Abstracts of Poster Presentations

April 27, 2023

1. *Existence of Numerical Semigroups on Faces of Kunz Polyhedra*
Jessica Wang (joint work with Harper Niergarth, Levi Borevitz, and Daniel Pocklington)
Worcester Polytechnic Institute
jwang22@wpi.edu
A numerical semigroup is a cofinite subset of the non-negative integers that is closed under addition and contains zero. The Kunz Polyhedra, P_m, are a family of rational polyhedra whose integer points are in bijection with numerical semigroups with fixed smallest nonzero element m. Each face of P_m corresponds to a poset that describes the structure of the numerical semigroups found on that face. Interestingly, some faces of P_m do not have any integer points on them. In this work, we present conditions to classify which faces of P_m contain integer points by examining their poset. Based on these conditions, we provide an algorithm that locates an integer point on the face if any exist.

2. *Irreducible components of Hilbert scheme of points on non-reduced curves*
Yuze Luan
UC Davis
yuzluan@ucdavis.edu
We classify the irreducible components of the Hilbert scheme of n points on non-reduced algebraic plane curves. The irreducible components are indexed by partitions and all have dimension n.

3. *Characterizing some graphs which are recognizable by spectrum*
Emily Barranca
University of Rhode Island
emily_barranca@uri.edu
A graph is said to be determined by its spectrum (DS) if its adjacency spectrum is not shared by any nonisomorphic graphs. As an extension to the question of determining which graphs are DS, we say a graph G is recognizable by spectrum (RS) if any graph's spectrum determines if it contains G as an induced subgraph. We present examples of graphs which are RS and, in our attempt to characterize small RS graphs, we will share results focusing on trees and graphs with special cycles and cliques.

4. *Lattice Path Matroids and Ehrhart Theory*
Dania Morales
University of Kansas
dania.morales@ku.edu
Given an integer polytope, $P \subseteq \mathbb{R}^n$, the Ehrhart polynomial, is a fundamental invariant that counts the number of integer lattice points contained in all nonnegative integer dilations of P. We are interested in

computing the Ehrhart polynomial for the base polytope of a lattice path matroid. The Ehrhart polynomials for lattice path matroids of rectangle, hook and ribbon shapes are given by Stanley/Katzman, Ferroni and Knauer-Sandoval-Alfonsín, respectively. We give an inclusion-exclusion formula for the Ehrhart polynomial for a lattice path matroid of any skew shape in terms of ribbon shapes. We then give the Ehrhart polynomial for any ribbon shape by considering its h^*-polynomial.

5. *The Characteristic Polynomial and Its Applications*
Kate Howell and Rongwei Yang
University at Albany
khhowell@albany.edu
Given matrices $A_1, ..., A_n$, the characteristic polynomial is defined as $\det(z_0 I + z_1 A_1 + ... + z_n A_n)$. We consider the braid group B_4 with respect to the Burau representation through the lens of the characteristic polynomial in hopes to better understand the open faithful problem. We also prove that the characteristic polynomial is a complete unitary invariant for pairs of projection matrices.

6. *A partially commutative generalization of the immaculate functions and their duals*
Spencer Daugherty
North Carolina State University
sdaughe@ncsu.edu
We define a partially commutative generalization of the immaculate and dual immaculate functions following Doliwa's notion of colored NSym and QSym. The colored dual immaculate functions form a basis of colored QSym and have positive expansions into the colored monomial functions and colored fundamental functions. We also give expansions of the colored complete homogeneous and colored ribbon bases into the colored immaculate basis. These results extend to the row-strict immaculate functions and their dual.

7. *Recursive local amoeba construction*
Denae Ventura, Laura Eslava, Adriana Hansberg and Tonatiuh Matos
Institute of Mathematics UNAM
dventura@im.unam.mx
Amoeba graphs were born as examples of balanceable graphs, which are graphs that appear in any 2-edge coloring of the edges of a large enough K_n with a sufficient amount of red and blue edges. As they were studied further, interesting aspects were found. An edge-replacement $e \to e$ in a labeled graph G means to take an edge $e \in E(G)$ and replace it with $e' \in E(\overline{G}) \cup \{e\}$. If $G - e + e'$ is isomorphic to G then we say $e \to e'$ is a *feasible edge-replacement*. Every edge-replacement yields a set of permutations of the labels in G. The set of all permutations associated with all feasible edge-replacements in G generates the group $fer(G)$. A graph G of order n is a *local amoeba* if $fer(G) \cong S_n$ and a *global amoeba* if $fer(G \cup tK_1) \cong S_{n+t}$, for a large enough t. One might think local and global amoebas are hard to find. However, in this poster we will go over a recursive construction of infinite families of local amoebas.

