The feasibility of producing a photovoltaic module with a concentrator of solar radiation is considered. A geometrical concentration of 3.5–3.7, providing safe focal spot illumination, as well as thermal, energy design parameters of the module are described. The proposed module with the efficiency of solar cell density of 1000 W/m² allows one to obtain 82–84% cost of the established power at a level of 3 dollars/m² surface.
with a prismocon (Fig. 1). A prismocon represents a symmetrical prism made from an optically transparent material (glass or organic glass) with a reflecting layer on the external surface (broken line). The prismocon operates as follows: a light beam \( L \) falls on a receiving surface, passes into the prismocon, reflects from the back surface, where the reflecting layer is deposited, and comes to the receiving surface at an angle of the complete internal reflection. Beams remain in the prismocon and are distributed to the outlet surface, where the effect of the radiation concentration arises.

**Fig. 1.** Cross section of the prismocon: \( a \) and \( b \), respectively, the dimensions of the half of the inlet and outlet cross sections, cm; \( h_0 \), the height of the central part of the prismocon, cm; \( H \), the total height of the prismocon, cm; \( \beta \), the angle at the apex of the prismocon, deg.; \( \gamma \), the angle of inclination of the inner face of the prismocon, deg.; \( \delta \), angle of the beam maximum deflection from the axis of the prismocon \( \pm 24^\circ \) (deg. of arc).

The \( \beta \) and \( \gamma \) angles are selected so that with the deflection of a flow of solar beams from the axis of symmetry (the sighting direction to the Sun) within \( \beta = \pm 24^\circ \) (declination), the prismocon continues to catch radiation. So, when establishing the plane of the prismocon inlet by the direction to the south and under the angle of the latitude of the area to the horizon, it provides stationary operation the year round.

Knowing \( b \), \( \beta \), \( \delta \), and the refractive index \( n \) of the material from which the prismocon is manufactured, we can calculate all the parameters of the prismocon by the following formulas:

\[
\gamma = \arcsin \left( n \times \sin \left( \arcsin \frac{1}{n} - 2 \times \beta \right) \right) + \delta, \quad h_0 = b \times \tan \left( \arcsin \frac{1}{n} + \gamma - \frac{\pi}{2} \right),
\]

\[
a = \frac{b \times \sin \left( 2 \times \gamma + \arcsin \frac{1}{n} - \frac{\pi}{2} \right)}{\sin \beta \times \sin \left( \gamma + \arcsin \frac{1}{n} \right)}, \quad H = a \times \sin (\beta + \gamma). \tag{1}
\]

The geometrical concentration of the prismocon is calculated by the equation:

\[
K = \frac{a}{b} \tag{2}
\]

Calculations demonstrate that a maximum concentration obtained by equations (1) and (2) when \( n = 1.49 \) is reached with \( \beta = 8^\circ \) and is \( K = 3.18 \).

We can improve this result at the expense of the rise of the height \( h_0 \) of the central part of the prismocon. In doing so, insignificant losses of radiation arise and prismocons with the geometrical concentration \( K = 3.5 - 3.7 \) appear.

The real concentration of the prismocon is calculated by the equation:

\[
K_{\text{real}} = K \times \tau \times \cos \delta, \tag{3}
\]
where $K$ is the geometrical concentration and $\tau$ is the light transmission coefficient of the prismocon which is calculated by the equation:

$$\tau = (1 - \tau_1) \times \tau_2 \times \tau_3 \times (1 - \tau_4),$$  

(4)

where $\tau_1$ is the mean coefficient of the Fresnel reflection when beams pass inside the prism depending on the angles of incidence of radiation on the receiving faces of the prismocon; $\tau_2$ is the mean reflection factor of the reflecting layer including the reflection factor raised to the power of the mean amounts of reflections; $\tau_3$ is the mean transmission coefficient of the prismocon for radiation including the unit transmission coefficient of the material raised to the power of the beam path length in the optical material, and $\tau_4$ is the coefficient of possible admissible losses of radiation (arises at the expense of the rise of the height $h_0$ of the central part of the prismocon), allowing one to increase the concentration and to smooth the energy distribution at the prismocon outlet.

Table 1 lists values of the coefficients and the value of the concentration really reached at different angles of the beam deflection from the axis of the prismocon. In doing so, the geometrical concentration $K = 3.504$ is considered.

