University at Albany
System Dynamics Group

Sensitivity [or the lack of it] may either be a *virtue* or a *curse*.

PAD824, October 4, 2001

Ignacio J. Martinez, Peter Otto

Agenda

- Introduction
- Use and approaches in sensitivity analysis
- Monte Carlo Sensitivity Simulations
- Options available
- How to read the results
- Conclusion

Not invented here…

- Sensitivity Analysis (SD) was first applied in electrical engineering in 1843 by Wheatstone for the resistance measurement.
- Today, SD is an important aspect of any engineering and economy research to study the effects of uncertainty on project valuation.

Introduction

- The parameter values and assumptions of any economic model are subject to change and error. Sensitivity analysis, broadly defined, is the investigation of these potential changes and errors and their impacts on conclusions to be drawn for the model (Baird 1989).

In a Nutshell

- Sensitivity Analysis is used to increase the confidence in the model and its predictions, by providing and understanding how the model response variables respond to changes in the inputs.

Use of Sensitivity Analysis in Model Development

- Testing the model for validity or accuracy
- Searching for errors in the model
- Simplifying the model
- Calibrating the model
- Coping with poor or missing data
- Prioritizing acquisition of information

Why Sensitivity Analysis

- In all models, parameters are more-or-less uncertain. The modeler is likely to be unsure of their current values, and to be even more uncertain about their future values.
- This applies to things such as prices, costs, productivity, and technology.

Why Sensitivity Analysis cont.

Uncertainty is one of the primary reasons why sensitivity analysis is helpful in making decisions or recommendations. If parameters are uncertain, sensitivity analysis can give information such as:

- How robust the optimal solution is in the face of different parameter values
- Under what circumstances the optimal solution would change
- How the optimal solution changes in different circumstances

Decision Variables and Strategy

- A decision variable is a variable over which the decision maker has control and wishes to select a level, whereas strategy refers to a set of values for all the decision variables of a model.
- An optimal strategy is the strategy which maximizes the value of the decision maker's objective function (e.g. profit, social welfare, expected utility).

Theoretical Framework for Using Sensitivity Analysis

- It is assumed that the modeler has subjective beliefs (internal beliefs, hunches or guesses) about the performance of different strategies and about what is the objective of the decision maker who will use the information generated by the model.
- The modeler's subjective beliefs are influenced by the model but also by other factors; these beliefs may or may not be close to the objective truth.

Approaches to Sensitivity Analysis

- In principle, sensitivity analysis is a simple idea: change the model and observe its behavior.
- One might choose to vary any or all of the following:
  - The objective (e.g. minimize risk of failures instead of maximizing profit)
  - A constraint limit (e.g. maximum availability of a resource)
  - Technical parameter

What to observe?

- Whichever items the modeler chooses to vary, there are many different aspects of a model output to which attention might be paid:
  - The Value of the objective function for the optimal strategy
  - The values of decision variables
  - The difference in objective function values between two strategies (e.g. between the optimal strategy and a particular strategy suggested by the decision maker)

How to do a Sensitivity Analysis

- Most system dynamics and simulation software packages include automated sensitivity tools
- However, comprehensive sensitivity analysis is generally impossible even when restricted to parametric sensitivity
- Vensim® applies Monte Carlo simulation to generate dynamic confidence intervals for the trajectories of the variables in the model

Options in MCSS

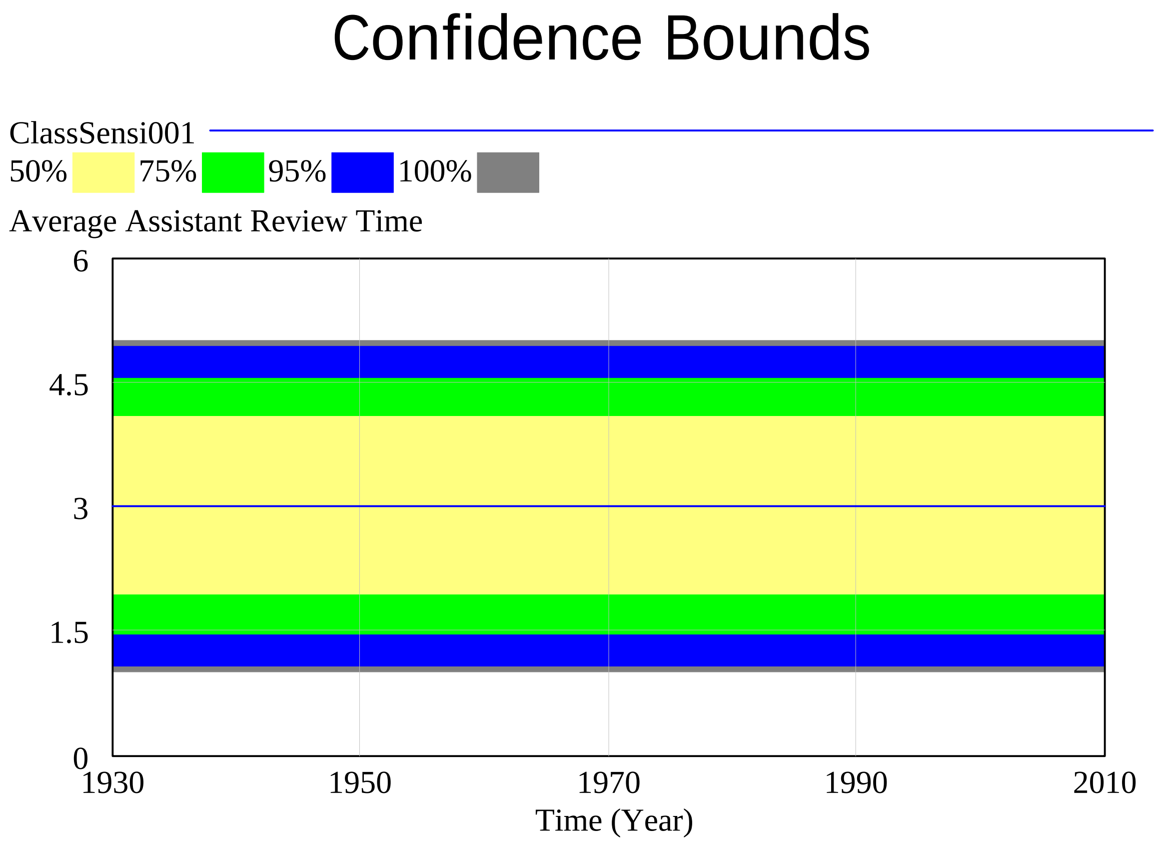
- Univariate
- Multivariate
- Latin Hypercube

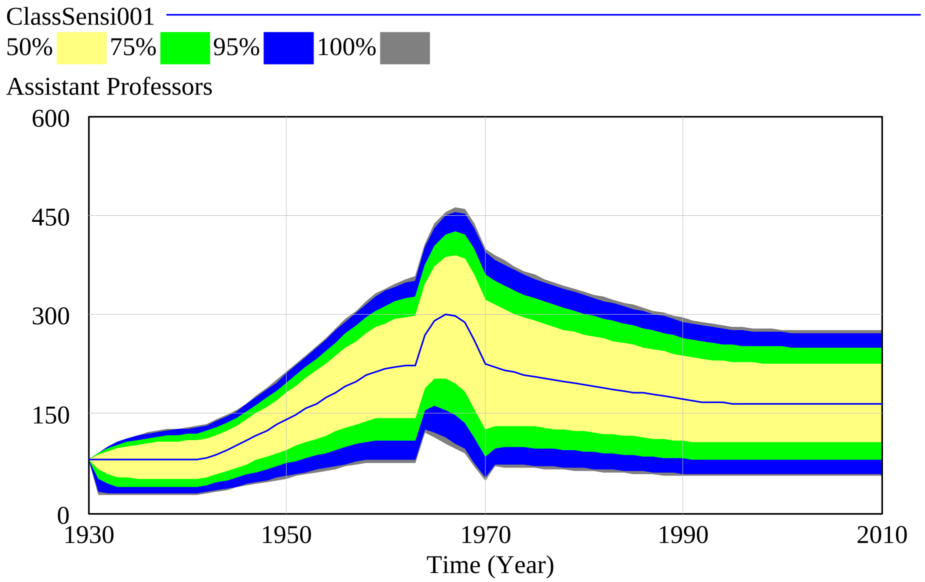
- Beta
- Binomial
- Exponential
- Gamma
- Lookup
- Negative Binomial
- Normal
- Poisson
- Triangular
- Uniform
- Weibull
- Vector

Different Distributions

See Reference Material

How to read the results?





ClassSensi001
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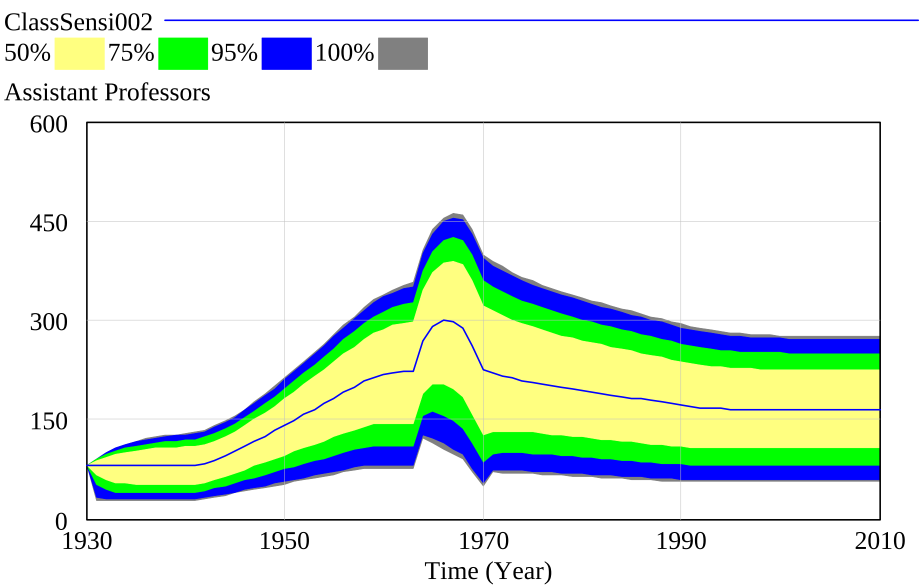

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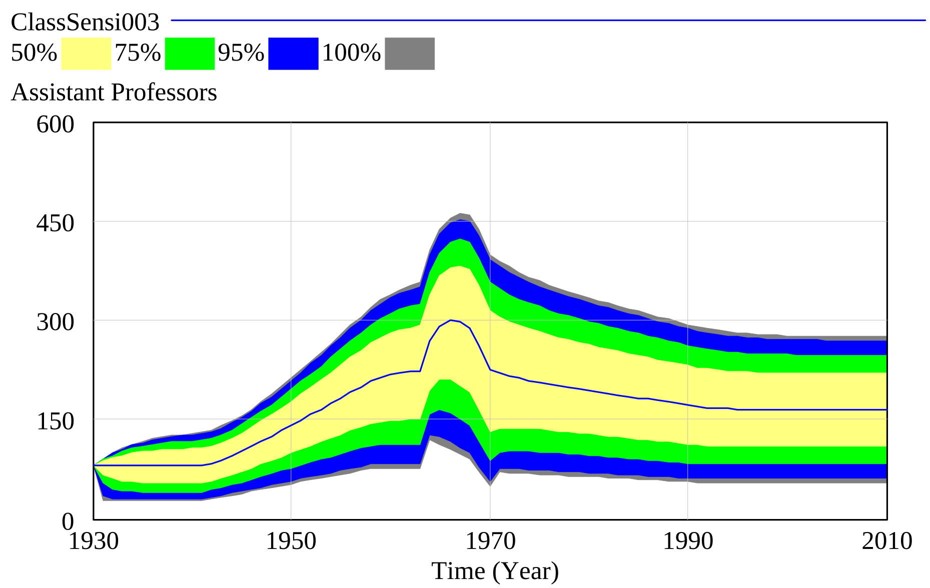

