Math 220 - Rank and Nullity - Fall 2000

1. Find a basis for Null(A) where

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 & 0 \\ -1 & 0 & -4 & 3 & -1 \\ 2 & -1 & 6 & 0 & 1 \\ -1 & 2 & 0 & -1 & 1 \end{bmatrix}$$

Find the dimension of Null(A).
2. The matrix A takes a vector in $\mathbb{R}^5$ and returns a vector in what space?

Definition: the **nullity** of a matrix A is the dimension of Null(A).
Definition: The **Column space** of A (Col(A)), is the space spanned by the columns of the matrix A. The **rank** of A is the dimension of Col(A).The **Row space** of A is the space spanned by the rows of A.

3. For the above matrix, find a basis for Col(A) and the rank of A.
4. Let e_1, e_2, e_3, e_4, e_5 be the standard basis of $\mathbb{R}^5$. Find Ae_1, Ae_2, Ae_3, Ae_4 and Ae_5. What did you get? What is span($Ae_1, Ae_2, Ae_3, Ae_4, Ae_5$)?

Theorem 1. *dim Col(A) = dim Row (A)*

This is because dim(Col A) is the number of pivot colums of A, which is the same as the number of pivot rows of A.

Corollary 2. *A and A^t have the same rank.*

Theorem 3. *(Rank Theorem) Rank(A)+Nullity(A) = number of columns of A.*

5. Find basis for Null(A) and Col(A), and verify the Rank Theorem for

$$A = \begin{bmatrix} 1 & 2 & 2 & -1 \\ 1 & 3 & 1 & -2 \\ 1 & 1 & 3 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

Homework Sec 4.6: Read Section 4.6, do problems: 1–9 odds, 19–23 odds, 31, 33, 37, 39, 41, 45, 52, 53.