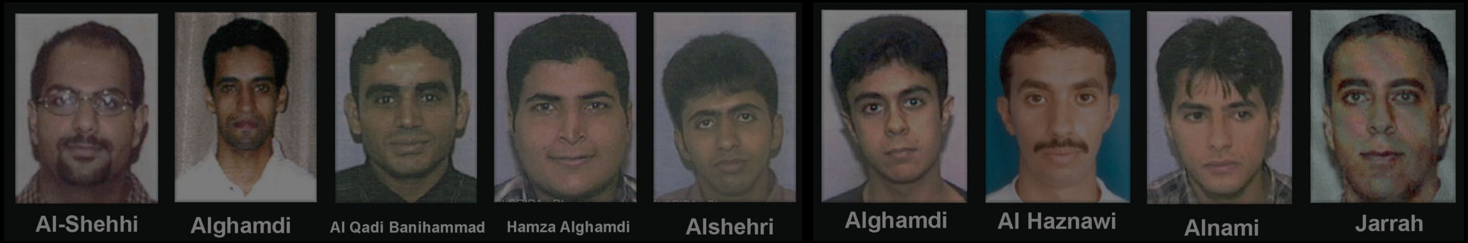
Ranking terrorists in networks:

A sensitivity analysis of Al Qaeda's 9/11 attack

By Bart Husslage, Peter Borm, Twan Burg, Herbert Hamers, & Roy Lindelauf

(2015)

Presentation by Paul Tomchik for CS 660



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- Peter Borm
  - Professor in Mathematics and Game Theory at Tilburg University
- Twan Burg
  - Tilburg University
- Herbert Hamers
  - Professor in Game Theory and Operations Research at Tilburg University
- Roy Lindelauf
  - Military Operations Researcher at Netherlands Defense Academy

Bart Husslage

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Peter Borm

Peter E. M. Borm

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- [j62] Soesja Grundel, Peter Borm, Herbert Hamers: **Resource allocation games: a compromise stable extension of bankruptcy games.** Math. Meth. of OR 78(2): 149-169 (2013)
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1010697	Burg, Twan			
Federation	Netherlands			
FIDE title	Grand Master			
Rating	std. 2500	rapid Not rated	blitz Not rated	
Online Rating	bullet. 2477	rapid 2507	blitz 2469	
B-Year	1990			
Sex	Male			

Profile | Rating Progress | Game Statistics

View Games of a Player online

World Rank	
World (all players):	**936**
World (active players):	**775**
National Rank NED	
National (all players):	**24**
National (active players):	**21**
Continent Rank Europe	
Rank (all players):	**710**
Rank (active players):	**604**
FIDE Titles (year)	
Grand Master (GM)	**2015**
International Master (IM)	**2010**
FIDE Master (FM)	**2006**

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[j42] Herbert Hamers, Silvia Miquel, Henk Norde:
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Roy Lindelauf

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Overview: Traditional Approach to Terrorist Network Analysis

- Intelligence is viewed and analyzed as network data.
  - Members of networks -> nodes
  - Their interactions -> (weighted) edges
- Analysts use standard centrality measures (degree, closeness, and betweenness) to find key network members.
- Results are used in decision support systems.

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- *These methods take only this network structure into account.*

Intelligence gathered often includes information regarding the nature of relations between members and their individual skills/resources.

Overview: This paper’s contributions

- Introduces a new game theoretic centrality measure for ranking players in a terrorist network. (Elaborating Linderlauf’s earlier work.)
- This centrality measure incorporates individual and coalitional characteristics, such as special skills, and relational characteristics, like frequency of communication.
- This measure takes into account the operational strength of connected subnetworks, potentially providing a more suitable model of real-life networks.
- The robustness of the rankings is tested by performing a sensitivity analysis on the rankings of the terrorists in the 9/11 attack.