### Table 1. Values for the Light Transmission Coefficients and the Values for the Concentration at Different Angles of the Beam Deflection from the Axis of the Prismocon

<table>
<thead>
<tr>
<th>The angle $\delta$ of the beam deflection from the axis of prismocon, deg. of arc</th>
<th>24</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K \times \cos \delta$</td>
<td>3.201</td>
<td>3.292</td>
<td>3.384</td>
<td>3.450</td>
<td>3.490</td>
<td>3.503</td>
</tr>
<tr>
<td>$1 - \tau_1$</td>
<td>0.936</td>
<td>0.890</td>
<td>0.873</td>
<td>0.875</td>
<td>0.879</td>
<td>0.881</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>0.805</td>
<td>0.815</td>
<td>0.826</td>
<td>0.833</td>
<td>0.837</td>
<td>0.838</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>0.970</td>
<td>0.971</td>
<td>0.971</td>
<td>0.972</td>
<td>0.972</td>
<td>0.972</td>
</tr>
<tr>
<td>$1 - \tau_4$</td>
<td>0.950</td>
<td>0.961</td>
<td>0.970</td>
<td>0.976</td>
<td>0.980</td>
<td>0.981</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.690</td>
<td>0.677</td>
<td>0.680</td>
<td>0.691</td>
<td>0.700</td>
<td>0.704</td>
</tr>
<tr>
<td>$K_{\text{real}}$</td>
<td>2.222</td>
<td>2.228</td>
<td>2.298</td>
<td>2.385</td>
<td>2.445</td>
<td>2.466</td>
</tr>
</tbody>
</table>

![Fig. 2. Diagram for the radiation concentration distribution at the outlet of the prismocon.](image-url)
The energy distribution along the surface of the outlet close to the uniform distribution is an advantage of the prismocon compared to the Winston concentrator. The distribution of the energy concentration at the outlet of the prismocon depending on the angle of incidence of radiation for the beam deflection angles $\delta = 1^\circ$ and $24^\circ$ is depicted in Fig. 2. As is seen, even at the limiting declination angles $\delta$ the scatter in the values for the concentration is not over 4 which is ten or more times less than the corresponding scatter in Winston concentrators [3].

Small and comparatively uniform concentrations allow us to use the natural heat removal from solar cells to the environment at the expense of the back metal sheet (substrate), where solar cells are mounted, and to create stationary photovoltaic modules with a passive cooling system. Arrangement of this module is shown in Fig. 3. The photocell is in optical contact with the prismocon and in thermal contact with the aluminum substrate.

![Diagram of the photovoltaic module](image)

**Fig. 3. Scheme for the photovoltaic module:** 1, prism concentrator; 2, photocell; 3, aluminum substrate; 4, cover glass; h, substrate thickness, mm; 1 cm, photocell size; 3.5 cm, size of the input cross section of the prismocon; 4 cm, height of the prismocon.

We formulate the stationary problem of heat conduction for the photovoltaic module with the temperature determination on the solar cell as follows:

1. The photovoltaic module with the prismocon with $K = 3.5$; $n = 1.49$; $b = 0.5$ cm, and $a = 1.75$ cm;

2. The solar radiation density on the sensing surface $Q = 1000$ W/m$^2$;

3. The light transmission coefficient of the prismocon $\tau = 0.7$;

4. The energy distribution on the photocell (the gap $[-b, b]$) is uniform and the corresponding density of the radiation flux (with regard to the real concentration) is equal to $Q_{SC} = 2500$ W/m$^2$. The energy distribution outside the photocell (the gap $[b, a]$) is also uniform and the corresponding density of the heat flow equals $Q_h = 400$ W/m$^2$. The flow is formed from all the losses when the light passes through the prismocon. The density of the heat flow through the cover glass constitutes $Q_{gl} = 7 \times \Delta T$ [8];

5. The conversion efficiency of the radiation photocell is assumed to be equal to 12%.

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The density of the heat flow on the photocell after the conversion of part of the radiation to electricity is determined by the equation:

\[ Q_{\text{SCH}} = Q_{b} \times (1 - 0.12). \]

The stationary problem of heat conduction is divided into two parts. The first equation in the photocell region— \([-b, b]\) and the second equation—in the region free from the photocell—\([-b, 2 \times a-b]\). For reasons of the module symmetry, it will suffice to seek the solution of the problem in the gap \([0, a] = [0, b] \lor [b, a]\), which includes halves of corresponding regions. The problem will look like:

1. The equation in the \([0, b]\) region:
\[
\frac{d^2 \Delta T_1(x)}{dx^2} - C^2 \times \Delta T_1(x) = - \frac{Q_{\text{SCH}}}{\lambda \times h};
\]

