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Giving Feedback: Description of a method for fitting weights and displaying function forms for feedback to judges in judgment analysis

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When an investigator has decided to deliver cognitive feedback in a multiple cue probability learning task, two elements of such feedback are the relative importance weights of each of the cues and the display of function forms. The relative importance weights model cue use in judgment. The function forms illustrate how an individual's judgment changes over the range of values taken on by a single cue across all the profiles delivered for judgment.

The feedback of weights and the display of function forms relating participants' judgments to the range of cue values presented to them requires attention to the appropriate statistics, to the constraints on computer generated models, and to the capacity of the participants to interpret the displays. Tom Stewart has developed the following technique to simplify the feedback of weights and function forms to individuals.

Let Y_i be the judgment on a case profile i described by j cues with values, X_{ji}. A regression model with a linear and quadratic term in each cue can be fitted to yield parameters b_{j1} and b_{j2}, the coefficients of the linear and quadratic terms respectively, associated with each cue j.

The choice to model judgment with linear and quadratic terms in all cues is not a necessary one. The investigator might wish to add quadratic terms for some cues and not for others. For example, if there are only a few cases to judge (each quadratic term in effect adds another cue), a cue has only two values, or there is no reason to examine non-linear function forms, then the quadratic term should not be used.

If an investigator has decided to use quadratic terms, it is necessary to transform the squared cue before proceeding. Because X_{ji} and X_{ji}^2 are highly correlated as predictors, with the potential to crash some regression programs, it is recommended that the quadratic term be shifted by the average for the cue $(\bar{X}_j)$ before it is squared.

The regression model of the judgment against the cue values to be fit is as follows:

$$\hat{Y}_i = b_0 + \sum_j b_{j1} X_{ji} + b_{j2} (X_{ji} - \overline{X}_j)^2 \qquad (1)$$

where j indexes the particular cue
i indexes the case profile
b_0 is the fitted intercept
b_{j1} is the fitted coefficient of the linear term
b_{j2} is the fitted coefficient of the quadratic term
X_{ji} is the value of the jth cue on the ith profile
$\hat{Y}_i$ is the modeled judgment value

During the course of this description of the technique of displaying weights and function forms, simplified steps will be presented along with examples of the layout of the spreadsheets for quick reference.

Step 1: For each cue, create two columns in a spreadsheet. The first column should have cue values for all the cases. The second column should have the squared deviations of the cue values from their mean over cases. Add a final column with the participants' judgments for each case.

Possible Layout of Spreadsheet:

Pro-file	**Value of Cue 1**	**Quadratic for Cue 1**	**Value of Cue 2**	**Quadratic for Cue 2**	**Value of Cue 3**	**Quadratic for Cue 3**	**Judg-ments**
i	X_{1i}	$(X_{1i}$- \`$X_1)^2$	X_{2i}	$(X_{2i}$- \`$X_2)^2$	X_{3i}	$(X_{3i}$- \`$X_3)^2$	Y_i

Step 2: Do a multiple regression analysis of the judgments on all the cue values in the spreadsheet. Keep track of the raw regression weights, b_{j1} and b_{j2}, coefficients for the linear and quadratic predictors for each cue.

Possible Layout of Spreadsheet:

	Intercept	**Cue 1 (j=1)**	**Cue 2 (j=2)**	**Cue 3 (j = 3)**
Constant (b_0)	b_0			
Linear regression weight (b_{j1})		b_{11}	b_{21}	b_{31}
Quadratic regression weight (b_{j2})		b_{12}	b_{22}	b_{32}

As it is important to feed back only one weight for each cue to the participants to interpret, it is necessary to develop a combined weight for each cue to represent a pooling of the linear and quadratic terms. In addition, in order to describe the relative weight of each cue, it is necessary to standardized these pooled coefficients so that they are on the same scale, regardless of the initial scale values for each cue.

Pooling the linear and quadratic terms:

Let X_{j1i} be the linear predictor, above described as X_{ji}.
Let X_{j2i} be the quadratic predictor, above described as $(X_{ji} - \overline{X}_j)^2$.

Substituting in Equation (1) above:

$$\hat{Y}_i = b_0 + \sum_j b_{j1} X_{j1i} + b_{j2} X_{j2i} \quad (2)$$

$$= b_0 + \sum_j 1 * f(X_{ji}), \quad (3)$$

$$\text{where } f(X_{ji}) = b_{j1} X_{j1i} + b_{j2} X_{j2i} \quad (4)$$

Note that the regression fit of Equation (3) yields a coefficient of 1, as the coefficients from Equation (2) have already been built into the function $f(X_{ji})$.