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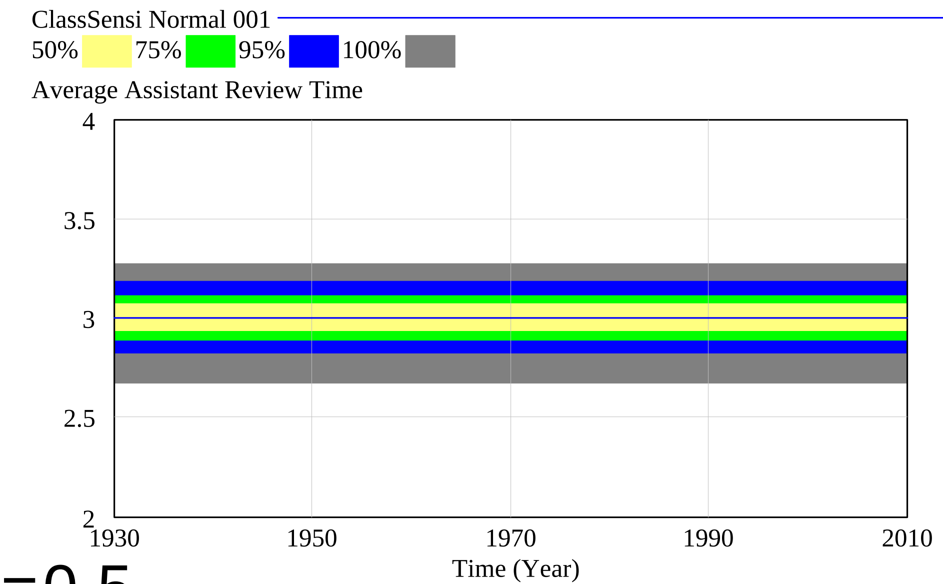
SD=0.1



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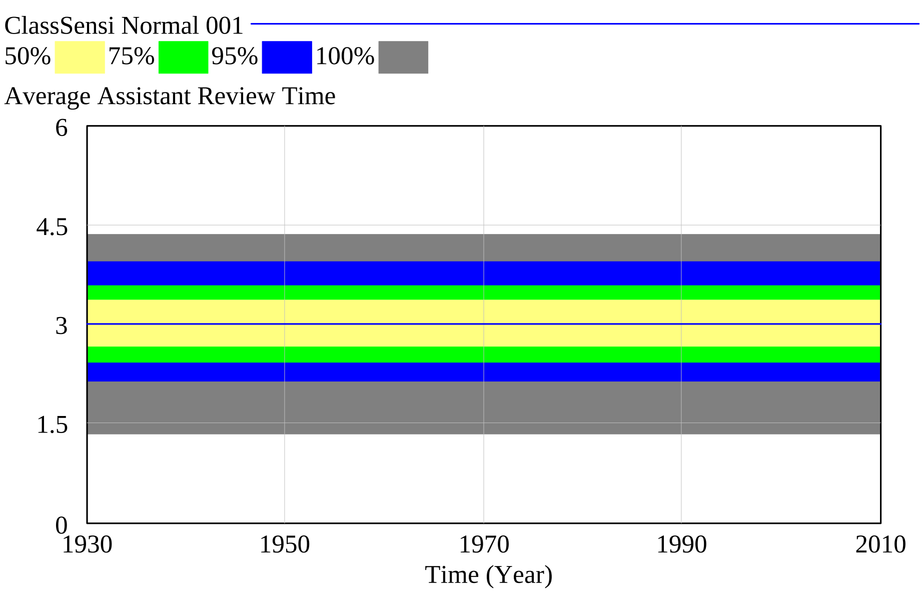

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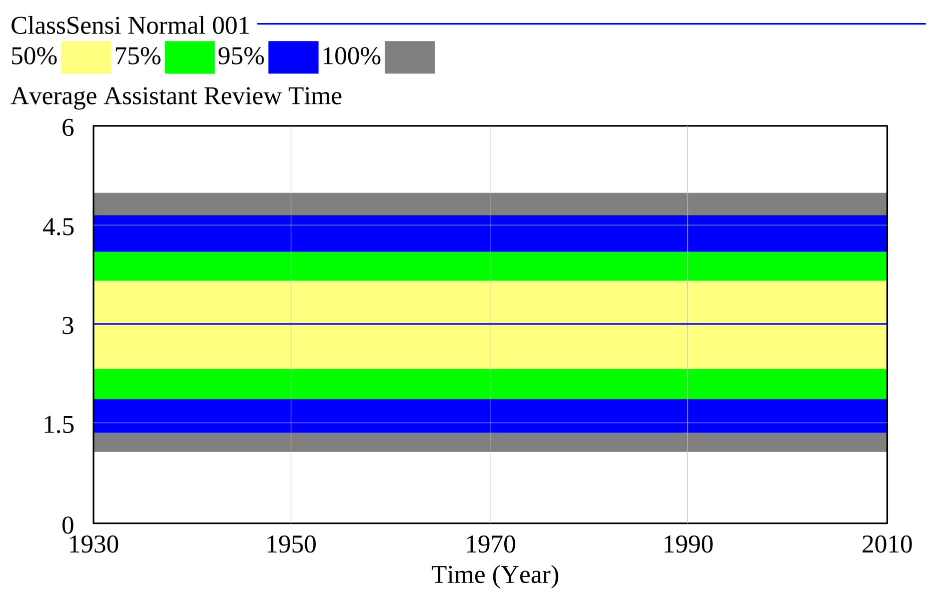
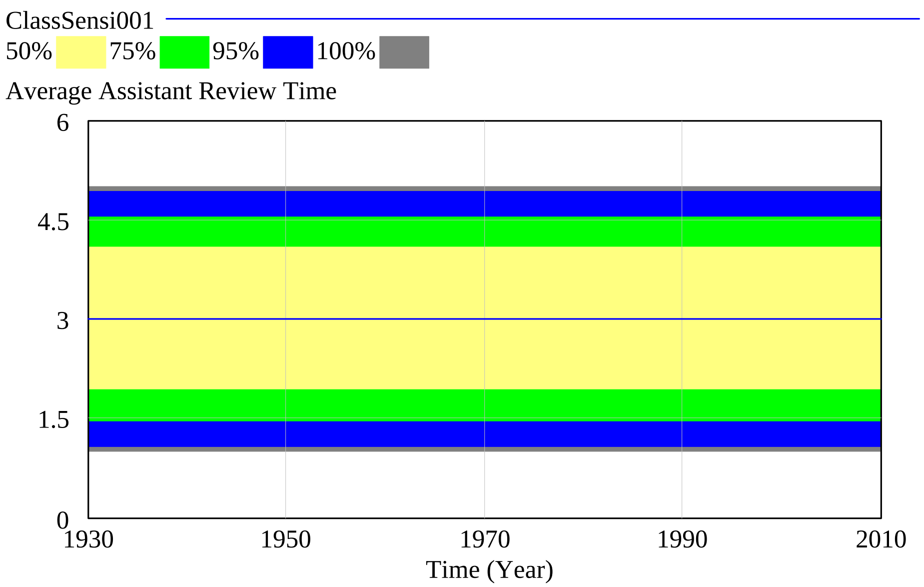
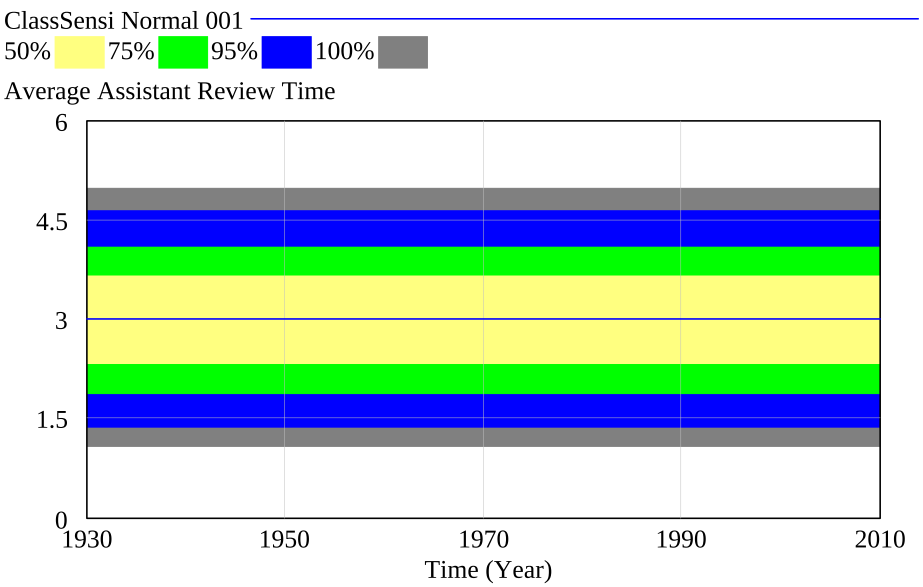
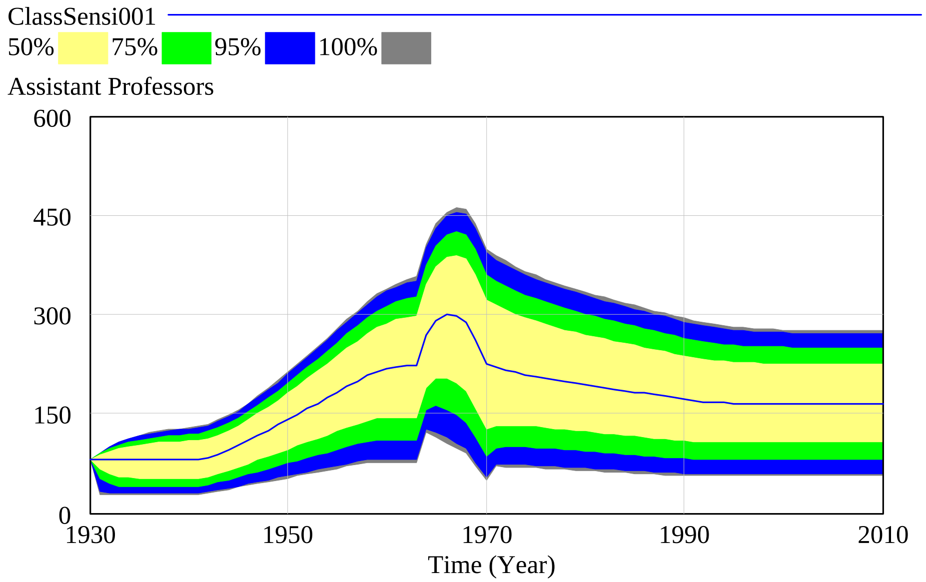