2. The equation in the \([b, a]\) region:
\[
\frac{d^2 \Delta T_2(x)}{dx^2} - C^2 \times \Delta T_2(x) = - \frac{Q_{b}}{\lambda \times h};
\]

3. Conditions of continuity at point \(b\):
\[
\Delta T_1(b) = \Delta T_2(b); \quad \frac{d \Delta T_1}{dx} \bigg|_b = \frac{d \Delta T_2}{dx} \bigg|_b;
\]

4. Boundary conditions:
\[
\frac{d \Delta T_1}{dx} \bigg|_0 = 0; \quad \frac{d \Delta T_2}{dx} \bigg|_a = 0,
\]

\[
C^2 = \frac{\alpha}{\lambda \times h} \times \left( 1 + \frac{4 \times \varepsilon \times \sigma \times T_0}{\alpha} \right) + 7,
\]

where \(C^2\) is the heat transmission coefficient between the aluminum, cover glass, and air with regard to the linearization of the part connected with the radiation; \(\lambda\) is the thermal conductivity of the aluminum substrate; \(\alpha\) is the convective heat transfer coefficient between the aluminum and air; \(h\) is the aluminum substrate thickness; \(\varepsilon\) is the coefficient of the aluminum blackness; \(\sigma\) is the emission coefficient of the absolutely black body; \(T_0\) is the temperature of the outer air; \(\Delta T(x) = T_A1(x) - T_0\), where \(T_A1(x)\) is the aluminum substrate temperature.

The solution to the problem (5) and (6) is given as a sum total of the general solution with the hyperbolic functions and the partial solution of the differential equations

\[
\Delta T_1(x) = A_1 \times \sinh(C \times x) + B_1 \times \cosh(C \times x) + \frac{Q_{\text{SCH}}}{\lambda \times h \times C^2},
\]

\[
\Delta T_2(x) = A_2 \times \sinh(C \times x) + B_2 \times \cosh(C \times x) + \frac{Q_b}{\lambda \times h \times C^2},
\]

The coefficients are calculated by the substitution of equations (10) and (11) into conditions (7) and (8):
\[ A_1 = 0, \quad A_2 = \frac{(Q_h - Q_{SCH}) \times \text{sh}(C \times b)}{\lambda \times h \times C^2}, \]

\[ B_1 = \frac{(Q_{SCH} - Q_h) \times \left[ \text{sh}(C \times b) \times \text{ch}(C \times a) - \text{sh}(C \times a) \times \text{ch}(C \times b) \right]}{\lambda \times h \times C^2 \times \text{sh}(C \times a)}, \]

\[ B_2 = \frac{(Q_{SCH} - Q_h) \times \text{sh}(C \times b) \times \text{ch}(C \times a)}{\lambda \times h \times C^2 \times \text{sh}(C \times a)}. \]

The maximum temperature of the substrate by equations (10) and (11) at the air temperature \( T_0 = 30 \) °C and the absence of the wind is 75 °C. From the heat flow through the photocell zone, the photocell temperature is higher by 1° than the substrate temperature and is not over 76 °C.

The specific execution of the module allows us to obtain the following constructional parameters: the width of the solar cell \( 2b = 1 \) cm, the width of the input surface of the prismocon \( 2a = 3.5 \) cm, the height of the prismocon \( H = 4 \) cm, and the thickness of the aluminum substrate \( h = 3 \) mm.

The proposed module allows us to obtain 82–84 W of electric power from 1 m² of the sensing surface under standard conditions (\( E_0 = 1000 \) W/m², \( T_0 = 25 \) °C), the optical efficiency of the prismocon \( \tau = 0.68–0.7 \), and the SC efficiency \( \eta_{SC} = 0.12 \).

The estimated cost of the module constitutes 250 dollars/m², including the cost of the solar cells — 72.5 dollars (0.29 m² with the cost of 2.5 dollar/W), glass of solar quality — 10 dollars, organic glass of the prismocons — 134 dollars (9.2 kg), and the aluminum sheet — 5 dollars. The specific cost of the established power is $3 dollar/W with the existing prices of modules without concentrators from 4 to 5 dollar/W. With the SC efficiency of 15%, the cost drops to 2.5 dollar/W.

In summary, the feasibility of production of a photovoltaic module based on prism concentrators, operating the year round under steady conditions, with the mean geometrical concentration of 3.5, is shown. The cost of the established power is at a level of 3 dollar/W.

References

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