a chosen variable contributes most to the current behavior pattern of that variable?* This apparently simple question requires some mathematics to be answered. In doing so, the PPM calculates how much the net-flow could change given a small change in the state variable at the tail of the pathway. The magnitude of the changes in the net-flow is then distributed among pathway coming into the net-flow. The most influential pathway is defined as the one whose participation is the largest in magnitude and has the same sign as the total changes in the net-flow when it is disturbed by a infinitesimal change in the state variable at the tail of the pathway. It can also be shown that the derivative of the net-flow of a state variable with respect to the same state variable contains information curvature of the overtime behavior of the state variable. Thus, it is used to characterize the current pattern of behavior. A more formal definition of the mathematics underlying the PPM is given in Appendix A.

The calculation of one PPM, taken in isolation, does not do much to help anyone understand the overall relationship between system structure and system behavior. All that this calculation does is to identify for one phase of behavior and for one chosen variable which incoming pathway contributes most to that small piece of behavior. However, systematic and repeated calculations of the PPM can be very insightful, ultimately yielding a picture of what loops are controlling the evolution of the system over time.

A metaphor might help to understand why. Imagine that the overall structure of a system dynamics model is a lattice composed of many links fastened at nodes to form a complex pattern. A serious system dynamics model is made up of literally hundreds of these structural links (pathways). A single PPM calculation indicates that some single piece of system structure is most important for understanding (contributes most to) some specific piece of behavior. Color that link of structure bright orange so that it stands out from the complex mosaic of the whole model structure. As the PPM is repeatedly calculated a full set of related structural links are subsequently painted bright orange. Hence, application of the PPM calculations identifies a complete set of structural links (pathways) that contribute most to the overall behavior (defined as the linked set of distinct phases of behavior) of a selected variable of interest. Using this metaphor, a small set of links and loops known to contribute most to model behavior have been detected and painted bright orange because these are the loops that have the greatest influence on some selected pattern of behavior. It can be shown that if the PPM is applied systematically, it will always identify closed loops or exogenous time series that contribute most to system behavior.

The PPM is important because it identifies influential structure and hence supports dynamic thinking. It ultimately helps modelers craft insightful system stories that can help managers better to understand which feedback loops contribute most to the dynamics that they need to understand.

But how does this metaphor of painting links of structure bright orange one at a time actually work itself out in practice? How can a modeler compute even one PPM? In what order should a sequence of PPMs be calculated? How can a selected variable have its overall pattern of behavior sliced up into distinct and mathematically well-defined phases of behavior? These are the practical questions that are handled by the *Digest* software.

Digest: Experimental Software that Implements PPM

Digest is an experimental software package designed to automatically detect most influential system structure using the PPM method. *Digest* is not a simulation package such as iThink, Stella, Vensim, or Powersim. *Digest* cannot support most of the simulation functions that these languages can. *Digest* is designed to be used after the model has been constructed to detect and display influential structure. Of course, at some point in the future, the relevant and most useful features of *Digest* could be integrated into any of the commercial simulation packages.

Digest accepts model equations from any commercial simulation package in text form. In its present version, some hand editing of the text equations may be necessary if the model uses macros or functions that are not yet parsed by the *Digest* equation translator. Once a text version of the model equations has been edited and accepted by *Digest*, the software leads the modeler through a series of step-by-step procedures that uses the PPM calculation to first detect and then display model structure.

As stated immediately above, the first step in this process is having the user edit the model equations and then have *Digest* parse them. After *Digest* has loaded the simulation model, the modeler selects a model variable of interest. The behavior of this selected variable will be the object of subsequent PPM calculations and the influential structure associated with that behavior will be displayed by *Digest*.

Here in a nutshell is what happens (a complete step-by-step description showing sample screens is given below). *Digest* begins by automatically identifying all of the pathways that flow into the selected variable of interest. Next, *Digest* identifies the slice of time for the first phase of behavior of that model. *Digest* then uses the PPM calculation to identify the single pathways that is most influential in its contributions of this slice of behavior. The variable at the tail of the causal chain of the pathway just identified then becomes the selected variable of interest. In a recursive fashion, the most influential pathway leading into the newly selected variable is identified using the PPM calculation. This process is repeated until a closed loop has been identified. The set of pathways with one associated loop is then displayed as the most influential structure for the first phase of behavior.

This same process is repeated for each phase of behavior of the variable initially selected by the modeler. When the process is complete, *Digest* will have identified and displayed a series of loops that contribute most to the individual phases of behavior of the selected

variable of interest. Taken together, these loops help to tell a complete system story, indicating what portions of system structure have contributed most to what phases of behavior. Each of these steps is now described in more detail. In the discussion below, when *Digest* automatically produces a result, we will show the actual output from *Digest*. When the modeler must conceptualize or sketch a loop or other piece of structure, that artwork will appear in a rough hand-drawn version. This convention is designed to more clearly separate out what *Digest* does automatically from what the modeler must do in interpreting and responding to *Digest* output.