Uniform



Normal SD=1

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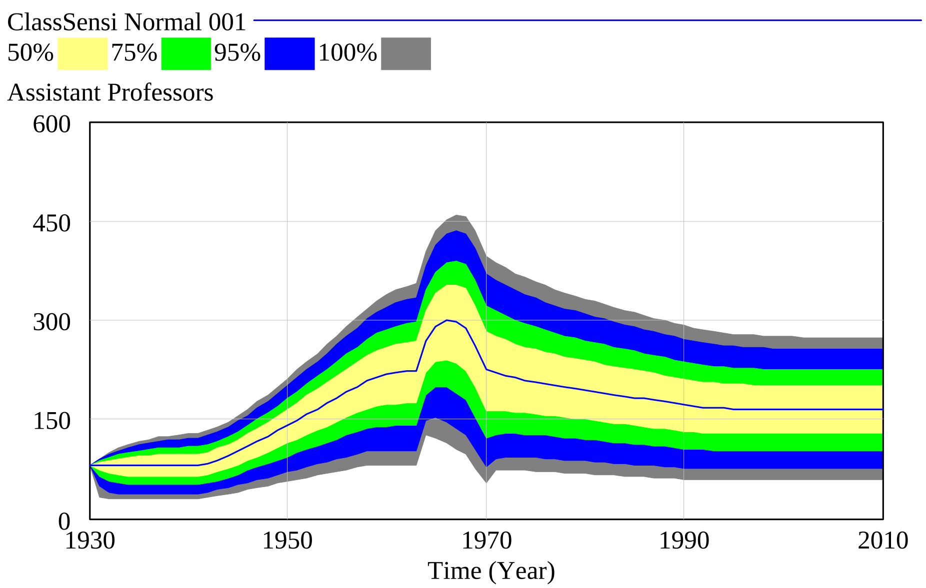
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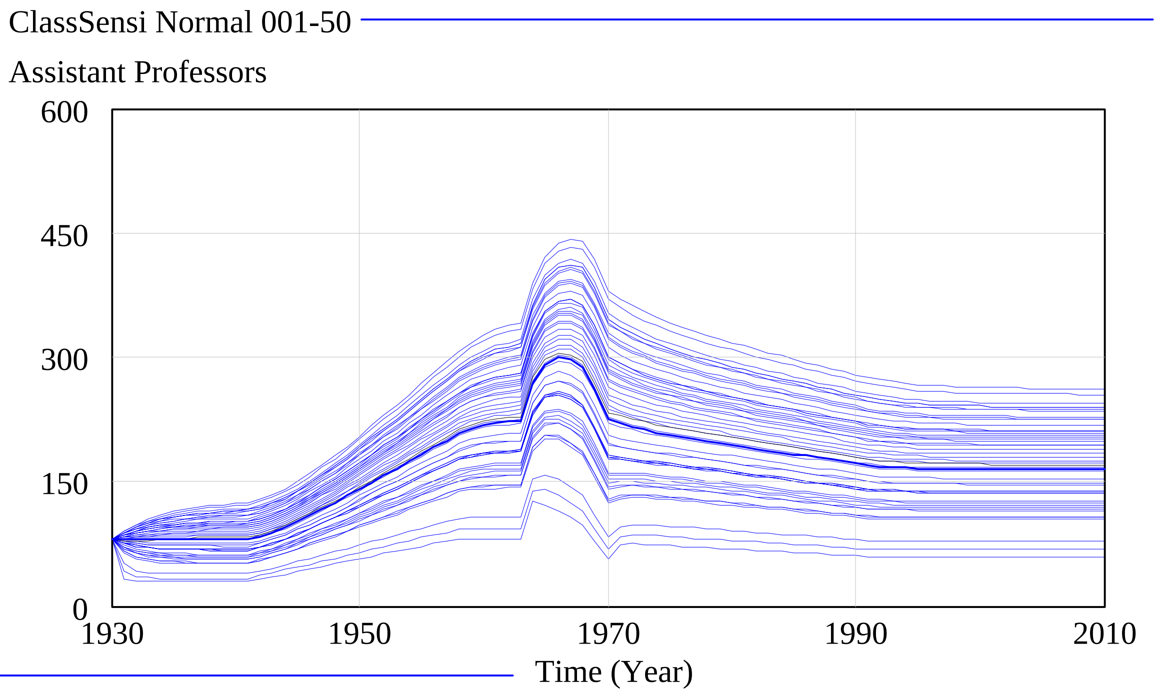
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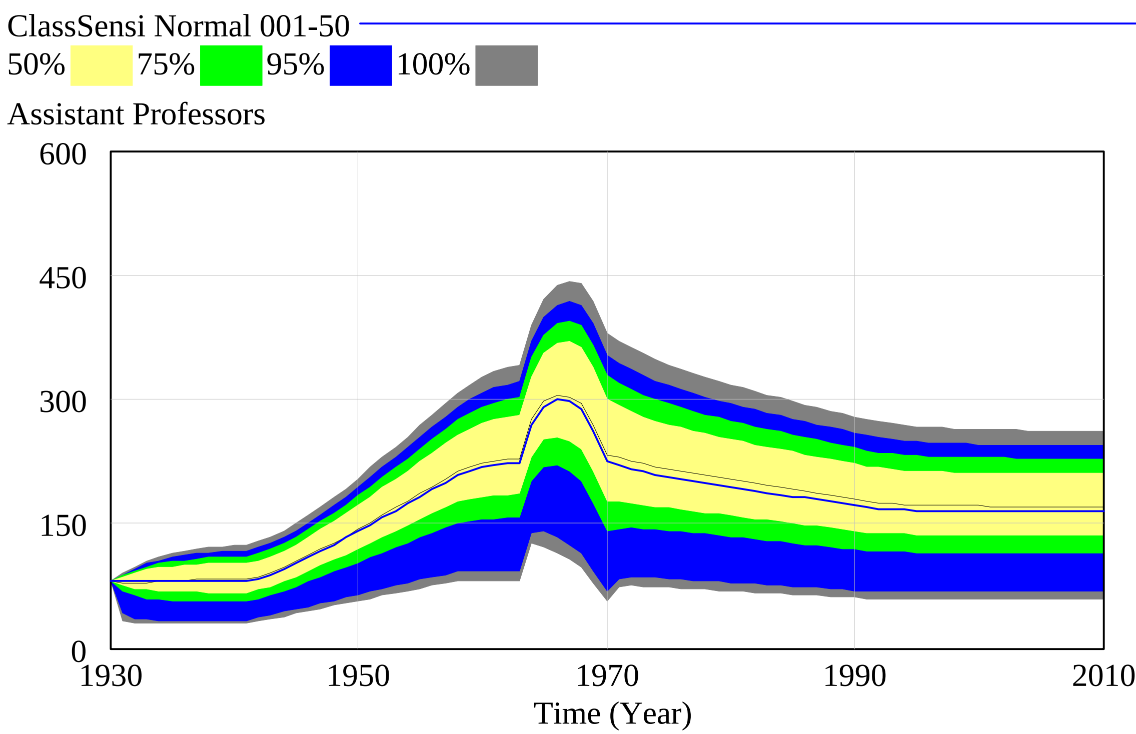