8. *Arithmetical structures on bidents*
Kassie Archer, Abigail C. Bishop, Alexander Diaz-Lopez, Luis D. Garcia Puente, Darren Glass and Joel Louwsma
Niagara University
jlouwsma@niagara.edu
An arithmetical structure on a finite, connected graph without loops is given by an assignment of

positive integers to the vertices such that, at each vertex, the integer there is a divisor of the sum of the integers at adjacent vertices, counted with multiplicity if the graph is not simple. Associated to each arithmetical structure is a finite abelian group known as its critical group. We study arithmetical structures and their critical groups on bidents, which are graphs consisting of a path with two "prongs" at one end. We give a process for determining the number of arithmetical structures on the bident with n vertices and show that this number grows at the same rate as the Catalan numbers as n increases. We also completely characterize the groups that occur as critical groups of arithmetical structures on bidents.

9. *Minimal skew SSYT and the Hillman-Grassl correspondence*
Alejandro Morales, Greta Panova, GaYee Park
University du Quebec a Montreal
`park.ga_yee@uqam.ca`
Standard tableaux of skew shape are fundamental objects in enumerative and algebraic combinatorics and no product formula for the number is known. Naruse presented a formula as a positive sum over excited diagrams of products of hook-lengths. Shortly after, Morales, Pak, and Panova gave a q-analogue of Naruse's formula for semi-standard tableaux of skew shapes in terms of restricted excited arrays. They also showed, partly algebraically, that the Hillman-Grassl map restricted to skew shapes is the bijection between skew SSYTs and excited arrays. We study the problem of making this argument completely bijective. For a skew shape, we define a new set of semi-standard Young tableaux, called the minimal SSYT, that are equinumerous with excited diagrams via a new description of the Hillman–Grassl bijection and have a version of excited moves. The minimal skew SSYT are the natural objects to compare with the terms of the Okounkov-Olshanski formula for counting SYT of skew shape. We prove that the number of summands in the Okounkov-Olshanski formula is larger than the excited diagrams in NHLF.

10. *Frames for signal processing on Cayley graphs*
Kathryn Beck, Mahya Ghandehari, Skyler Hudson and Jenna Paltenstein
University of Delaware
`kbeck@udel.edu`
The spectral decomposition of graph adjacency matrices is an essential ingredient in the design of graph signal processing (GSP) techniques. When the adjacency matrix has multi-dimensional eigenspaces, it is desirable to base GSP constructions on a particular eigenbasis (the 'preferred basis'). We provide an explicit and detailed representation-theoretic account for the spectral decomposition of the adjacency matrix of a Cayley graph, which results in a preferred basis. Our method applies to all (not necessarily quasi-Abelian) Cayley graphs, and provides descriptions of eigenvalues and eigenvectors based on the coefficient functions of the representations of the underlying group. Next, we use such bases to build frames that are suitable for developing signal processing on Cayley graphs. These are the Frobenius–Schur frames and Cayley frames, for which we provide a characterization and a practical recipe for their construction.

11. *2-Caps in the Game of EvenQUADS*
Felicia Flores and Darrion Thornburgh
Bard College
`ff0811@bard.edu`
We studied EvenQUADS, a variant of the popular card game SET®. A Quad is 4 cards that satisfy a certain pattern. Our goal was to find and classify collections of cards that don't contain a Quad,

called 2-caps. In particular, for each k, we classified 2-caps that contain k distinct triples of cards in the 2-cap that determine the same fourth card. This game is modeled by the affine geometry $AG(n,2)$, allowing us to study this problem in higher dimensions.