How Digest Works, Step-by-Step

This section describes, step-by-step, how *Digest* identifies and displays a series of links and loops that contribute most to various phase of the behavior of a selected variable of interest. In doing that as mentioned before, we need the equations of a “simulatable” model saved in a text file format. The model used as an example is a classic structure that illustrates how Industrial Structure in a particular region grow over time until all the resources needed to support the growth of Industrial Structure are depleted. Figure 4 depicts the structure of the model. (A list of the equations of the model is provided in Appendix C).

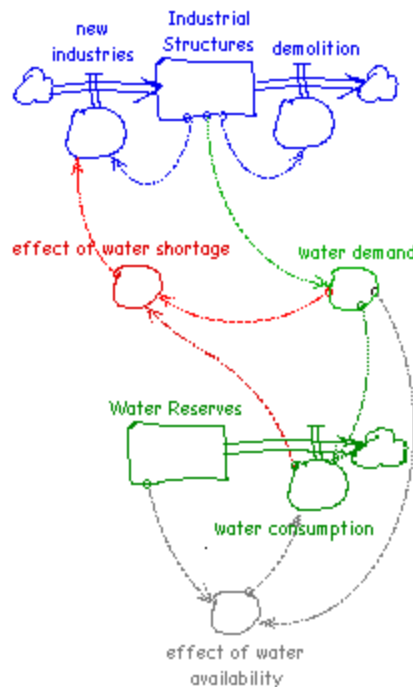


Figure 4: A simple model for the growth of Industrial Structures

The model captures three real-world processes:

1. Industrial Structures grow with new industries through a reinforcing loop and demolish by a balancing loop, (shown in blue),

2. Industrial Structures consume water which decreases water reserves (shown in green),
3. Water shortage (defined as the ratio of water consumption to water demand) affects new industries indirectly (shown in red),
4. Water availability (defined as the ratio of water reserves to water demand) controls water consumption (shown in gray).

For an appropriate set of parameters and initial values, the model generates an overshoot in the behavior of Industrial Structures, while Water Reserves follows an S-shape decline. In explaining the behavior of the model the question is what feedback loops are more influential in generating the behavior of the variable of interest. For example, what is making Water Reserves to decline rapidly and what controls it. What is deriving Industrial Structures to grow rapidly in the first few years? What part of the structure is responsible for the decline of Industrial Structures followed by its growth? For modelers who have worked with this sort of models, it is not difficult to explain the growth phase and the declining phase of the behavior of this simple structure. However, it may not be as easy to distinguish what part of the structure contribute most to the behavior of Industrial Structures in the transition from reinforcing growth to a balancing decline. Using *Digest* one can identify the most influential structure as the behavior of the model unfolds.

Step 1: Digest Accepts Equations from Any SD simulation software in Text Form

Once in the *Digest* environment, the user can click of file menu to open the text that contains the equations of the model. Files with txt extensions, which usually contain models developed in iThink or Powersim will appear as a default in the file selection dialogue box. The user should change the file type to mdl for models built in Vensim or to dyn for models built in Dynamo.

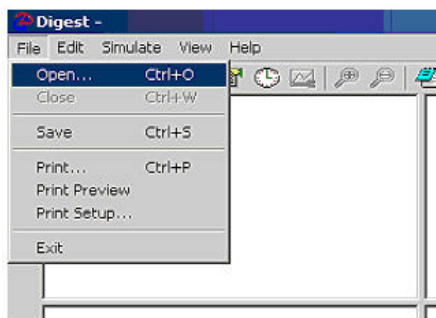


Figure 5: File menu in *Digest*

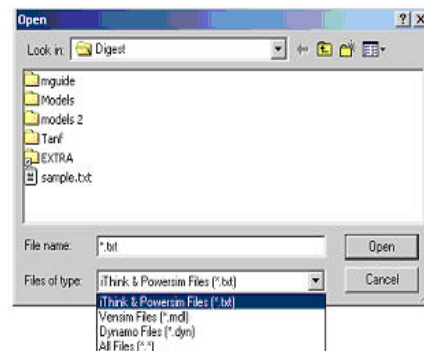


Figure 6: File selection dialogue box

Digest cannot interpret equations with built-in function unless they are defined in Macro. In case the software fails to read and/or interpret the equations, the user gets a message “can’t open the file. Check the model/Macro”.