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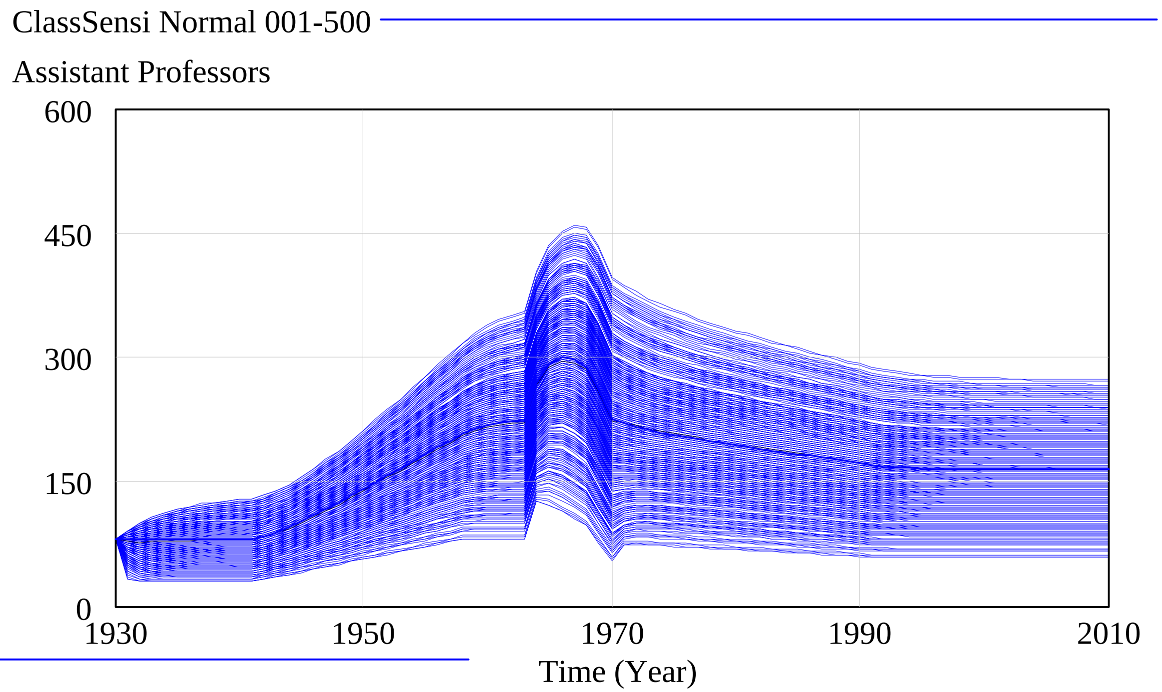
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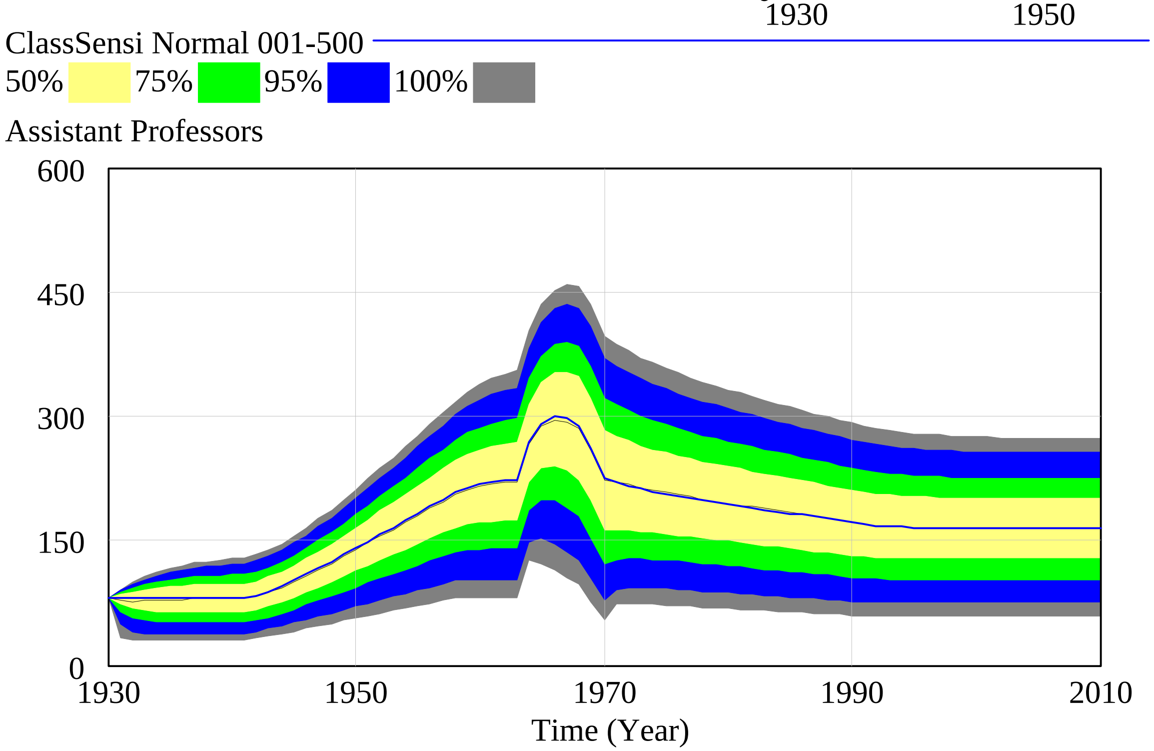
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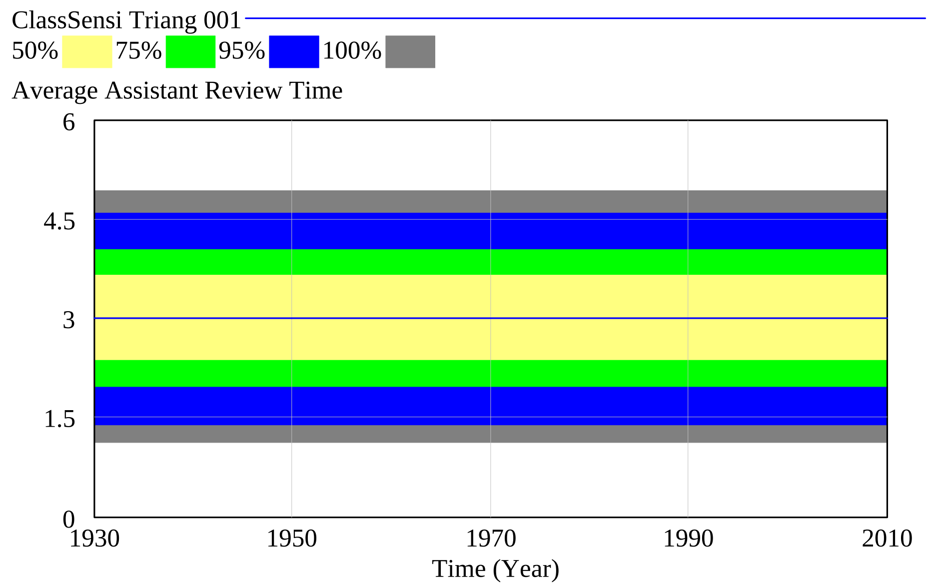
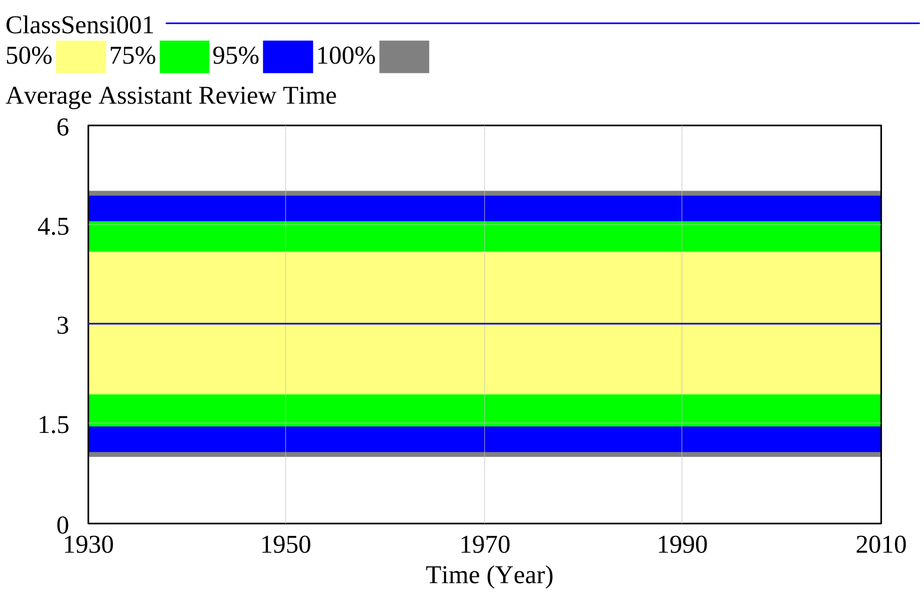
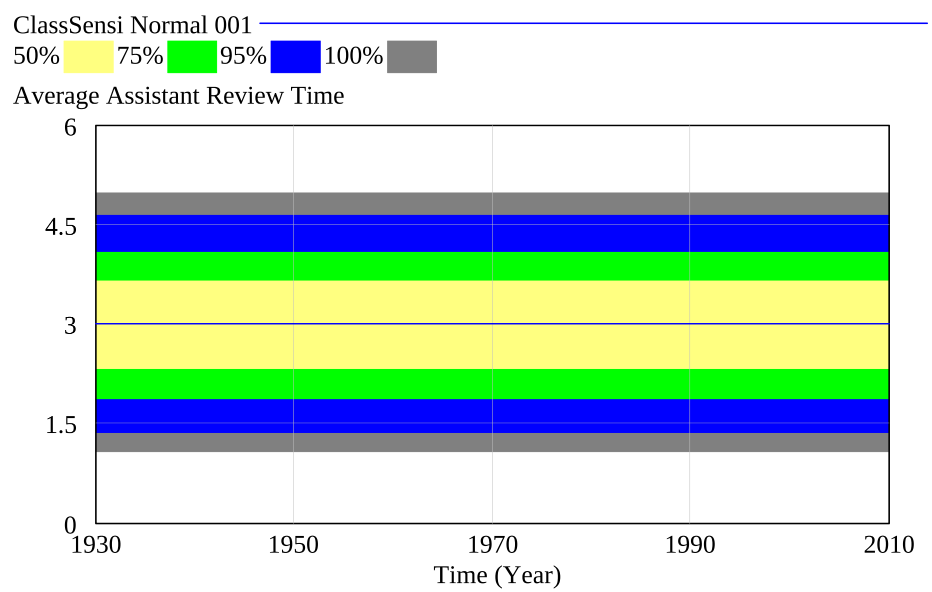