Overview: Roy Lindelauf’s earlier work

“**Context specific cooperative coalitional games** are defined that reflect the situation at hand *taking all available information about the network structure and the individual members and their relations into account.*

Next, the **Shapley value** is computed for the corresponding game to *measure the importance of members of the network in order to construct a ranking of these members.*

For each threat context a specific suitable game can be developed.”

(Bold face and italics added by presenter for emphasis.)

Cooperative/Coalitional Game Theory

- “An approach to modeling strategic situations that stands in contrast with Noncooperative Game Theory.”
- The essential difference is the *basic modeling unit*.

Cooperative/Coalitional Game Theory

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- The essential difference is the *basic modeling unit*.

Coalitional Game Theory

The basic modeling unit is the group and what they can accomplish.

Noncooperative Game Theory

The basic modeling unit is the individual and what he/she can accomplish *"playing a lone hand."*

Cooperative/Coalitional Game Theory

- Given a set of agents, a coalitional game defines how well each group (or coalition) of agents can do for itself.
- NOT concerned with how agents make choices within a coalition, or how they coordinate.

Cooperative/Coalitional Game Theory

Definition (Coalitional game with transferable utility)

A coalitional game with transferable utility is a pair (N, v), where

- N is a finite set of players, indexed by i; and
- $v : 2^N \mapsto \mathbb{R}$ associates with each coalition $S \subseteq N$ a real-valued payoff $v(S)$ that the coalition's members can distribute among themselves. We assume that $v(\emptyset) = 0$.

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- $\mathcal{V}$, called the "*characteristic function*", acts as utility function for a coalitional game.

http://www.game-theory-class.org/

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- v, called the "*characteristic function*", acts as utility function for a coalitional game.
- Says: "For every subset of the players, S, that could form in the game, what is the payoff $v(S)$ that the coalition can achieve?"

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Cooperative/Coalitional Game Theory

The questions we use coalitional game theory to answer:

- Which coalitions will form?
- How should that coalition divide its payoff amongst its members?

Cooperative/Coalitional Game Theory

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- Which coalitions will form?
- How should that coalition divide its payoff amongst its members?

Usually, all agents will agree to act together. However, sometimes this depends on how the coalition would divide its payoff amongst its members.

Cooperative/Coalitional Game Theory

The questions we use coalitional game theory to answer:

- How should that coalition divide its payoff amongst its members
  - in order to be fair?
  - in order to be stable?

Cooperative/Coalitional Game Theory

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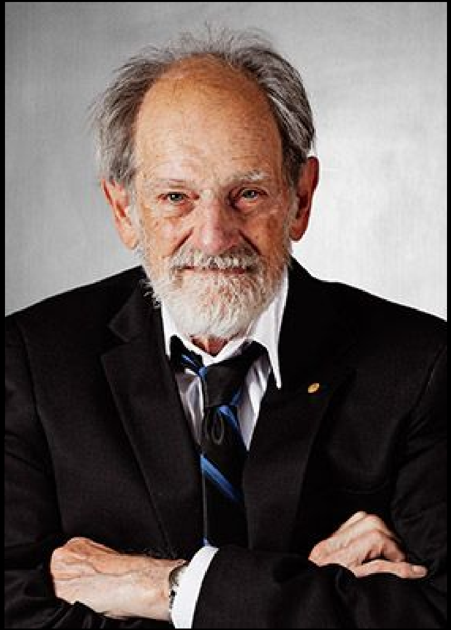
Shapley Value

One of the most prominent ways of dividing up the value of a set of individuals amongst its members.

Lloyd Shapley

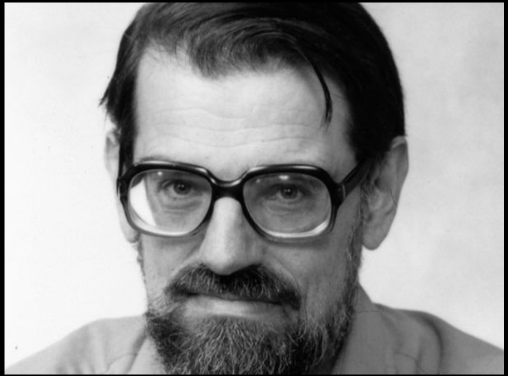
Lloyd Shapley has dominated game theory for the thirty-seven years since von Neumann and Morgenstern published their path-breaking book, *The Theory of Games and Economic Behavior*.