Step 2: The User Selects the System Variable of Interest and Digest Begins Analysis

In the *Digest* environment, there are four windows; once the software successfully opens the file containing the model equations, a list of variables of the model will appear. The left side of each variable indicates whether the variable is a stock (STK), flow (flw) or auxiliary (aux). Figure 7 depicts the first window of *Digest* when the Industrial Structure growth model's equations are loaded.

List of variables:	
STK	Industrial_Structures
flw	new_industries
flw	demolition
grph	effect_of_water_shortage
STK	Water_Reserves
flw	water_consumption
grph	effect_of_water_availability
aux	water_demand

Figure 7: The first window of *Digest*: A list of variables of the model

Step 2.A Digest Automatically identifies pathways associated with the user-selected variable of interest

When the variable of interest is chosen, the causal route associated with the behavior of interest will appear in the second window. This diagram reveals how the variable of interest is determined by other variables in the model. For the Industrial Structure as the variable of interest, Figure 8 shows the causal route diagram that is associated with Industrial Structures.

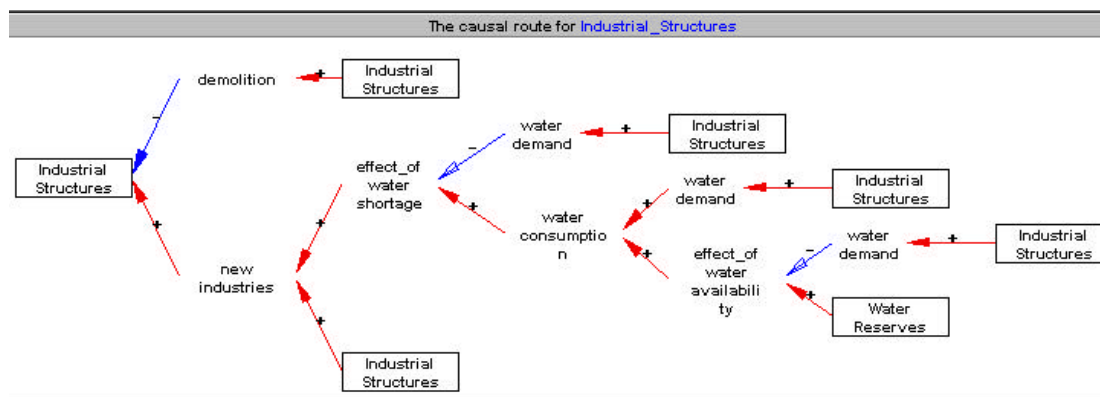


Figure 8: Causal route diagram for Industrial Structures

Arrows in red indicate a direct (positive) impact of the variable at the tail of the arrow at the dependent variable and a blue arrow refers to an indirect impact of the cause on the effect (a negative or indirect relationship).

Step 2.B Digest Automatically analyzes distinct phases in the behavior pattern of user selected variable of interest

There are four icons in the left side of the first window that show a list of models variables. By clicking on the second icon from the top, the first phase of the behavior of the variable of interest, Industrial Structures, appears in the third quadrant of the *Digest* environment. Colored in pink, the first phase of Industrial Structures is a reinforcing growth as shown in Figure 9. The reinforcing growth lasts for 24 years.

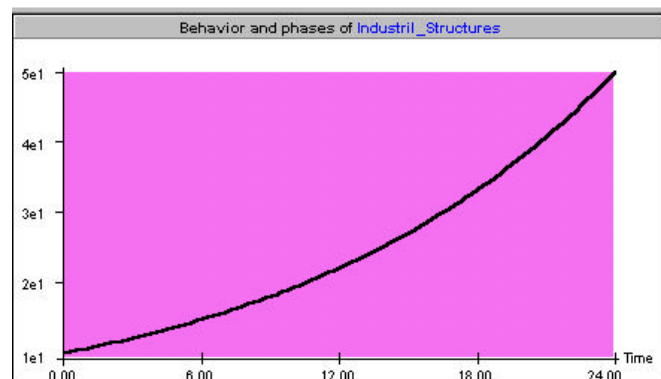


Figure 9: The first phase of behavior of Industrial Structures

During the first 24 years of simulation time, both slope (first time derivative) and curvature⁴ (second time derivative) of the variable of interest, Industrial Structures, remain positive. *Digest* recognizes the nature of the behavior of Industrial Structures and thus it colors the background of the over time behavior graph in pink.