Triangular

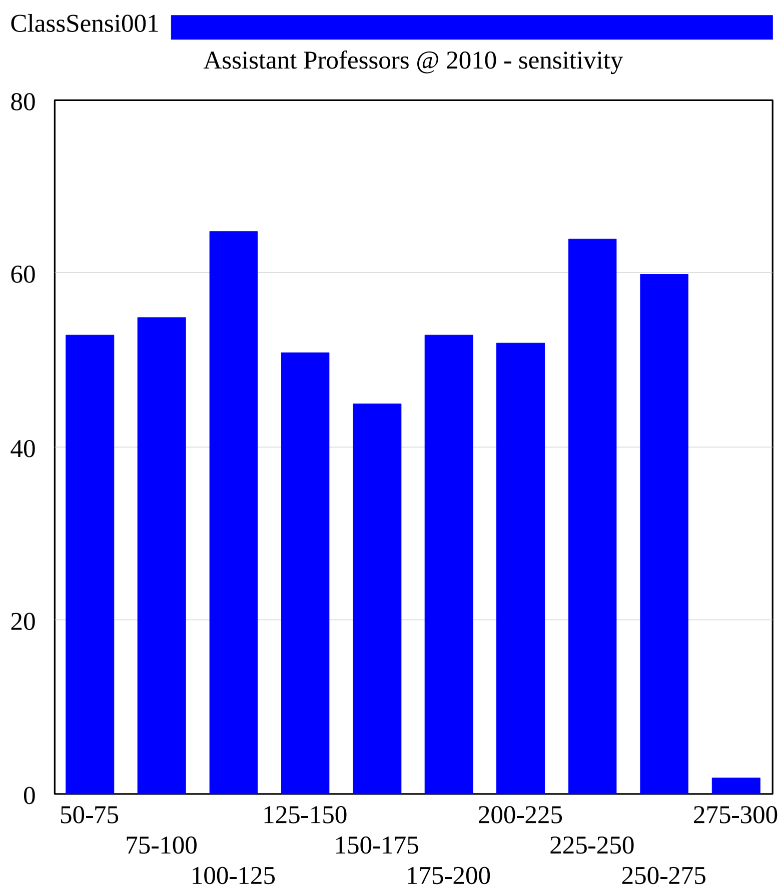
Uniform

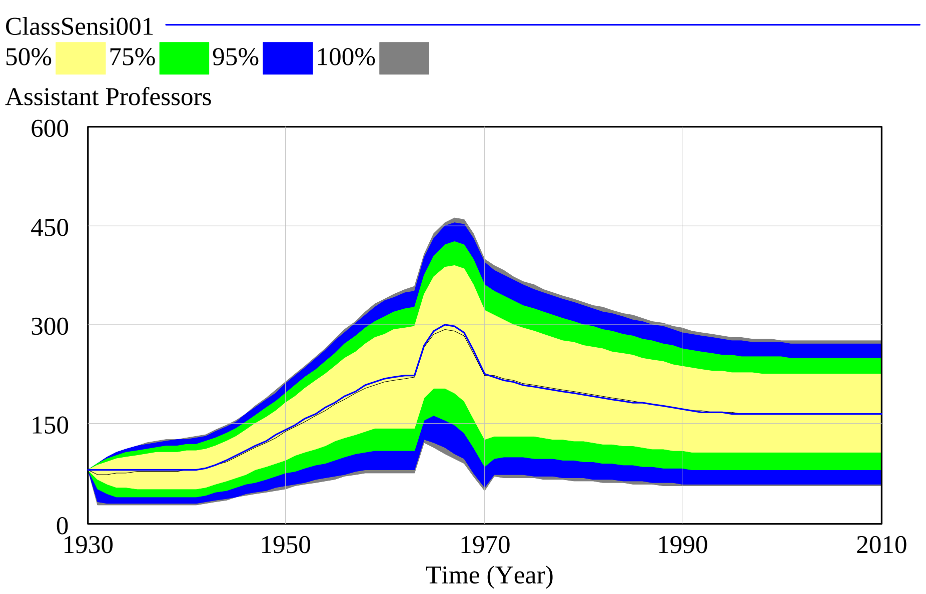
Normal SD=1



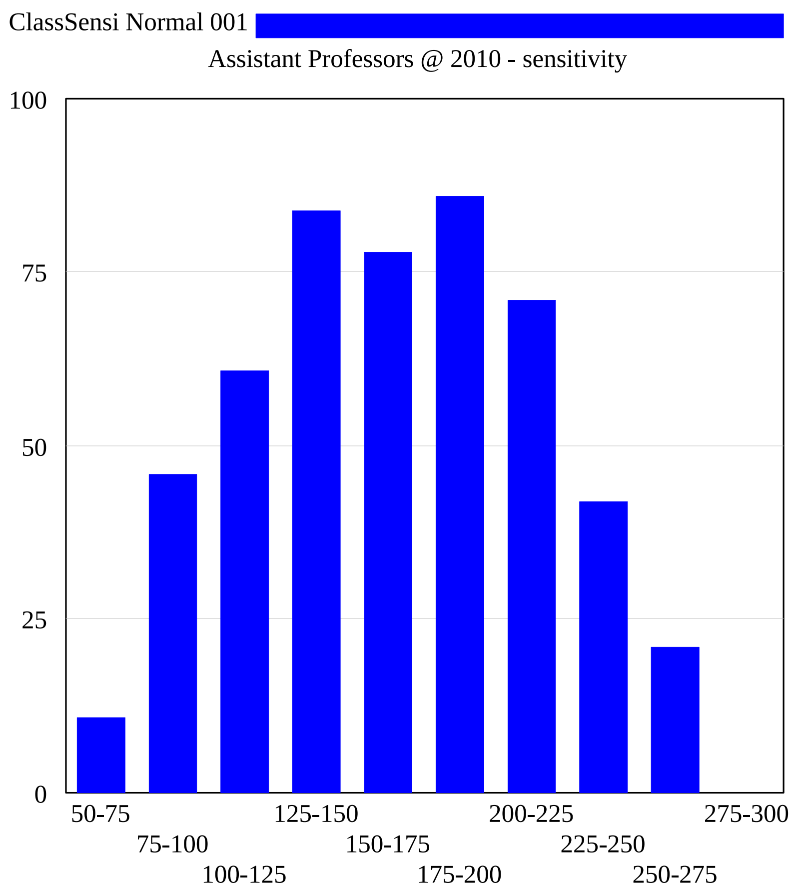


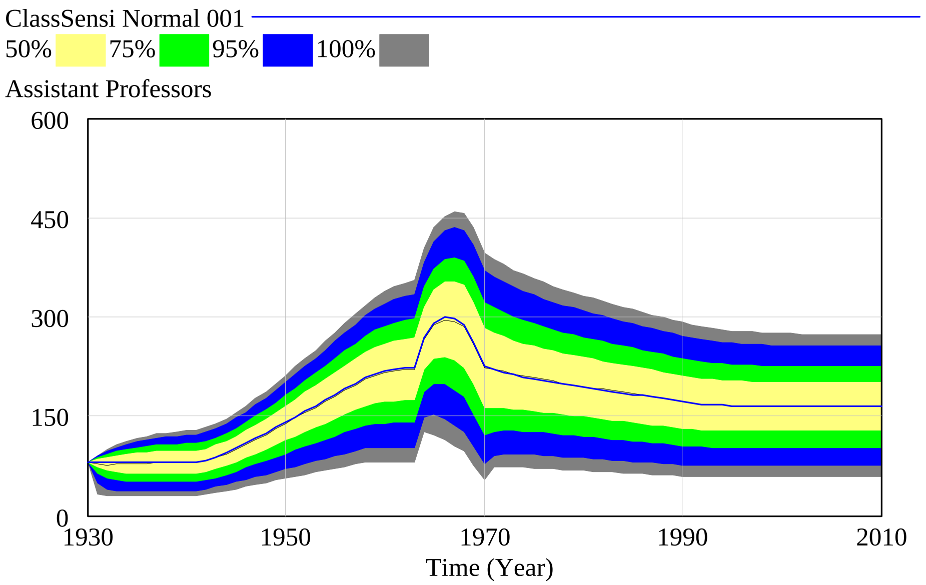
Two Types (Uniform)



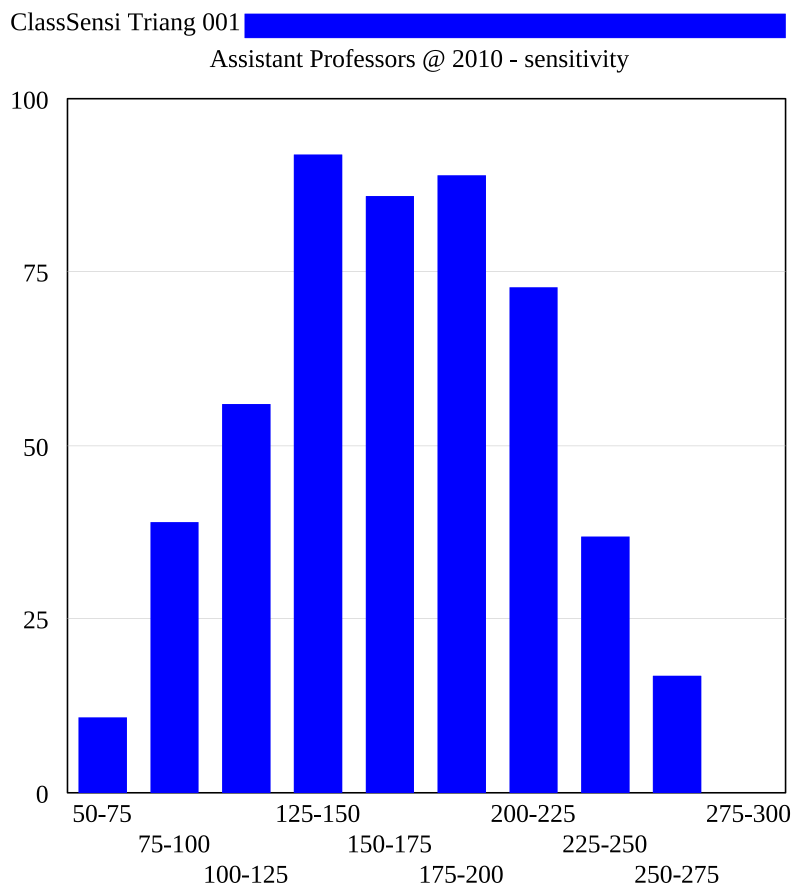


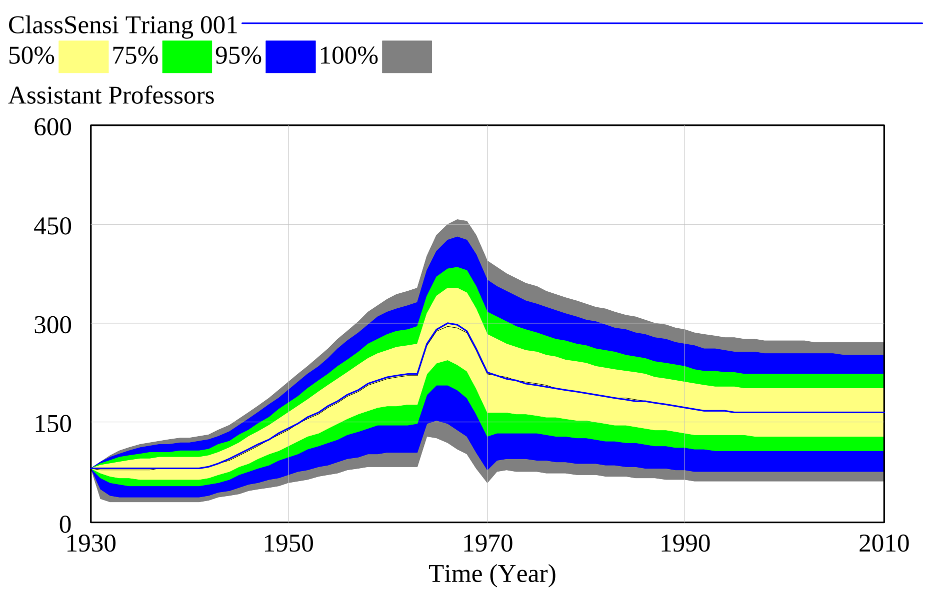
Two Types (Normal)

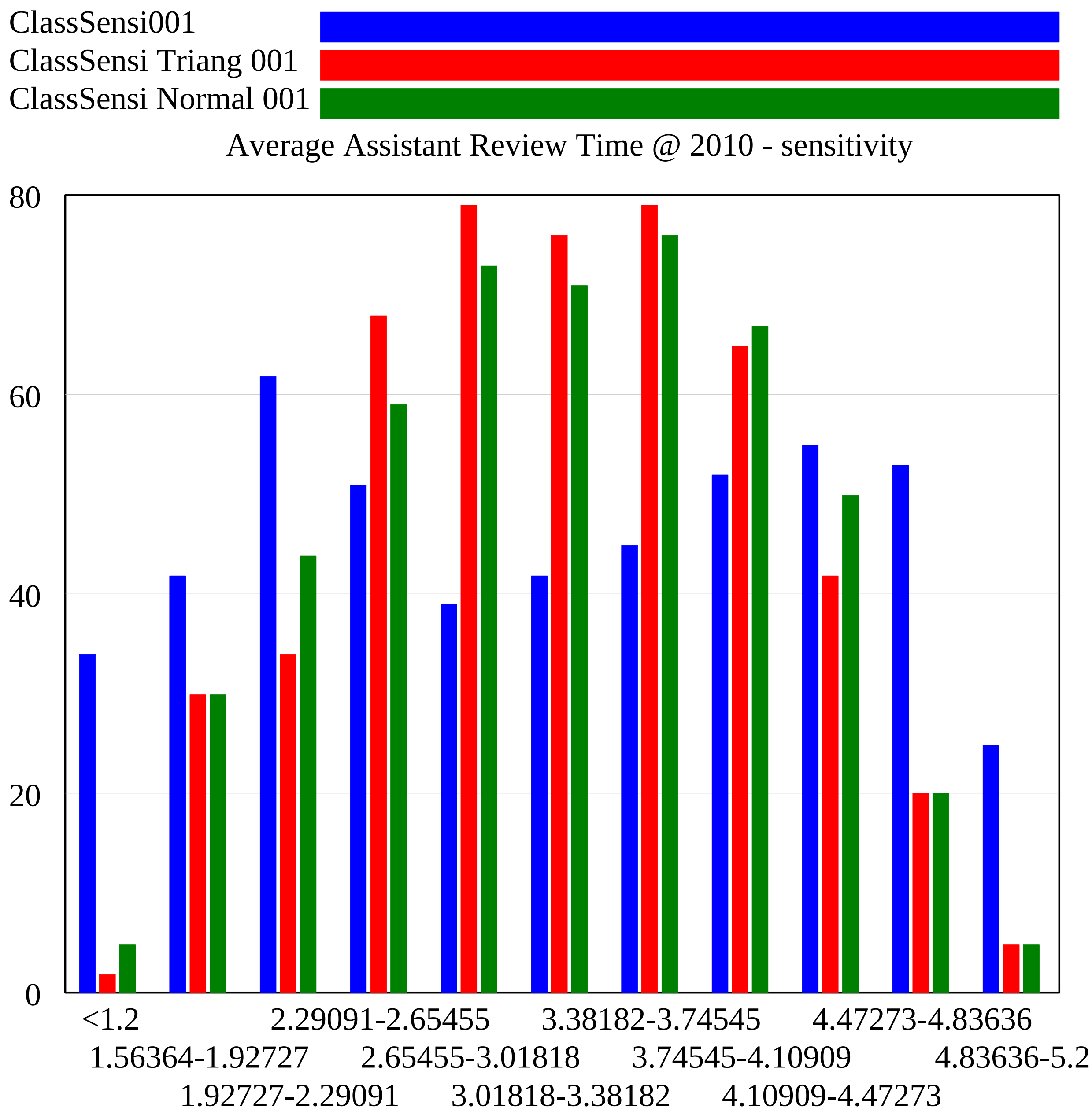




Two Types (Triangular)