--**John von Neumann Theory Prize Citation**



Awards and honors [edit]

- Bronze Star, U.S. Army Air Corps, 1944
- Procter Fellow, Princeton University, 1951–52
- Fellow, Econometric Society, 1967
- Fellow, American Academy of Arts and Sciences, 1974
- Member, National Academy of Sciences, 1979
- John von Neumann Theory Prize, 1981
- Honorary Ph.D., Hebrew University of Jerusalem, 1986
- Fellow, INFORMS (Institute for Operations Research and the Management Sciences), 2002
- Distinguished Fellow, American Economic Association, 2007
- Fellow, American Mathematical Society, 2012[7]
- Sveriges Riksbank Nobel Memorial Prize in Economic Sciences, 2012
- Golden Goose Award, 2013[8]



(Screencapture from Wikipedia.)

Shapley Value

One of the most prominent ways of dividing up the value of a set of individuals amongst its members.

A solution to the question:

"What is a fair way to divide a coalition's payout?"

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“What is a fair way to divide a coalition’s payout?”

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Shapley’s idea:

Members should receive payments or shares *proportional to their marginal contributions.*

Shapley Value

What does a person contribute when we add them to a group?

Their share of the group's values should reflect what they contribute to the group's value.

Members should receive payments or shares *proportional to their marginal contributions.*

Shapley Value

$$\phi_i(N,v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N|-|S|-1)!\Big[v(S \cup \{i\}) - v(S)\Big]$$

Members should receive payments or shares *proportional to their marginal contributions.*

http://www.game-theory-class.org/

Shapley Value

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Shapley Value

Summation over all possible coalitions that could be created from N without i.

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Marginal contribution

What does individual i add to coalitions that do not already include i?

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The number of ways we could have build the coalition before adding i.

Members should receive payments or shares *proportional to their marginal contributions.*

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The number of ways we could have build the coalition before adding i.

The number of ways that we could add the individuals who have yet to be added after i.

Members should receive payments or shares *proportional to their marginal contributions.*

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Shapley Value

Summation over all possible coalitions that could be created from N without i.

Marginal contribution

What does individual i add to coalitions that do not already include i?

$$\phi_i(N, v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N| - |S| - 1)! \left[v(S \cup \{i\}) - v(S) \right]$$

All the possible ways we could have done this.

The number of ways we could have build the coalition before adding i.

The number of ways that we could add the individuals who have yet to be added after i.

Members should receive payments or shares ***proportional to their marginal contributions.***

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Overview: Roy Lindelauf's earlier work

"**Context specific cooperative coalitional games** are defined that reflect the situation at hand *taking all available information about the network structure and the individual members and their relations into account.*

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A new game theoretic centrality measure

Preliminary definitions:

A coalition $S \subseteq N$ is called *a connected coalition* if the network G□ is connected, otherwise S is called *disconnected*.

In *monotonic weighted connectivity games* the effectiveness of a disconnected coalition is determined by the most effective connected subcomponent.

A new game theoretic centrality measure

The idea is to create a game that takes into account the structure of the network

$$G = (N, E)$$

individual strengths (e.g., special skills) of the members of the network

$$\mathcal{I} = \{w_i\}_{i \in N} \text{ with } w_i \geq 0$$

as well as the relational strength (e.g., communication) between members of the network.

$$\mathcal{R} = \{k_{ij}\}_{ij \in E} \text{ with } k_{ij} \geq 0$$

A new game theoretic centrality measure

We define a *monotonic weighted connectivity game* (N, v^{mwconn}) with respect to network $G = (N, E)$ based on $\mathcal{I}$ and $\mathcal{R}$ in the following way:

For a connected coalition S we define

$$v^{\text{mwconn}}(S) = \begin{cases} f(S, \mathcal{I}, \mathcal{R}) & \text{if } |S| > 1, \\ 0 & \text{otherwise,} \end{cases}$$

where f is a context specific and tailor-made non-negative function that measures the effectiveness of coalitions in the network.