Step 2.C Digest automatically detects and displays most influential structure contributing to behavior pattern identified in Step 2.B

Corresponding to the first phase of the behavior of Industrial structures, there is a reinforcing feedback loop that, according to *Digest*, is the most influential feedback loop in generating the reinforcing growth in Industrial Structures. The reinforcing feedback loop is shown in Figure 10. Based on this feedback loop a higher level of Industrial Structures attracts more new industries, which in turn increases Industrial Structures. By inspecting the structure of the models in Figure 4, one could identify about 6 feedback loops. Using pathway participation metrics, *Digest* automatically selects the reinforcing feedback among all the other loops in the model. At the present time, no other simulation software does that. Bear in mind that the iterative process of identifying most influential structures corresponding to a particular pattern of behavior in small models and complex models are principally the same.

⁴ Actually *Digest* calculates neither the first nor second time derivative of the variable of interest; it merely determines the derivative of net flow of the variable of interest with respect to the variable of interest at any time. The derivative is related to first and second time derivatives of the variable of interest. A positive sign of the derivative indicates that both slope and the curvature of the variable of interest have the same signs. (See Mojtahedzadeh 1996 for details).

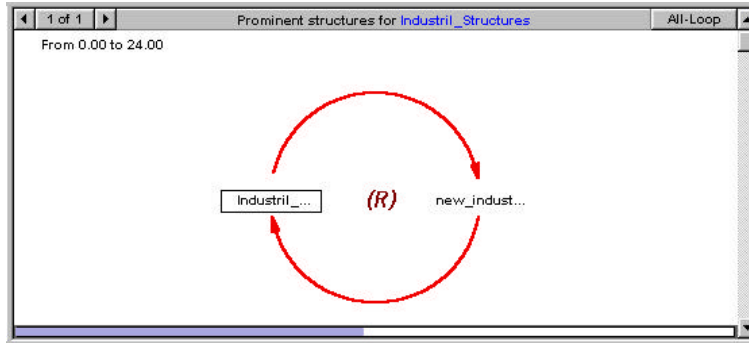


Figure 10: The most influential structure in creating the first phase of the behavior of Industrial Structures

Step 3: Digest Automatically Completes Analysis

During the first 24 years of simulation, the most influential structure in creating the behavior of variable of interest, Industrial Structures, remains unchanged. *Digest* stops simulating the model behavior when the most influential structure in behavior of interest is about to change⁵. To continue simulation, the user should click on the second icon from the top on the left side of screen. Repeating the same procedure used previously, *Digest* identifies the next phase of the behavior of Industrial Structures.

Step 3.A Digest automatically detects next distinct phase in behavior pattern of user selected variable of interest

The next distinct phase in the behavior of the variable of interest, Industrial Structures, identified by *Digest*, is a balancing growth followed by a reinforcing decline. Figure 11 shows the second and third phases in the behavior of Industrial Structure. Colored in blue in the period that the variable interest experience a balancing growth in its over time behavior. In the second while the first time derivatives of Industrial Structures is positive, its second time derivatives is negative. In the third phase, however, the variable of interest shows a reinforcing decline, which is colored in pink. Both first and second derivatives of Industrial Structures have the same sign during the third phase.

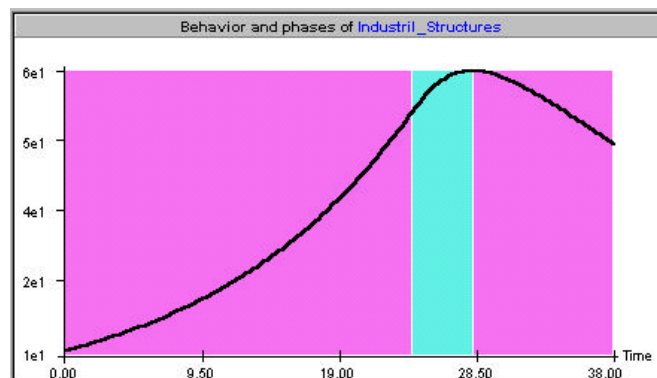


Figure 11: The behavior of Industrial Structures and its three phases

⁵ When the user click on the third icon (from the top) in the left side of the screen that is called “home run” simulation continue until the final time.

Step 3.B Digest automatically detects and displays most influential structure for phase of behavior detected in 3.A.

The fourth window in the *Digest* environments presents the most influential structure that is mainly responsible for creating the behavior of the variable of interest, Industrial Structures in its second and third phases. Figure 12 depicts a balancing loop that controls water consumption as Water Reserve continues to fall, along with a pathway that shifts the effect of the balancing feedback loop to the variable of interest, Industrial Structures.

