Lecture contents

• Magnetic field-2
  – Magnetization
  – Faraday’s law
Differential form of Ampere’s Law

• Applying Stokes’s theorem to Ampere’s Law

\[ \oint_C \vec{B} \cdot d\vec{l} = \int_S \nabla \times \vec{B} \cdot d\vec{s} \]

\[ = \mu_0 I_{encl} = \mu_0 \int_S \vec{J} \cdot d\vec{s} \]

• Because the above must hold for any surface \( S \), we must have

\[ \nabla \times \vec{B} = \mu_0 \vec{J} \]
Divergence of $B$-Field

- The $B$-field is *solenoidal*, i.e. the divergence of the $B$-field is identically equal to zero:

$$\nabla \cdot \vec{B} = 0$$

- From gauss theorem

$$\iiint_V \text{div} \vec{A} \, dV = \iint_S (\vec{A} \cdot \vec{n}) \, ds$$

field lines are closed.

- Physically, this means that magnetic charges (monopoles) do not exist.

- A magnetic charge can be viewed as an isolated magnetic pole.
Magnetization-1

• Similar to electric polarization due to dipole moment density, magnetization is magnetic dipole density

\[ \vec{M} = \sum_i^{\mu_i} \frac{A}{V} \]

Dimensionally as surface current density

• Similar to electric polarization, “bound” currents can be introduced:

\[ \vec{J}_b = \nabla \times \vec{M} \]

• In a medium with magnetization, magnetic field (induction) depends on macroscopic and bound “dipole” currents

\[ \nabla \times \frac{\vec{B}}{\mu_0} = \vec{J} + \vec{J}_b \equiv \vec{J} + \nabla \times \vec{M} \]

or

\[ \nabla \times \left( \frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J} \]

• Similar to electric displacement \( D \), “magnetic field” \( H \) is introduced which does not depend upon magnetization of material

• Similar to dielectrics in most cases magnetization is proportional to magnetic field. Historically, susceptibility \( \chi \) is introduced in the following way:

\[ \vec{M} = \chi \vec{H} \]

or

\[ \vec{M} = \frac{\chi}{\mu_0 (1 + \chi)} \vec{B} \]

• Also relative magnetic permittivity \( \mu_R \):

\[ \vec{B} = \mu_0 \mu_R \vec{H} \]

\[ \mu_R = 1 + \chi \]
Magnetization-2

- Magnetic induction field is the same in a uniformly magnetized cylinder as in a solenoid.

- Bound currents flow only along the surfaces across which magnetization change (such as surface of the cylinder).

\[ B = \mu_0 M \]

\[ B = \mu_0 nI \]

Magnetization equals surface current density

\[ M = nI \]

\[ \left\{ \frac{A}{m} \right\} \]
Fundamental Postulates of Magnetostatics

- Ampere’s law in differential form
  \[ \nabla \times \vec{B} = \mu_0 \vec{J} \]

- No isolated magnetic charges
  \[ \nabla \cdot \vec{B} = 0 \]

\( \vec{B} \) is solenoidal
The Three Experimental Pillars of Electrodynamics

- Electric charges attract/repel each other as described by *Coulomb’s law*.
- Current-carrying wires attract/repel each other as described by *Ampere’s law of force*.
- Magnetic fields that change with time induce electromotive force as described by *Faraday’s law*. 
Faraday’s Experiment

- Upon closing the switch, current begins to flow in the primary coil.
- A momentary deflection of the compass needle indicates a brief surge of current flowing in the secondary coil.
- The compass needle quickly settles back to zero.
- Upon opening the switch, another brief deflection of the compass needle is observed.

- Faraday’s Law of Electromagnetic Induction: “The electromotive force induced around a closed loop $C$ is equal to the time rate of decrease of the magnetic flux linking the loop.”

\[ V_{ind} = - \frac{d\Phi}{dt} \equiv - \frac{d}{dt} \int_C \vec{B} \cdot d\vec{s} \]
Faraday’s Law of Electromagnetic Induction

\[ V_{\text{ind}} = -\frac{d\Phi}{dt} \equiv -\frac{d}{dt} \int_{S} \vec{B} \cdot d\vec{s} \]

- Electromotive force = potential difference

\[ V_{\text{ind}} = \oint_{C} \vec{E} \cdot d\vec{l} \]

- Integral form of Faraday’s law

\[ \oint_{C} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{S} \vec{B} \cdot d\vec{s} \]

- Using Stokes’ theorem:

\[ \oint_{C} \vec{E} \cdot d\vec{l} = \int_{S} \nabla \times \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_{S} \vec{B} \cdot d\vec{s} = -\int_{S} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \]

- Faraday law in differential form:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

- Faraday’s law states that a changing magnetic field induces an electric field.
- The induced electric field is non-conservative.