Finding the Unit-Free Cue Weights, β:

$$\text{Let } Z_{f(X_{ji})} = \frac{f(X_{ji}) - \overline{f(X_{ji})}}{s(f(X_{ji}))}, \quad (5)$$

where s(f(Xji)) is the standard deviation of the $f(X_{ji})$

$$\text{Then } \hat{Z}_{\hat{Y}_i} = b_0 + \sum_j b_j Z_{f(X_{ji})} \quad (6)$$

These β_j values constitute, for each cue, the unit-free weight (sometimes referred to as "standardized weights") of that cue in the participants' judgment, pooling linear and quadratic terms for the cue. The β_j values are yielded as parameters in a regression model of standardized judgments against standardized values from Equation (5).

The β_j can also be calculated from the initial raw regression weights b_{j1} and b_{j2} found in Equation (2):

$$b_j = 1 * \sqrt{\frac{b_{j1}^2 s_{j1}^2 + b_{j2}^2 s_{j2}^2 + 2 b_{j1} b_{j2} r_{j12} s_{j1} s_{j2}}{s_{\hat{Y}}^2}} \qquad (7)$$

where the numerator is the standard deviation of $f(X_{ji})$ described by Equation (4) (with X_{j1} and X_{j2} not assumed to be independent)

the denominator is the standard deviation of the modeled judgments, $\hat{Y}_i$

s_{j1} is the standard deviation of the linear cue j values calculated over the cases i

s_{j2} is the standard deviation of the quadratic cue j values calculated over the cases i

r_{ji} is the correlation between the linear predictor X_{j1i} and the quadratic predictor X_{j2i}

Step 3: Calculate the standard deviation of the judgments. For each cue, calculate the standard deviation for the linear and quadratic columns of cue values and the correlation between the two columns. Fill in these values to calculate β for each cue according to Equation (7) (see table below)

Calculating the Relative Weight W_j:

In order to describe the weight in judgment of each cue j relative to the other cues, it is necessary to calculate relative weight, W_j.

$$W_j = 100 * \frac{b_j}{\sum_j b_j} \qquad (8)$$

Note that $0 \leq W_j \leq 100$ as the $\beta_j > 0$, by definition and they are normed in the above Equation (8).

Possible Layout of Spreadsheet

	Cue 1	**Cue 2**	**Cue 3**
$\mathbf{b_{j1}}$	b_{11}	b_{21}	b_{31}
$\mathbf{b_{j2}}$	b_{12}	b_{22}	b_{32}
$\mathbf{s_{Xj1}}$	s_{X11}	s_{X21}	s_{X31}
$\mathbf{s_{Xj2}}$	s_{X12}	s_{X22}	s_{X32}
$\mathbf{r(x_{j1},x_{j2})}$	$r(x_{11},x_{12})$	$r(x_{21},x_{22})$	$r(x_{31},x_{32})$
pooled $\boldsymbol{\beta_j}$	*pooled* b_1	*pooled* b_2	*pooled* b_3
$\mathbf{W_j}$	W_1	W_2	W_3
$\mathbf{p_{j1}}$	p_{11}	p_{21}	p_{22}
$\mathbf{p_{j2}}$	p_{21}	p_{22}	p_{23}

The Display of Function Forms

The Decision to Plot a Function Form

If a participant doesn't use a cue in his or her judgment, the value of W_j is likely to be below some arbitrary threshold, W_t (W_t = 5 has been used in practice). If so, a horizontal line should be plotted to feedback the participant's use of that cue j.

> Rule 1: If $W_j < W_t$, then present the image file nofunction.gif .

> **Step 4:** Calculate the W_j. Decide whether the value is high enough to plot a function form for each cue, by comparing it to some threshold W_t. Wt=5 has been used in practice.

> If $W_j \leq W_t$, plot a horizontal line (nofunction.gif)

Cues With Only Two Possible Values (Binary Cues)

For binary cues, only linear function forms are possible, so a linear model must be actively fit.

> Rule 2: For binary cues j,
> if $b_{1j} > 0$, present poslin.gif.
> if $b_{1j} < 0$, present neglin.gif.

> If Wj > Wt,

> **Step 5:** Decide whether the cue is binary or can take on multiple values. If binary, plot poslin.gif or neglin.gif.

Cues with Multiple Possible Values

In some cases, the fit may not require a quadratic term, because the fit is linear. If the fit is linear between judgments and cue j values, then the b_{2j} value should not be significantly different from 0. If the hypothesis $b_{2j} = 0$ yields a p_{2j} value such that $p_{2j} > p_t$, then the function form is linear. In practice a threshold p_t value of .01 or .05 has been used.

If the cue is non-binary,

Rule 3: Given $p_{2j} > p_t$,
if $b_{1j} > 0$, present poslin.gif.
if $b_{1j} < 0$, present neglin.gif.

Step 6: Decide whether the cue use is linear based on significance of weight of quadratic term. If so, plot poslin.gif or neglin.gif.