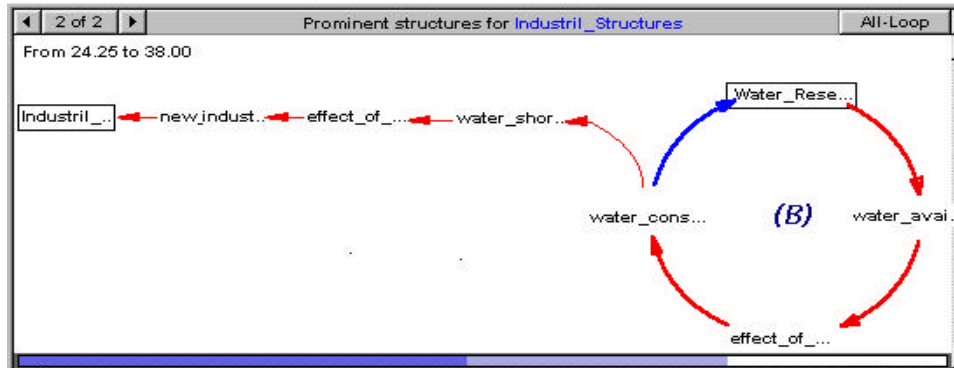


Figure 12: The most influential structure in the second and third phases of the behavior of Industrial Structures

It may not be difficult for the modelers to draw attention to the balancing feedback loop that controls water consumption, when comes to explain why Industrial Structures is generating a balancing growth in its second phase. Water availability due to a continuous fall of water reserves is dropping and, therefore, new industries cannot grow any more. The subtlety in explaining the behavior of the variable of interest is the subsequent reinforcing decline in the behavior of Industrial Structures in phase four. Some novices may even look for a positive feedback loop to explain the reinforcing decline. Digest reveals that what forces Industrial Structures to fall faster and faster is exactly the same process that controls it. The balancing loop that controls water consumption pushes Industrial Structures to the edge by continuously plunging new industries and once new industries falls behind industrial demolition, the Industrial Structures generates a reinforcing decline.

Step 3.C Steps 3.A and 3.B are repeated until influential structures for all phases of behavior pattern of selected variable have been detected and displayed.

Clicking on the second icon from the top in the left side of the screen reveals the fourth and the last phase of the behavior of the variable of interest, Industrial Structure. The variable of interest experiences a balancing decline in it behavior during the fourth phase. In this phase while the first time derivative of Industrial Structures is negative, its second time derivative remain positive. Figure 13 depicts the four phases of the behavior of Industrial Structures.

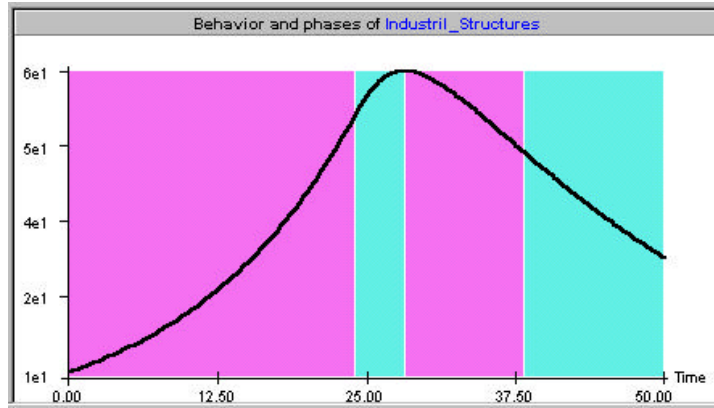


Figure 13: The behavior of Industrial Structures and its four phases

The most influential structure that is mainly responsible for the balancing growth in the behavior of the variable of interest identified by *Digest* is depicted in Figure 14.

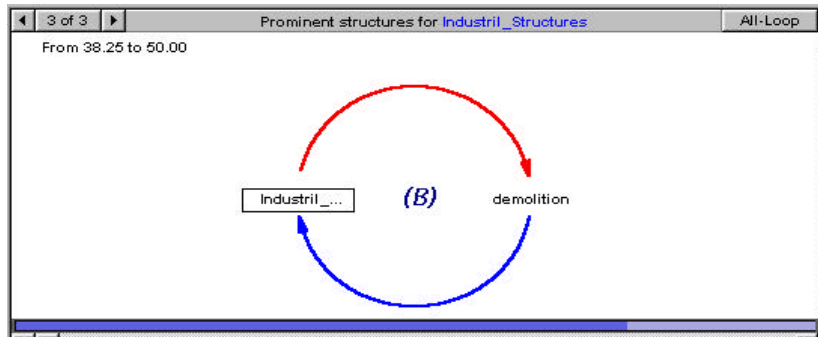


Figure 14: The most influential structure in the fourth phase of the behavior of Industrial Structures

Step 4: User Reviews Digest Output and Creates a System Story

This section needs to be written. Return to figure 3 and use some sort of a shading convention to show how the most influential loops have been moving around the model during the various phases of behavior.