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ClassSensi Triang 001
ClassSensi Normal 001
Average Assistant Review Time @ 2010 - sensitivity
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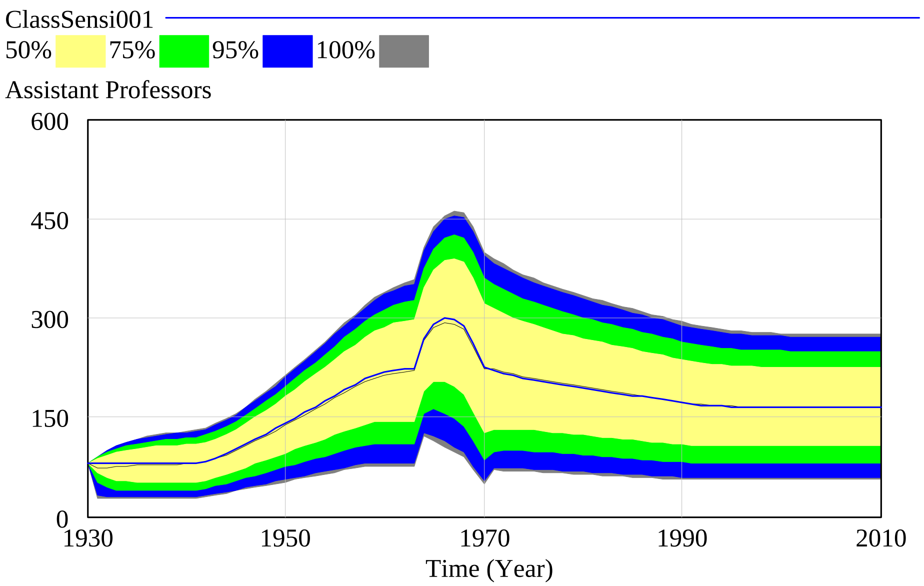
60
40
20
0
<1.2
1.56364-1.92727
1.92727-2.29091
2.29091-2.65455
2.65455-3.01818
3.01818-3.38182
3.38182-3.74545
3.74545-4.10909
4.10909-4.47273
4.47273-4.83636
4.83636-5.2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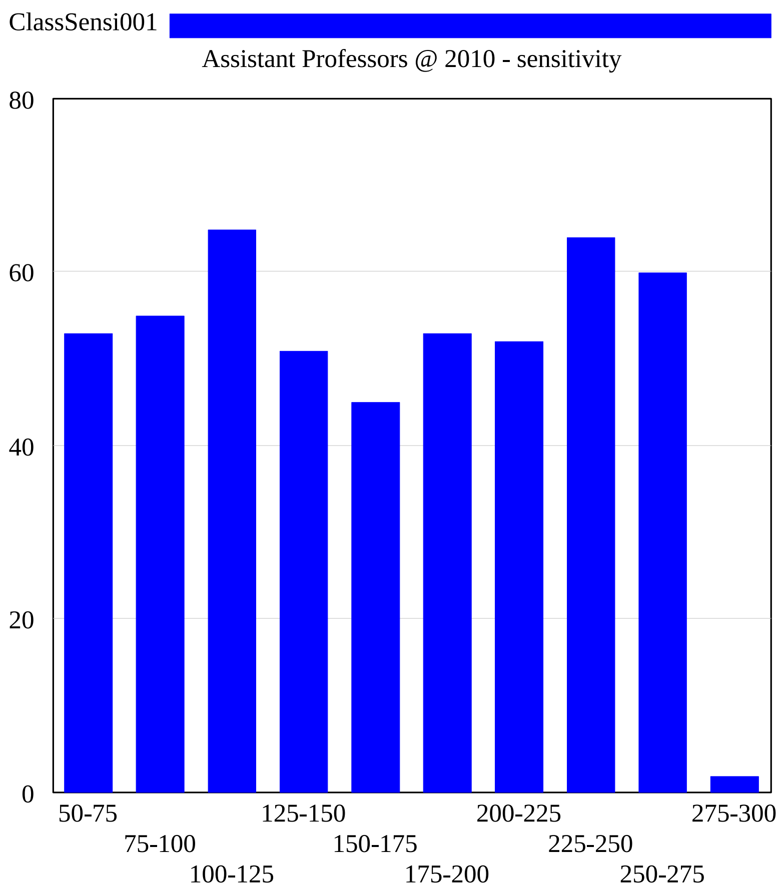
Time (Year)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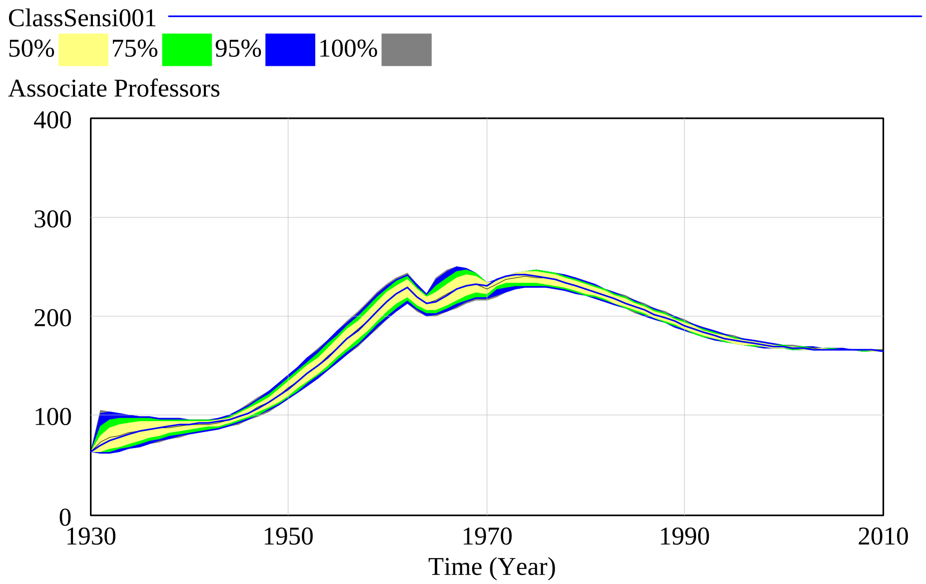
Assistant Professors @ 2010 - sensitivity
80
60
40
20
0
50-75
75-100
100-125
125-150
150-175
175-200
200-225
225-250
250-275
275-300


ClassSensi001
50% 75% 95% 100%
Associate Professors
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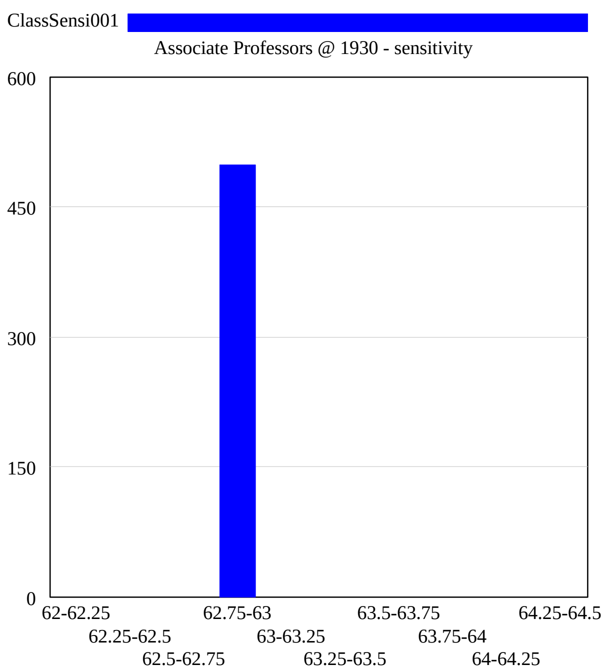
Time (Year)


ClassSensi001
Associate Professors @ 1930 - sensitivity
600
450
300
150
0
62-62.25
62.25-62.5
62.5-62.75
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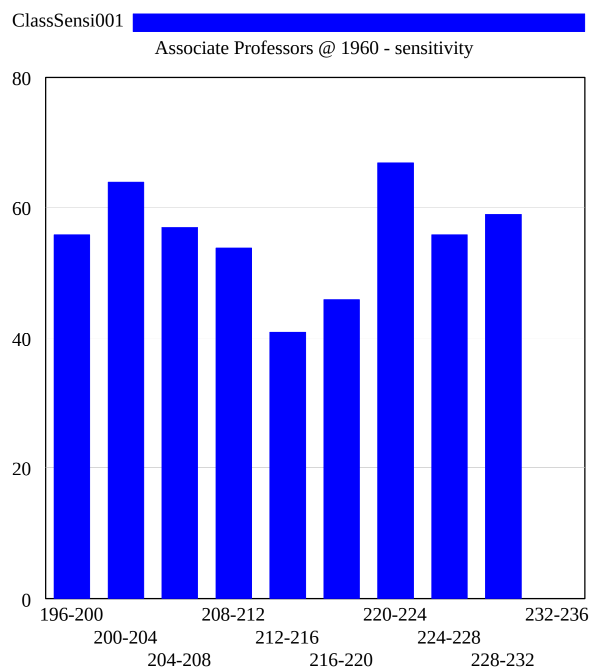
63-63.25
63.25-63.5
63.5-63.75
63.75-64
64-64.25
64.25-64.5


ClassSensi001
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60
40
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224-228
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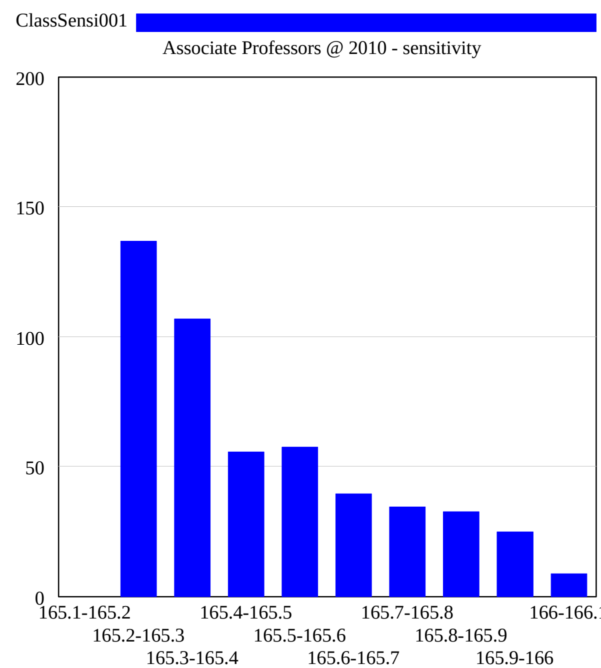
232-236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Associate Professors @ 2010 - sensitivity
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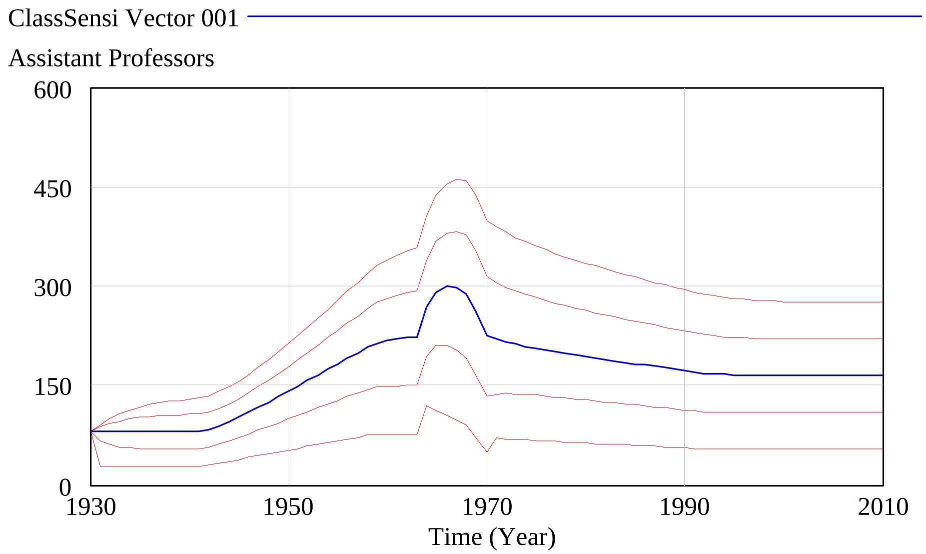
150
100
50
0
165.1-165.2
165.2-165.3
165.3-165.4
165.4-165.5
165.5-165.6
165.6-165.7
165.7-165.8
165.8-165.9
165.9-166
166-166.1