A new game theoretic centrality measure

We define a *monotonic weighted connectivity game* (N, v^{mwconn}) with respect to network $G = (N, E)$ based on $\mathcal{I}$ and $\mathcal{R}$ in the following way:

For a disconnected coalition S we define

$$v^{\text{mwconn}}(S) = \max_{T \subset S,\; T \text{connected}} v^{\text{mwconn}}(T)$$

A new game theoretic centrality measure

The game theoretic centrality measure C^{m} of member i

in network $G = (N, E)$ based on $\mathcal{I}$ and $\mathcal{R}$ is defined by

$$C^{\mathrm{m}}(i) = \varphi_i(v^{\mathrm{mwconn}})$$

where $\varphi_i(v^{\mathrm{mwconn}})$ is the **Shapley value** of member i in the game.

The corresponding ranking of all members in N is denoted by R^{m}.

An example coalition effectiveness function

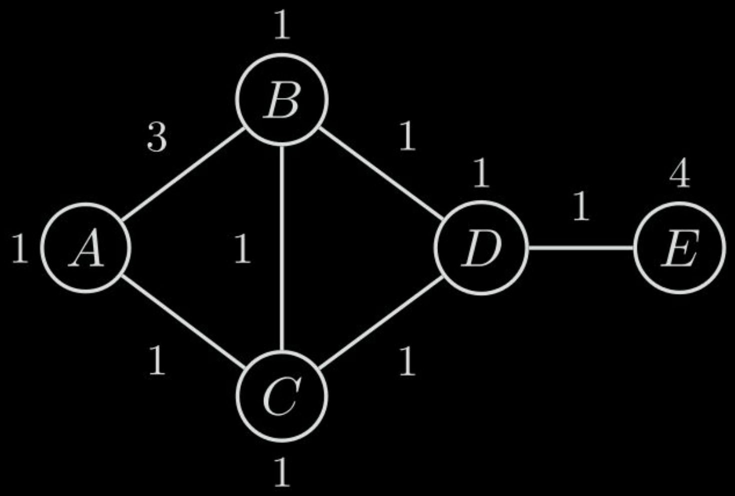
$$f(S, \mathcal{I}, \mathcal{R}) = \left(\sum_{i \in S} w_i \right) \cdot \max_{ij \in E_S} k_{ij}.$$

This specific choice can be motivated for terrorist cells in which we need to focus on the total operational strength of the cell as well as the most prominent line of communication between members.

(This function is used throughout the paper in the analyses.)

Example

$$f(S, \mathcal{I}, \mathcal{R}) = \left(\sum_{i \in S} w_i\right) \cdot \max_{ij \in E_S} k_{ij}.$$



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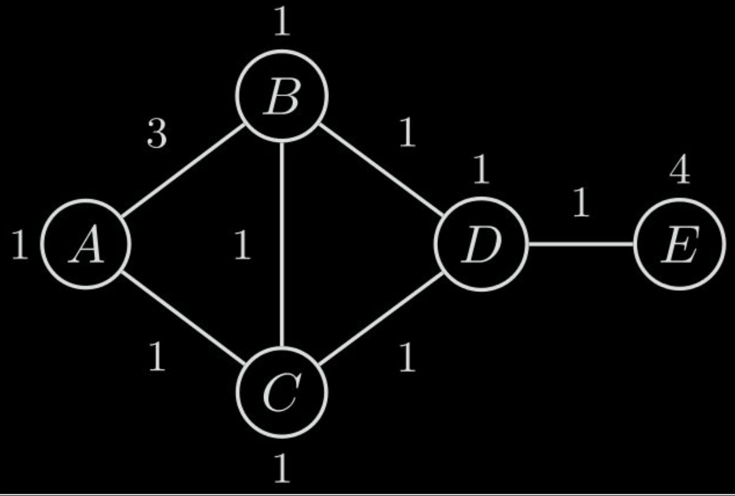