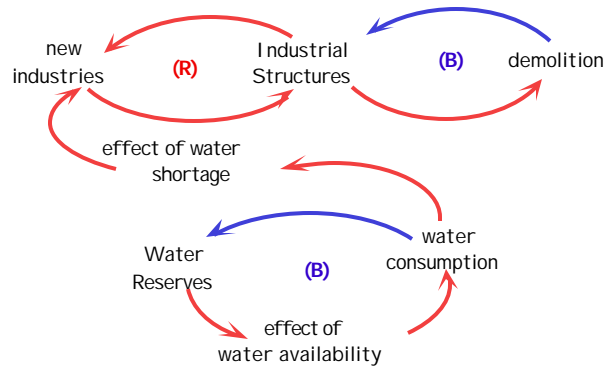


Figure 15: The essential structure for explaining Industrial Structures growth model

Acknowledge that this is a very different mode of explanation and understanding than comes from dominant loop or from eigenvalue analysis. Spend some time ruminating on the qualitative aspects of this behavior. Play up the fact that it is more akin to repeated simulation analysis than to eigenvalue analysis in quality of explanation delivered, but more like eigenvalue analysis in rigor of approach (i.e., best of both worlds.)

Next Steps: Further Research Into How Modelers and Managers Use Digest and PPM

This is a new section that needs to be drafted Points to include

See how this works with many classes of systems (especially oscillatory)

See how it works with very high order and complex systems

Experiment with different algorithms in getting from the raw pathway matrix to identified dominant structure

Explore ways to automate, support and enrich step 4, the derivation of system stories from digest output

See how actual users gain value from using this approach (the experiment we did not do with Hassan)

WAIT AND SEE WHO TAKES FIRST CRACK AT THIS.

Appendix A: A Mathematical Definition of Pathway Participation Metrics

This appendix introduces the mathematics of pathway participation metrics (PPM).

Consider the following n-order non-linear system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}; \mathbf{p}) \quad [1]$$

Where \mathbf{x} is the vector of state variable, $\dot{\mathbf{x}}$ is the vector of derivative of \mathbf{x} with respect to time, and \mathbf{p} is the vector of the parameters of the system. The equation of the k^{th} state variable as the variable of interest may look like:

$$\dot{x}_k = f_k(x_1, x_2, \dots, x_n; \mathbf{p}) \quad [2]$$

Taking the derivative of the net changes in the state variable of interest, \dot{x}_k , with respect to the state variable of interest, x_k , yields:

$$\frac{d\dot{x}_k}{dx_k} = \frac{\partial f_k}{\partial x_1} \frac{dx_1}{dx_k} + \frac{\partial f_k}{\partial x_2} \frac{dx_2}{dx_k} + \dots + \frac{\partial f_k}{\partial x_k} \frac{dx_k}{dx_k} + \dots + \frac{\partial f_k}{\partial x_n} \frac{dx_n}{dx_k} \quad [3]$$

Or simply,

$$\frac{d\dot{x}_k}{dx_k} = \sum_{i=1}^n \frac{\partial f_k}{\partial x_i} \frac{\dot{x}_i}{\dot{x}_k} \quad (\text{for } \dot{x}_k \neq 0) \quad [4]$$

Each term in equation [4] represents all minor feedback loops and pathways leaving i^{th} state variable and coming into the variable of interest, x_k . We can decompose the effect of each minor feedback and pathway coming into the state variable x_k .

$$\frac{d\dot{x}_k}{dx_k} = \sum_{i=1}^n \sum_{j=1}^{m(i)} \frac{\partial f_k^j}{\partial x_i} \frac{\dot{x}_i}{\dot{x}_k} \quad [5]$$

Where $m(i)$ is number of minor loops and pathways that leave a i^{th} state variable and come into the k^{th} state variable, and $\partial f_k^j / \partial x_i$ is the polarity of the pathway or minor feedback loop. The ratio \dot{x}_i / \dot{x}_k represents the net changes in the i^{th} state variable and the net changes in k^{th} state variable. The total effect infinitesimal change in x_k of the net rate of x_k is the not only driven by the polarity of the feedback loops and pathways but also the ratio of net changes in the two state variables.

The effect of each pathway could be normalized in such a way that it varies between -1 and 1 . Thus for each pathway coming into variable of interest we could have a metric

that measure the impact of that pathway (or minor feedback loop) in creating the behavior of the variable of interest. This metric is called pathway participation metrics (PPM).

$$PPM(i,j) = \frac{\frac{\partial f_k^j}{\partial x_i} \dot{x}_i}{\sum_{i=1}^n \sum_{j=1}^{n(i)} \left| \frac{\partial f_k^j}{\partial x_i} \dot{x}_i \right|} \quad [6]$$

The most influential pathway (or minor feedback loop) is defined as the one whose participation metrics (PPM) is the largest and has the same sign as $d\dot{x}_k/x_k$. For $i \neq k$ the same calculation is done until there is a feedback loop.