If cue use is non-linear,

Finding X-Value at which $f(X_{ji})$ Takes on its Maximum or Minimum Value

From Equation (4), for a cue j,

$$f(X_{ji}) = b_{j1}X_{j1i} + b_{j2}X_{j2i}$$

$$= b_{j1}X_{j1i} + b_{j2}(X_{j1i} - \bar{X}_{j1})^2 \text{, by definition of } X_{j2i} \qquad (9)$$

Taking derivative with respect to X_{ji},

$$f'(X_{ji}) = b_{j1} + 2b_{j2}(X_{j1i} - \bar{X}_{j1}) \qquad (10)$$

Set the derivative to 0 and solve for X_{jm} to find the maximum or minimum value of the function for judgments with respect to cue j.

$$0 = b_{j1} + 2b_{j2}(X_{j1i} - \bar{X}_{j1}) \qquad (11)$$

$$X_{jm} = \bar{X}_{j1} - \frac{b_{j1}}{2b_{j2}} \text{, where } X_{jm} = X_{j1} \text{ at the peak of } f(X_{ji}) \qquad (12)$$

Intuition Behind Selection of Function Form

If X_{jm} lies midway between X_{jmin} and X_{jmax}, then a U shaped or inverted-U shaped function form should be displayed (see Figure 1). If X_{jm} lies in between Xjmin or Xjmax but not at the middle, then the judgment function on the cue range results which has a part that is a plateau and a part that slopes. Finally, if X_{jm} lies far outside of X_{jmin} and X_{jmax}, then the cue range judgment function appears nearly linear.

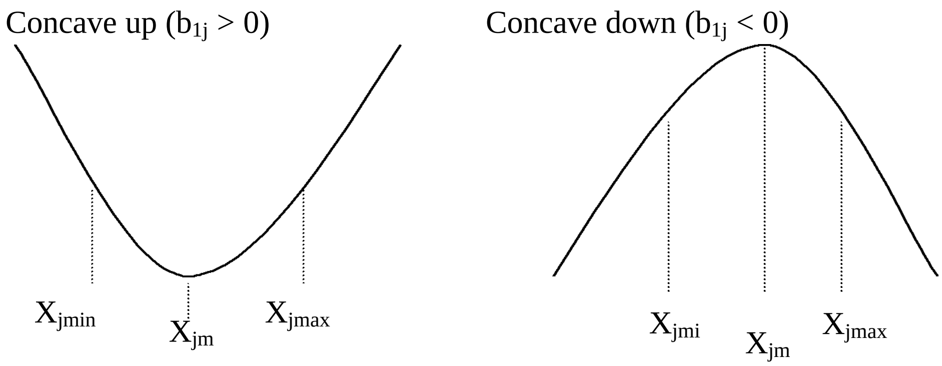

Figure 1. Concave up or down, with X_{jm} inside cue range.

Calculating $X_{jm\%}$ or location of X_{jm}, relative to cue range

$$X_{jm\%} = 100 * \frac{(X_{jm} - X_{j\min})}{(X_{j\max} - X_{j\min})}, \qquad (13)$$

where X_{jmax} is maximum value on cue range
X_{jmin} is minimum value on cue range
X_{jm} is value of cue at which judgment takes on its maximum or minimum value

Step 7: Calculate X_{jm} using Equation (12), and then calculate $X_{jm\%}$ using Equation (13). Follow Rule 4 to decide which function form to display.

Rule 4:

If $b_{j2} < 0$ and

$-\infty \le X_{jm\%} \le -50$, use neglin.gif

$-50 \le X_{jm\%} \le 150$, use neg##.gif, with ## taken from the table below

$150 \le X_{jm\%} \le \infty$, use poslin.gif

If $b_{j2} > 0$ and

$-\infty \le X_{jm\%} \le -50$, use poslin.gif

$-50 \le X_{jm\%} \le 150$, use pos##.gif, with ## taken from the table below

$150 \le X_{jm\%} \le \infty$, use neglin.gif

Range of $X_{jm\%}$	##	Description of Curve
$-50 < X_{jm\%} < -10$	01	If $b_{j2} > 0$, Positive Concave Up; If $b_{j2} < 0$, Negative Concave Down
$-10 < X_{jm\%} < 0$	02	
$0 < X_{jm\%} < 10$	03	
$10 < X_{jm\%} < 20$	04	
$20 < X_{jm\%} < 30$	05	
$30 < X_{jm\%} < 40$	06	
$40 < X_{jm\%} < 46$	07	If $b_{j2} > 0$, U-shaped; If $b_{j2} < 0$, Inverted U-shaped
$46 < X_{jm\%} < 54$	08	
$54 < X_{jm\%} < 60$	09	
$60 < X_{jm\%} < 70$	10	If $b_{j2} > 0$, Negative Concave Up; If $b_{j2} < 0$, Postive Concave Down
$70 < X_{jm\%} < 80$	11	
$80 < X_{jm\%} < 90$	12	
$90 < X_{jm\%} < 100$	13	
$100 < X_{jm\%} < 110$	14	
$110 < X_{jm\%} < 150$	15	