The value $v^{\text{mwconn}}(S)$ for each coalition

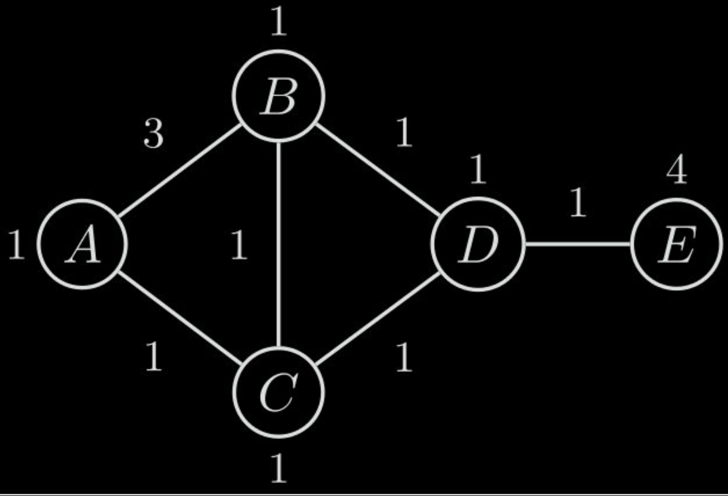
Coalition S	$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{D\}$	$\{E\}$	$\{A, B\}$
$v^{\text{mwconn}}(S)$	0	0	0	0	0	0	6
Coalition S	$\{A, C\}$	$\{A, D\}$	$\{A, E\}$	$\{B, C\}$	$\{B, D\}$	$\{B, E\}$	$\{C, D\}$
$v^{\text{mwconn}}(S)$	2	0	0	2	2	0	2
Coalition S	$\{C, E\}$	$\{D, E\}$	$\{A, B, C\}$	$\{A, B, D\}$	$\{A, B, E\}$	$\{A, C, D\}$	$\{A, C, E\}$
$v^{\text{mwconn}}(S)$	0	5	9	9	0	3	2
Coalition S	$\{A, D, E\}$	$\{B, C, D\}$	$\{B, C, E\}$	$\{B, D, E\}$	$\{C, D, E\}$	$\{A, B, C, D\}$	$\{A, B, C, E\}$
$v^{\text{mwconn}}(S)$	5	3	2	6	6	12	9
Coalition S	$\{A, B, D, E\}$	$\{A, C, D, E\}$	$\{B, C, D, E\}$	$\{A, B, C, D, E\}$			
$v^{\text{mwconn}}(S)$	21	7	7	24			

Example

$$f(S, \mathcal{I}, \mathcal{R}) = \left(\sum_{i \in S} w_i \right) \cdot \max_{ij \in E_S} k_{ij}.$$

The centrality measure C^m for the example

Member	Centrality measure C^m
A	6.1667
B	6.4167
C	1.7500
D	5.5833
E	4.0833



The value $v^{\text{mwconn}}(S)$ for each coalition

Coalition S	$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{D\}$	$\{E\}$	$\{A, B\}$
$v^{\text{mwconn}}(S)$	0	0	0	0	0	0	6
Coalition S	$\{A, C\}$	$\{A, D\}$	$\{A, E\}$	$\{B, C\}$	$\{B, D\}$	$\{B, E\}$	$\{C, D\}$
$v^{\text{mwconn}}(S)$	2	0	0	2	2	0	2
Coalition S	$\{C, E\}$	$\{D, E\}$	$\{A, B, C\}$	$\{A, B, D\}$	$\{A, B, E\}$	$\{A, C, D\}$	$\{A, C, E\}$
$v^{\text{mwconn}}(S)$	0	5	9	9	0	3	2
Coalition S	$\{A, D, E\}$	$\{B, C, D\}$	$\{B, C, E\}$	$\{B, D, E\}$	$\{C, D, E\}$	$\{A, B, C, D\}$	$\{A, B, C, E\}$
$v^{\text{mwconn}}(S)$	5	3	2	6	6	12	9
Coalition S	$\{A, B, D, E\}$	$\{A, C, D, E\}$	$\{B, C, D, E\}$	$\{A, B, C, D, E\}$			
$v^{\text{mwconn}}(S)$	21	7	7	24			