If the variable of interest is a non-state variable, we need to determine the net changes the variable of interest and the follow the same procedure. Suppose \mathbf{a} presents the vector of non-state variables and it is related to state variables through \mathbf{g} and a vector of parameters \mathbf{q} . Thus we have,

$$\mathbf{a} = \mathbf{g}(\mathbf{x}; \mathbf{q})$$

If the a_k is the variable of interest, that is a non state variable, the net changes in a_k over the period of dt will be,

$$\dot{a}_k = \sum_{i=1}^n \frac{\partial g_k}{\partial x_i} \dot{x}_i \quad [7]$$

Taking the derivative of the net changes in the variable of interest, \dot{a}_k , with respect to the state variable of interest, a_k , yields:

$$\frac{d\dot{a}_k}{da_k} = \sum_{i=1}^n \left(\frac{\partial^2 g_k}{\partial x_i \partial a_k} \dot{x}_i + \frac{\partial g_k}{\partial x_i} \frac{d\dot{x}_i}{da_k} \right) \quad [8]$$

Which can be rearranged as:

$$\frac{d\dot{a}_k}{da_k} = \sum_{i=1}^n \left(\sum_{j=1}^n \left(\frac{\partial^2 g_k}{\partial x_i \partial x_j} \frac{\dot{x}_j}{\dot{a}_k} \right) \dot{x}_i + \frac{\partial g_k}{\partial x_i} \frac{d\dot{x}_i}{dx_i} \frac{\dot{x}_i}{\dot{a}_k} \right) \quad [9]$$

The pathway participation metrics can be determined after decomposing the impact of each pathway leaving a particular state variable and coming into the variable of interest.

Appendix B. Pathway Participation Metrics and Eigenvalues

In linear systems there is a close relationship between the pathway participation metrics and eigenvalue analysis. In fact we show that

- *In the steady state condition, the total participation metrics is equal to the largest eigenvalue of the system.*

In doing so, we use a second order linear system and derive pathway participation metrics for a state variable. Then, we show that the sum of the pathway participation metrics, or total participations metrics, for any state variable equal the largest eigenvalue of the system.

Consider the following second order system:

$$\dot{x} = ax + by \quad [1]$$

$$\dot{y} = cx + dy \quad [2]$$

The pathway participation metrics for state variable x is:

$$\frac{d\dot{x}}{dx} = a + b \frac{\dot{y}}{\dot{x}} \quad [3]$$

There are two pathways coming to the state variable x whose participation metrics are:

Participation metrics for Pathway 1: a

Participation metrics for Pathway 2: $b \frac{\dot{y}}{\dot{x}}$

The pattern of behavior of x is determined by the total participation metrics, which is the sum of participation metrics for these two pathways. If total participation metrics is positive the state variable x experiences a reinforcing growth and if it is negative, x shows a balancing behavior. The most influential pathway then is the one whose participation metrics is the largest in magnitude and has the same sign as the total participation metrics.

Now we calculate \dot{y} / \dot{x} through the response of state variables x and y . We can rewrite the second order linear system presented in [1] and [2] as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad [4]$$

The above system has two eigenvalues, λ_1 and λ_2 . For each eigenvalue we have:

$$ar_{i1} + br_{i2} = \lambda_i r_{i1} \quad [5]$$

$$cr_{i1} + dr_{i2} = \lambda_i r_{i2} \quad [6]$$

Where r_{i1} and r_{i2} are the elements of the right eigenvector associated with λ_i . The time response of the state variables is:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \mathbf{j}(t) \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad [7]$$

Where x_0 and y_0 are the initial values of the state variables and $\mathbf{j}(t)$ with the dimension of 2×2 is the transition matrix of the system which can be calculated as:

$$\mathbf{j}(t) = \sum_{i=1}^2 e^{I_i t} \begin{bmatrix} r_{i1} \\ r_{i2} \end{bmatrix} \begin{bmatrix} f_{i1} & f_{i2} \end{bmatrix} \quad [8]$$

Where f_{i1} and f_{i2} are the elements of the left eigenvector associated with I_i . Substituting [8] in [7] and expanding it yields:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = e^{I_1 t} \begin{bmatrix} r_{11}f_{11} & r_{11}f_{12} \\ r_{12}f_{11} & r_{12}f_{12} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + e^{I_2 t} \begin{bmatrix} r_{21}f_{21} & r_{21}f_{22} \\ r_{22}f_{21} & r_{22}f_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad [9]$$