Vectors

$$A = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix}$$

Where c_1 is what Vensim Calls minimum value and c_n is the maximum value Using a certain increment.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Assistant Professors
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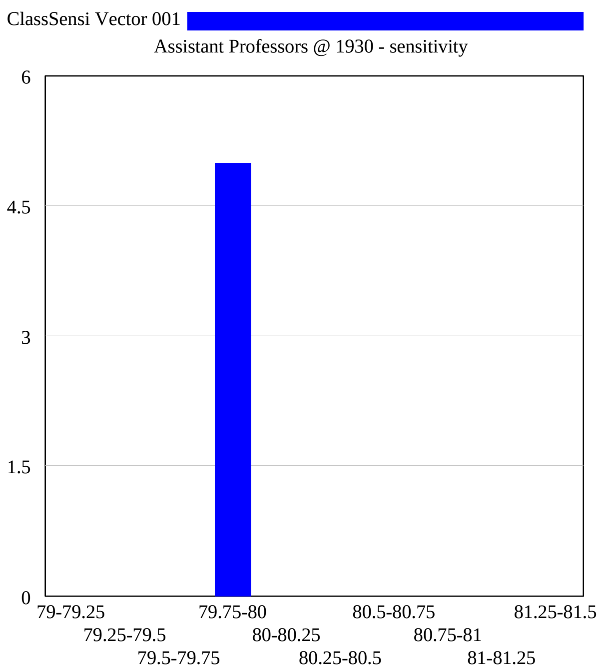
Time (Year)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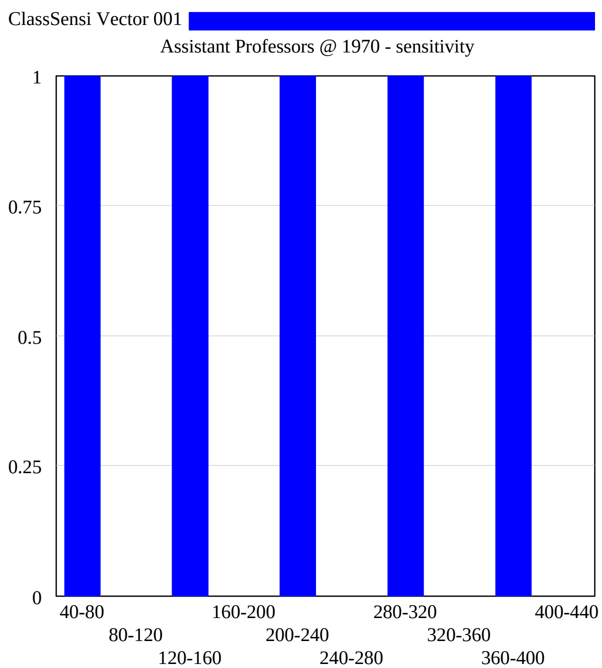
6
4.5
3
1.5
0
79-79.25
79.25-79.5
79.5-79.75
79.75-80
80-80.25
80.25-80.5
80.5-80.75
80.75-81
81-81.25
81.25-81.5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0.5
0.25
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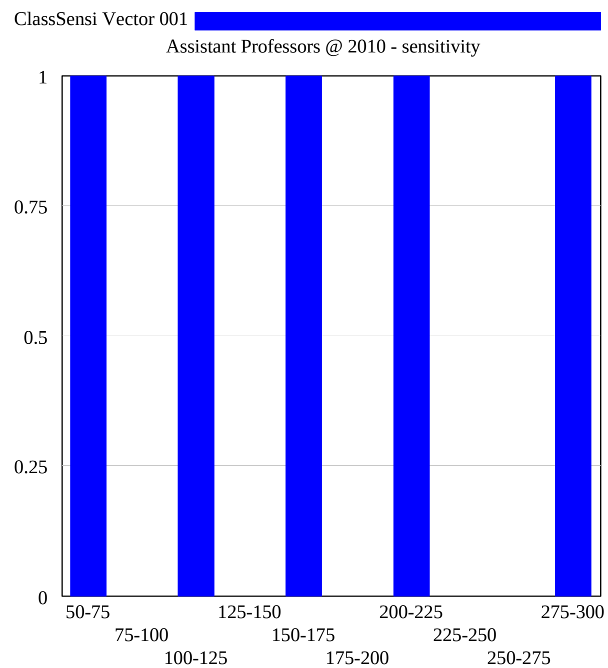
320-360
360-400
400-440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0.5
0.25
0
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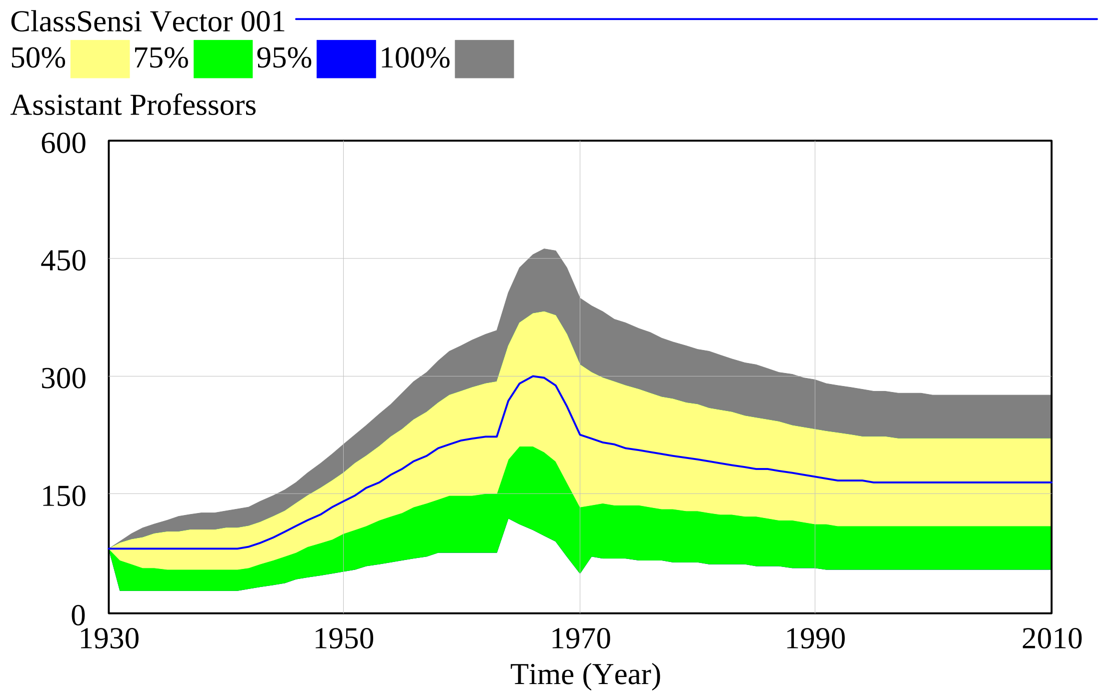
225-250
250-275
275-300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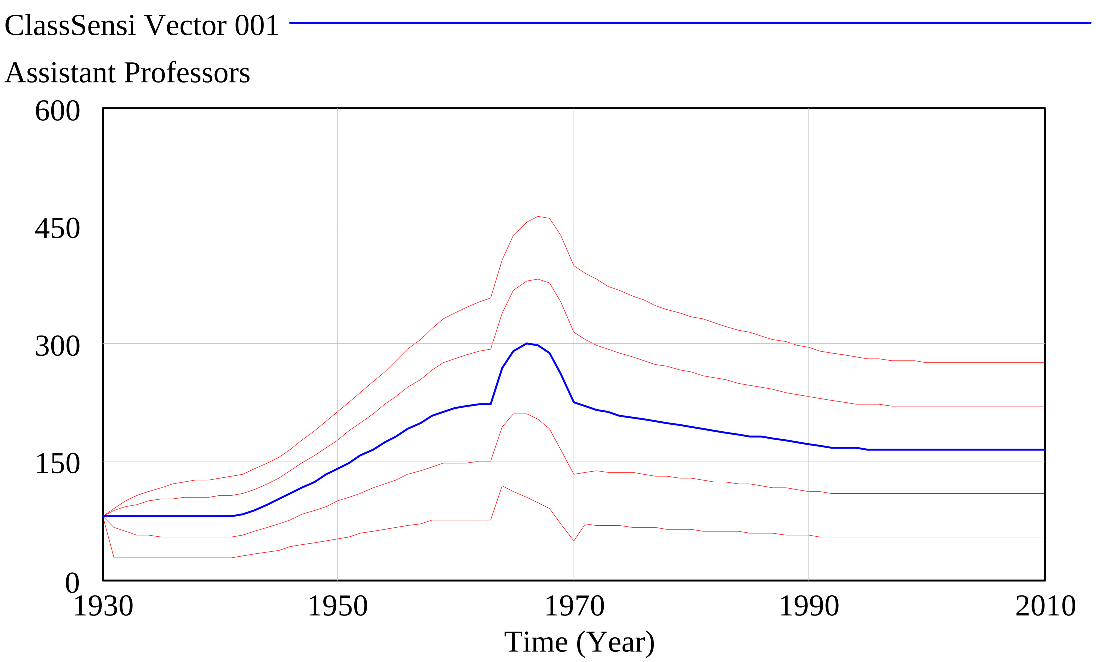
Time (Year)