Sensitivity analysis of Al Qaeda’s 9/11 attack

A sensitivity analysis is run to investigate the robustness of the ranking obtained.

- To model individual strength, the data on individuals are expressed as weights on the nodes of the network.

- Not all interactional data between members may be completely known.

- Finally, some of the interactions between members may be considered more important than others.

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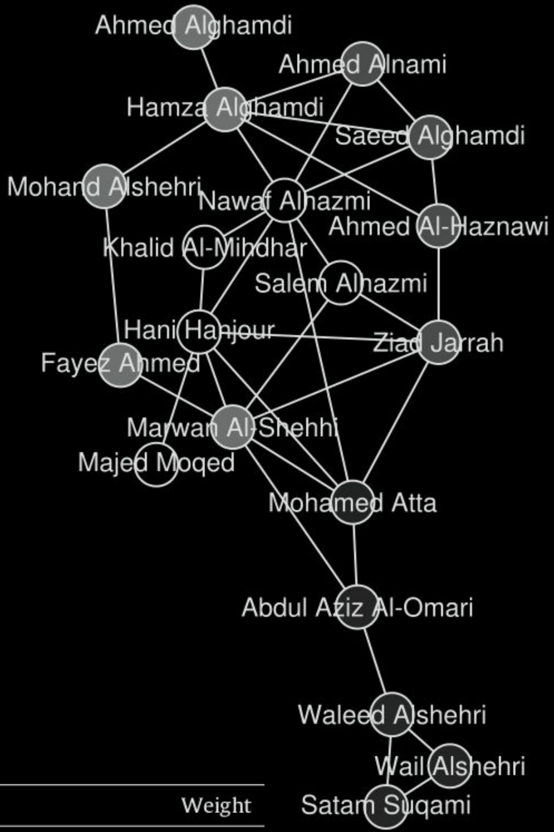
Sensitivity analysis of Al Qaeda's 9/11 attack



Weight assigned to each hijacker of Al Qaeda's 9/11 attack.

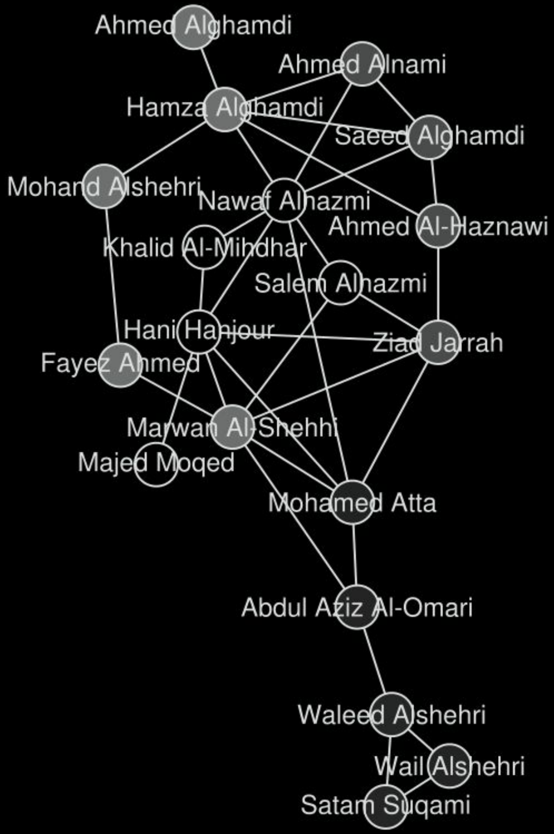
Hijacker	Weight	Hijacker	Weight
Ahmed Alghamdi	1	Nawaf Alhazmi	2
Hamza Alghamdi	1	Khalid Al-Mihdhar	3
Mohand Alshehri	1	Hani Hanjour	1
Fayez Ahmed	1	Majed Moqed	1
Marwan Al-Shehhi	3	Mohamed Atta	4
Ahmed Alnami	1	Abdul Aziz Al-Omari	1
Saeed Alghamdi	1	Waleed Alshehri	1
Ahmed Al-Haznawi	1	Satam Suqami	1
Ziad Jarrah	4	Wail Alshehri	1
Salem Alhazmi	1		

