Math 220 - Change of Basis - Fall 2000

Suppose you have two bases for the same vector space:

$$\beta = \{v_1, v_2, \ldots, v_n\}, \quad \beta' = \{v_1', v_2', \ldots, v_n'\}$$

Each vector in the first basis has a different name in the second basis. Let $P_{\beta,\beta'}$ be the matrix whose columns are these names

$$P_{\beta,\beta'} = [[v_1]_{\beta'}, [v_2]_{\beta'}, \ldots, [v_n]_{\beta'}]$$

The is called the **change of basis** (from basis β to basis β') matrix.

If v is any other vector, for which you know the name in the basis β, then the name of v in the basis β' is obtained by doing:

$$[v]_{\beta'} = P_{\beta,\beta'}[v]_\beta$$

1. Let β the standard basis of $\mathbb{R}^2$, Let $\beta' = \{(1,1),(1,-1)\}$.
   1. Compute the change of basis matrix from β to β'.
   2. Compute the change of basis matrix from β' to β.
   3. Let $v = (4,-2)$. Find $[v]_\beta$.
   4. Verify $[v]_{\beta'} = P[v]_\beta$.

2. Let β the standard basis of P_3, Let $\beta' = \{1, x, -\frac{1}{2} + \frac{3}{2}x^2, -\frac{3}{2}x + \frac{5}{2}x^3\}$.
   1. Compute the change of basis matrix from β to β'.
   2. Compute the change of basis matrix from β' to β.
   3. Let $p = x - x^2 + x^3$. Find $[v]_{\beta'}$.
   4. Let p be the polynomial $[p]_{\beta'} = (3,2,4,2)$. Find $[p]_\beta$.

Note 1: change of basis matrix from β' to β is the **inverse** of change of basis matrix from β to β'.

$$P_{\beta,\beta'} = P_{\beta',\beta}^{-1}$$

Note 2. If β' = standard basis, then

$$P_{\beta,\beta'} = [v_1, v_2, \ldots, v_3]$$

Note 3. If β and β' are any given bases, then, $P_{\beta,\beta'}$ is the last n–columns of

$$\text{rref}\left(\underbrace{[v_1', v_2', \ldots, v_n']}_{\beta'} \mid \underbrace{v_1, v_2, \ldots, v_n]}_{\beta}\right)$$

Homework: Read section 4.5. Do problems 1–19 odds.