ClassSensi Vector 001
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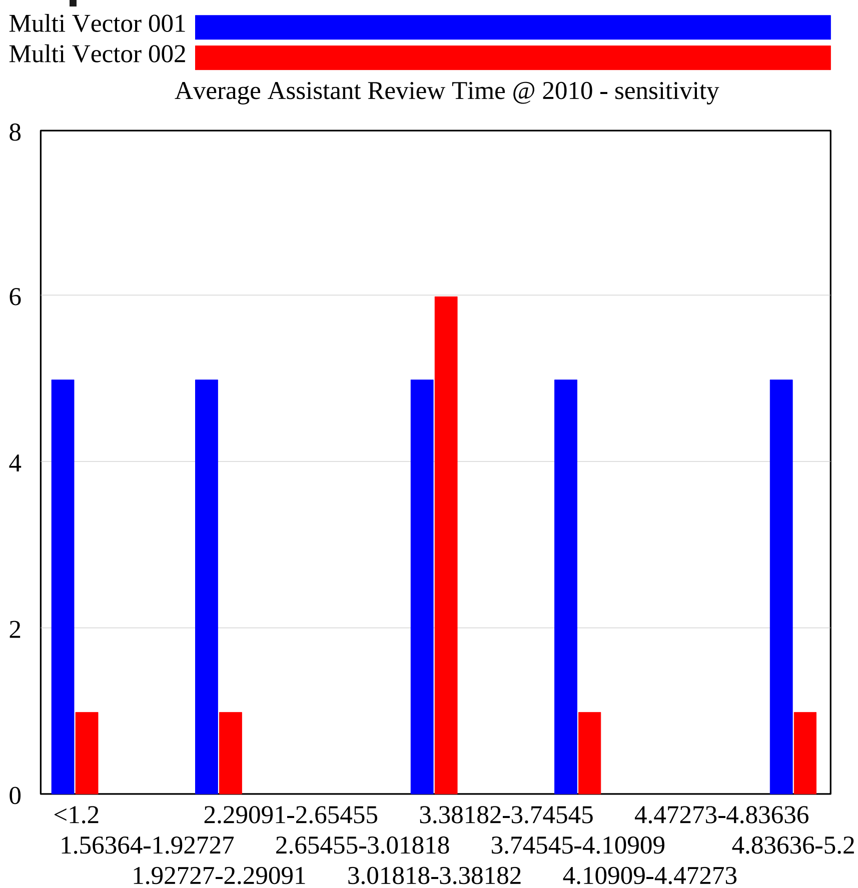
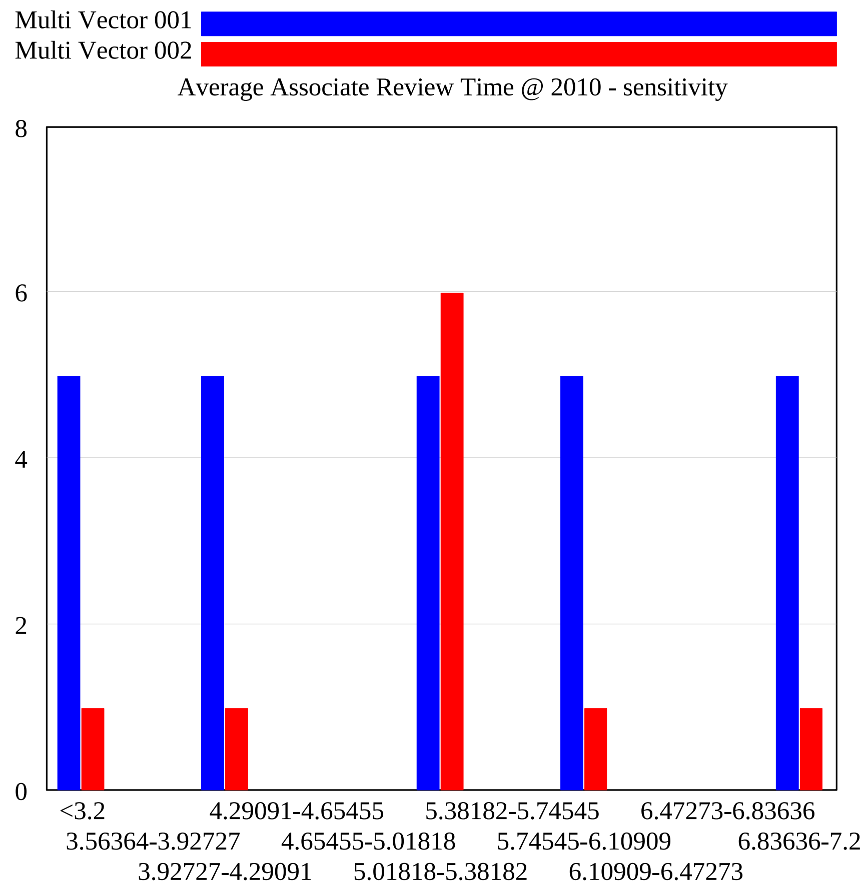
Time (Year)

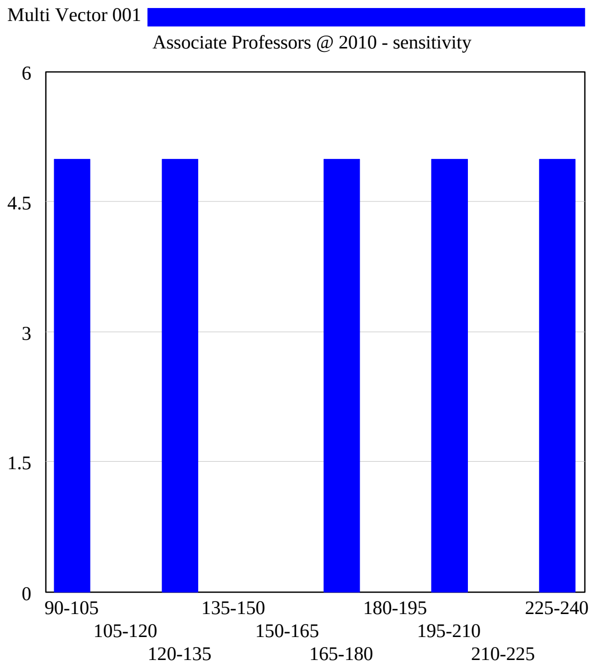
Multivariate

- More than one variable at a time.
  - Random variables
  - Vector variables (combinatorial)
- Latin Hypercube Sampling
  - Full range of the variable being sampled

Combinatorial






Multi Vector 001
Associate Professors @ 2010 - sensitivity
6
4.5
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135-150
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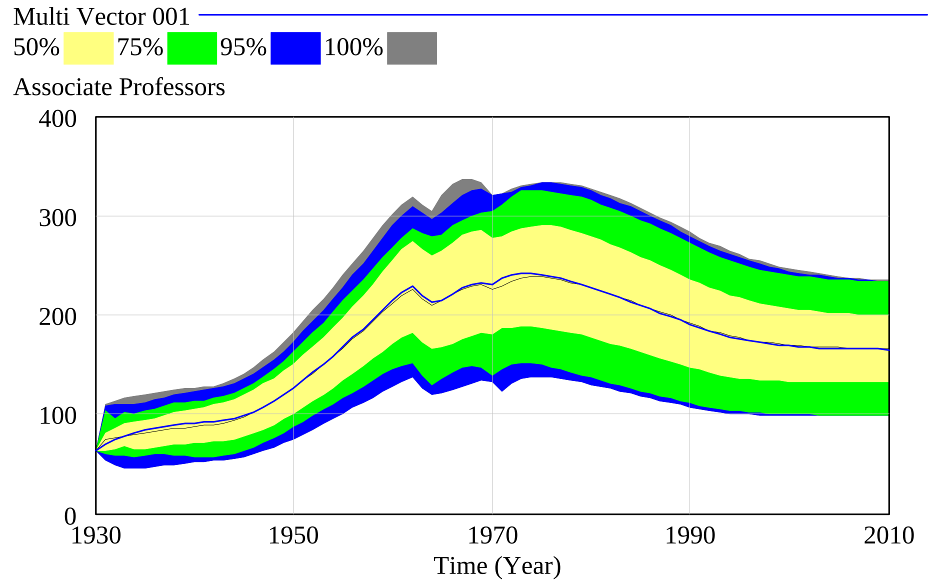
195-210
210-225
225-240


Multi Vector 001
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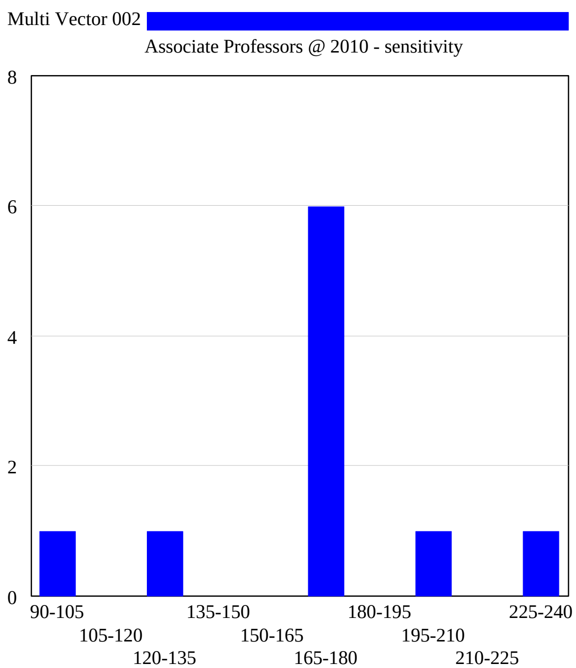
Time (Year)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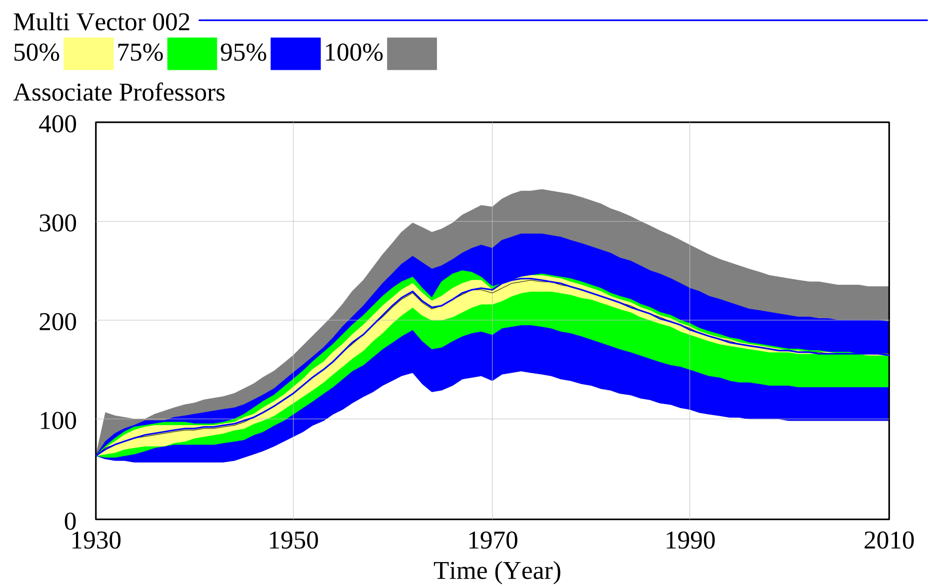
210-225
225-240


Multi Vector 002
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1950
1970
1990
2010
Time (Year)

Multivariate

- Full Factorial Method
  - A model with 10 parameters with 3 levels each becomes a 3^{10} array or 59,047 trials.
- Taguchi Method (sample)
  - A model with 10 parameters with 3 levels each becomes a $L_{27}3^{10}$ orthogonal array or 27 trials.[for weak interactions among levels]
  - Limited to 63 factors

Arrays for 3 factors with 2 levels

$$c = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix} \mapsto c_{i(\max)}$$

Full Factorial array

$$c = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \mapsto c_{i(\max)}$$

Taguchi array

Concluding...It is important

- There are a large number of model parameters that require testing.
- There are a large number of output variables that might need to be monitored.
- The output generated is dynamic.
- Many models are constructed without a clear statement of purpose.

(Ford, et al 1983)

Assignment

Go to:

www.albany.edu/cpr/sdgroup/HIMS/index.htm

And conduct sensitivity analysis on the concept model 3 and/or the Trust1 model using at least:

- Univariate, Multivariate, and Latin hypercube MCSS
- 3 different random distributions for the parameters
- 1 combinatorial process with at least 2 parameters as vectors.
- Design a small experiment using 3 parameters with 3 levels each. (You can use a full factorial design or a Taguchi design).
- Report on the findings.