Description indicator	Example(s)	Hijacker(s)	Weight
Attending meetings on terror attack planning	Kuala Lumpur meeting January 2000	Nawaf Al-Hazmi Khalid Al-Midhar	+1
Signs of radicalization	Antisemitic and anti-American speech, talk about jihad and martyrdom, writing a will	Mohamed Atta Marwan Al-Shehhi Ziad Jarrah	+1
Affiliations	Al-Quds mosque Hamburg	Mohamed Atta Ziad Jarrah	+1
Accomplice to previous attacks	Attack on USS Cole	Khalid Al-Midhar	+1
Attending terrorist training camps	Traveling to training camps in Pakistan and Afghanistan	Mohamed Atta Marwan Al-Shehhi Ziad Jarrah	+1

Sensitivity analysis of Al Qaeda's 9/11 attack

Rankings for Al Qaeda's 9/11 network based on game theoretic and standard centrality.

Mwconn	Degree	Betweenness	Closeness
M. Atta	N. Alhazmi	N. Alhazmi	N. Alhazmi*
Z. Jarrah	M. Al-Shehhi*	A. Aziz Al-Omari	M. Atta*
M. Al-Shehhi	H. Alghamdi*	M. Atta	M. Al-Shehhi•
N. Alhazmi	H. Hanjour*	M. Al-Shehhi	H. Hanjour•
H. Hanjour	M. Atta•	Wd. Alshehri	Z. Jarrah
K. Al-Midhar	Z. Jarrah•	H. Alghamdi	H. Alghamdi◇
A. Aziz Al-Omari	S. Alghamdi	H. Hanjour	S. Alhazmi◇
H. Alghamdi	A. Aziz Al-Omari◇	Z. Jarrah	A. Aziz Al-Omari
Wd. Alshehri	Wd. Alshehri◇	F. Ahmed	S. Alghamdi
A. Al-Haznawi	A. Al-Haznawi◇	M. Alshehri	A. Al-Haznawi
S. Alhazmi	S. Alhazmi◇	A. Al-Haznawi	F. Ahmed⋆
F. Ahmed	A. Alnami◇	S. Alhazmi	A. Alnami⋆
S. Alghamdi	F. Ahmed⋆	S. Alghamdi*	K. Al-Midhar
M. Alshehri	M. Alshehri⋆	A. Alnami*	M. Alshehri
A. Alnami	K. Al-Midhar⋆	K. Al-Midhar*	M. Moqed
M. Moqed	S. Suqami⋆	S. Suqami*	Wd. Alshehri
A. Alghamdi	W. Alshehri⋆	W. Alshehri*	A. Alghamdi
S. Suqami*	A. Alghamdi°	A. Alghamdi*	W. Alshehri°
W. Alshehri*	M. Moqed°	M. Moqed*	S. Suqami°



Sensitivity analysis of Al Qaeda's 9/11 attack

Performing a sensitivity analysis on the ranking R^{m} (the original ranking):

The difference between the rankings R^{m} and R_1 is represented by the number $\rho(R^{\mathrm{m}}, R_1) \geq 0$ where R_1 is a ranking obtained by slightly perturbing the data.