12. *Large scale geometry of graphs of polynomial growth*
Anton Bernshteyn and Jing Yu
Georgia Institute of Technology
`jingyu@gatech.edu`
In 1995, Levin and Linial, London, and Rabinovich conjectured that every connected graph G of polynomial growth admits an injective homomorphism to the n-dimensional grid graph for some n. Moreover, they conjected that if every ball of radius r in G contains at most $O(r^\rho)$ vertices, then one can take $n = O(\rho)$. The first part of the conjecture was proved by Krauthgamer and Lee in 2007. However, the second part of the conjecture is false, and the best possible upper bound on n is $O(\rho \log \rho)$. Prompted by these results, Papasoglu asked whether a graph G of polynomial growth admits a coarse embedding into a grid graph. As I will explain in this talk, the answer to Papasoglu's question is ""yes."" Moreover, it turns out that the dimension of the grid graph only needs to be linear in the asymptotic growth rate of G, which confirms the original Levin–Linial–London–Rabinovich conjecture ""on the large scale."" Besides, we find an alternative proof of the result of Papasoglu that graphs of polynomial growth rate $\rho < \infty$ have asymptotic dimension at most ρ. Furthermore, our proof works in the Borel setting and shows that Borel graphs of polynomial growth rate $\rho < \infty$ have Borel asymptotic dimension at most ρ, and hence they are hyperfinite, which answers a question of Marks. This is joint work with Anton Bernshteyn.

13. *Combinatorial Formulas for the Equivariant Cohomology of Peterson Varieties*
Swan Klein and Quincy Frias; Advisor is Dr. Rebecca Goldin
George Mason University
`hklein2@gmu.edu`
Our goal was to verify a conjecture about the decomposition of the restriction of Schubert classes associated with transpositions to the Peterson variety into a linear combination of Peterson classes. Using a corollary of the AJS/Billey formula, we reduced the conjecture to a more concise combinatorial question about counting reduced words for transpositions embedded into long words. We uncovered an elegant visual framework for understanding these combinatorial questions and proved our conjecture in a specific subcase. With future work, we hope to prove the remaining cases of the conjecture and extend our combinatorial strategy to as many types of Schubert classes as possible.

14. *Local Rational Formulae for Computing Topological Invariants*
Mattie Ji and Kexin Ding
Brown University
`mattie_ji@brown.edu`
Many rational invariants of characteristics classes (such as the Euler characteristics and Pontryagin numbers) can be computed with a universal "local combinatorial formula". This idea was first given by Gaifullin to compute the first rational Pontryagin classes of a combinatorial manifold that depended only on the link of each vertex in its triangulation.

15. *On the shifted Littlewood-Richardson coefficients and the Littlewood-Richardson coefficients*
Khanh Nguyen Duc
University at Albany

khanh.mathematic@gmail.com
We give a new interpretation of the shifted Littlewood-Richardson coefficients $f^{\nu}_{\lambda\mu}$ (λ, μ, ν are strict partitions). The coefficients $g_{\lambda\mu}$ which appear in the decomposition of Schur Q-function Q_λ into the sum of Schur functions $Q_\lambda = 2^{l(\lambda)} \sum_{\mu} g_{\lambda\mu} s_\mu$ can be considered as a special case of $f^{\nu}_{\lambda\mu}$ (here λ is a strict partition of length $l(\lambda)$). We also give another description for $g_{\lambda\mu}$ as the cardinal of a subset of a set that counts Littlewood-Richardson coefficients $c^{\tilde{\lambda}}_{\mu^t\mu}$. This new point of view allows us to establish connections between $g_{\lambda\mu}$ and $c^{\tilde{\lambda}}_{\mu^t\mu}$. More precisely, we prove that $g_{\lambda\mu} = g_{\lambda\mu^t}$, and $g_{\lambda\mu} \leq c^{\tilde{\lambda}}_{\mu^t\mu}$. We conjecture that $g^2_{\lambda\mu} \leq c^{\tilde{\lambda}}_{\mu^t\mu}$ and formulate some conjectures on our combinatorial models which would imply this inequality if it is valid.

16. *Combinatorial models for type A specialized non-symmetric Macdonald polynomials and affine Demazure crystals*
Sam Spellman and Cristian Lenart
University at Albany
sspellman@albany.edu
The non-symmetric Macdonald polynomials are a family of orthogonal polynomials with parameters q and t. There are two well-known combinatorial models for computing these polynomials: a tableau model in type A, due to Haglund, Haiman and Loehr, and the type-independent model due to Ram and Yip, based on alcove walks. In this talk, we establish a bijection between these two models in the case $t = 0$ (in type A). Furthermore, we construct a crystal structure on alcove walks, and we conjecture that it is compatible with Assaf's crystal on semistandard key tabloids via our bijection; these crystals are colored directed graphs encoding the structure of certain affine Demazure modules. The bijection has applications to translating calculations from the alcove model (involving general concepts) to the simpler tableau model.