The value of x and y at any time is:

$$x_t = (r_{11}f_{11}x_0 + r_{11}f_{12}y_0)e^{I_1 t} + (r_{21}f_{21}x_0 + r_{21}f_{22}y_0)e^{I_2 t} \quad [10]$$

$$y_t = (r_{12}f_{11}x_0 + r_{12}f_{12}y_0)e^{I_1 t} + (r_{22}f_{21}x_0 + r_{22}f_{22}y_0)e^{I_2 t} \quad [11]$$

We can calculate \dot{x} and \dot{y} by taking derivatives of [10] and [11] with respect to time.

$$\dot{x} = I_1(r_{11}f_{11}x_0 + r_{11}f_{12}y_0)e^{I_1 t} + I_2(r_{21}f_{21}x_0 + r_{21}f_{22}y_0)e^{I_2 t} \quad [12]$$

$$\dot{y} = I_1(r_{12}f_{11}x_0 + r_{12}f_{12}y_0)e^{I_1 t} + I_2(r_{22}f_{21}x_0 + r_{22}f_{22}y_0)e^{I_2 t} \quad [13]$$

Using [12] and [13], we calculate the ratio of \dot{y} / \dot{x} .

$$\frac{\dot{y}}{\dot{x}} = \frac{I_1(r_{12}f_{11}x_0 + r_{12}f_{12}y_0)e^{I_1 t} + I_2(r_{22}f_{21}x_0 + r_{22}f_{22}y_0)e^{I_2 t}}{I_1(r_{11}f_{11}x_0 + r_{11}f_{12}y_0)e^{I_1 t} + I_2(r_{21}f_{21}x_0 + r_{21}f_{22}y_0)e^{I_2 t}} \quad [14]$$

Or,

$$\frac{\dot{y}}{\dot{x}} = \frac{I_1(r_{12}f_{11}x_0 + r_{12}f_{12}y_0)e^{t(I_1 - I_2)} + I_2(r_{22}f_{21}x_0 + r_{22}f_{22}y_0)}{I_1(r_{11}f_{11}x_0 + r_{11}f_{12}y_0)e^{t(I_1 - I_2)} + I_2(r_{21}f_{21}x_0 + r_{21}f_{22}y_0)} \quad [15]$$

Assuming I_2 is the largest eigenvalue, when time approach infinity terms $I_1(r_{12}f_{11}x_0 + r_{12}f_{12}y_0)e^{t(I_1 - I_2)}$ and $I_1(r_{11}f_{11}x_0 + r_{11}f_{12}y_0)e^{t(I_1 - I_2)}$ in [15] approaches zero. Thus, for \dot{y} / \dot{x} we have

$$\frac{\dot{y}}{\dot{x}} = \frac{I_2(r_{22}f_{21}x_0 + r_{22}f_{22}y_0)}{I_2(r_{21}f_{21}x_0 + r_{21}f_{22}y_0)} \quad [16]$$

The above equation can be rewritten as:

$$\frac{\dot{y}}{\dot{x}} = \frac{r_{22}}{r_{21}} \quad [17]$$

Now we can substitute [17] in [3],

$$\frac{d\dot{x}}{dx} = a + b \frac{r_{22}}{r_{21}} \quad [18]$$

Equation [18] according to [5] is equal to \mathbf{I}_2 .

$$\frac{d\dot{x}}{dx} = \mathbf{I}_2 \quad [19]$$

It can be easily shown that the above proposition is true for an n-order system.

Appendix C: A list of the equations of the Industrial Structures Growth Model (iThink version)

Industrial_Structures(t) = Industrial_Structures(t - dt) + (new_industries - demolition) * dt
INIT Industrial_Structures = 10
new_industries = Industrial_Structures*effect_of_water_shortage*normal_growth
demolition = Industrial_Structures*dem_frc
Water_Reserves(t) = Water_Reserves(t - dt) + (- water_consumption) * dt
INIT Water_Reserves = 10000
water_consumption = effect_of_water_availability*water_demand
dem_frc = .05
normal_growth = .12
water_demand = Industrial_Structures*water_demand_per_industry
water_demand_per_industry = 10
effect_of_water_availability = GRAPH(0.1*Water_Reserves/water_demand)
(0.00, 0.00), (0.1, 0.06), (0.2, 0.14), (0.3, 0.255), (0.4, 0.395), (0.5, 0.535), (0.6, 0.685), (0.7, 0.825), (0.8, 0.92), (0.9, 0.975), (1, 1.00)
effect_of_water_shortage = GRAPH(water_consumption/water_demand)
(0.00, 0.00), (0.1, 0.06), (0.2, 0.14), (0.3, 0.255), (0.4, 0.395), (0.5, 0.535), (0.6, 0.685), (0.7, 0.825), (0.8, 0.92), (0.9, 0.975), (1, 1.00)

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