Highly ranked hijackers that leave the top-5, and lowly ranked hijackers that enter the top-5, result in a large value of ρ.

Sensitivity analysis of Al Qaeda's 9/11 attack

Network structure: 1000 simulations run in which up to 4 links were randomly added or deleted.

Individual strength: 1000 simulations where the weight for each of 4 randomly selected individuals was set randomly equal to 1, 2, 3 or 4.

Relational strength: The weight of a single link was set to 4, with the rest of the weights kept to 1.

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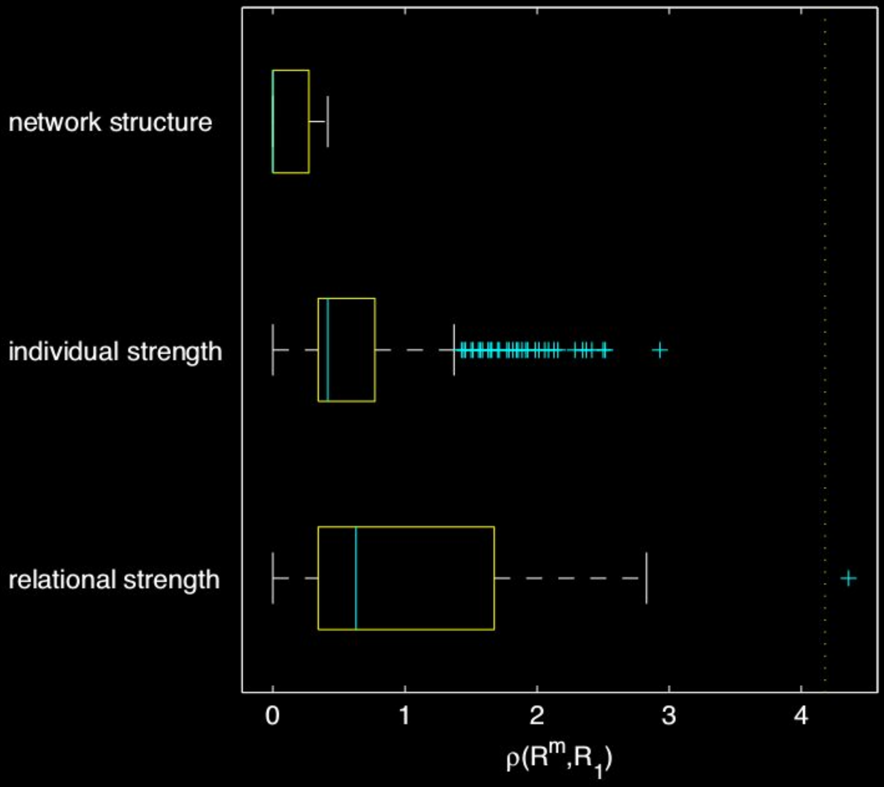

Conclusions

The new game theoretic centrality measure takes all available information about the members of the network and their relations into account, incorporates the strength of connected subnetworks, and is robust to small changes in the available data, which makes it a promising measure to construct rankings of terrorists in real-life networks.

Extra Resources

Tomasz P. Michalak:

- http://users.ecs.soton.ac.uk/tr/pub/DefeatingTerroristNetworks.pdf
- http://www.jair.org/media/3806/live-3806-6969-jair.pdf

The Value of an n-person Game by Lloyd Shapley

- http://www.rand.org/content/dam/rand/pubs/research_memoranda/2008/RM670.pdf

Theory Of Games And Economic Behavior by von Neumann & Mortgenstern

- https://archive.org/details/theoryofgamesand030098mbp

Game Theory In Economics by Shapley and Shubik

- https://www.rand.org/content/dam/rand/pubs/reports/2006/R904.6.pdf

Game Theory Online Courses

- http://www.game-theory-class.org/

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9/11 Suspected Terrorists:
http://cdn.history.com/sites/2/2014/01/picture-of-suspected-hijackers.jpg

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