Ranking terrorists in networks:
A sensitivity analysis of Al Qaeda’s 9/11 attack

By Bart Husslage, Peter Borm, Twan Burg, Herbert Hamers, & Roy Lindelauf

(2015)

Presentation by Paul Tomchik for CS 660
Authors:

- Bart Husslage
  - Lecturer Of Applied Mathematics at Fontys Hogescholen

- Peter Borm
  - Professor in Mathematics and Game Theory at Tilburg University

- Twan Burg
  - Tilburg University

- Herbert Hamers
  - Professor in Game Theory and Operations Research at Tilburg University

- Roy Lindelauf
  - Military Operations Researcher at Netherlands Defense Academy
Bart Husslage, Peter Borm, Twan Burg, Herbert Hamers, Roy Lindelauf:

Roy Lindelauf, Herbert Hamers, Bart Husslage:

Edwin R. van Dam, Bart Husslage, Dick den Hertog:

Edwin R. van Dam, Gijis Rennen, Bart Husslage:

Edwin R. van Dam, Bart Husslage, Dick den Hertog, Hans Melissen:

Bart Husslage, Edwin R. van Dam, Dick den Hertog, Peter Stehouwer, Erwin Stinstra:
2015

- M. Musegaas, Peter E. M. Borm, Marieke Quant: 

- Peter Borm, Yuan Ju, David Wettstein: 

- Estela Sánchez-Rodríguez, Peter Borm, Arantza Estévez-Fernández, M. Gloria Fiestras-Janeiro, Manuel A. Mosquera: 

- Sybren Haaijer, Peter E. M. Borm, John Kleppe, J. H. Regenbergs: 

- Bart Hustlage, Peter Borm, Twan Burg, Herbert Hamers, Roy Lindelaufen: 

2014

- Carlos González-Altocín, Peter Borm, Ruud Hendrickx: 
  Nash Equilibria in 2 × 2 × 2 Trimatrix Games with Identical Anonymous Best-Responses. IGTR 10(4) (2014)

- Edwin Lohmann, Peter Borm, Marco Stikker: 

- O. Tejada, Peter Borm, Edwin Lohmann: 

2013

- Søsja Grundel, Baris Çiftçi, Peter Borm, Herbert Hamers: 

- Søsja Grundel, Peter Borm, Herbert Hamers: 

- John Kleppe, Peter Borm, Ruud Hendrickx: 
  Fall back equilibrium for 52 times n5 bimatrix games. Math. Meth. of OR 78(2): 171-180 (2013)
Herbert Hamers

2015

2014

2013

2012
<table>
<thead>
<tr>
<th>Year</th>
<th>Title</th>
<th>Conference/Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>Power grid defense against malicious cascading failure.</td>
<td>AAMAS 2014: 813-820</td>
</tr>
<tr>
<td></td>
<td>models: The cases of Jemaah Islamiyah and Al Qaeda.</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>Understanding Terrorist Network Topologies and Their Resilience</td>
<td>Counterterrorism and Open Source Intelligence 2011: 61-72</td>
</tr>
<tr>
<td></td>
<td>Against Disruption.</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>Key Player Identification: A Note on Weighted Connectivity Games and</td>
<td>ASONAM 2010: 356-359</td>
</tr>
<tr>
<td></td>
<td>the Shapley Value.</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>The influence of secrecy on the communication structure of covert</td>
<td>Social Networks 31(2): 126-145</td>
</tr>
<tr>
<td></td>
<td>networks.</td>
<td></td>
</tr>
</tbody>
</table>
Overview: Traditional Approach to Terrorist Network Analysis

- Intelligence is viewed and analyzed as network data.
  - Members of networks -> nodes
  - Their interactions -> (weighted) edges
- Analysts use standard centrality measures (degree, closeness, and betweenness) to find key network members.
- Results are used in decision support systems.
Overview: Traditional Approach to Terrorist Network Analysis

- Intelligence is viewed and analyzed as network data.
  - Members of networks -> nodes
  - Their interactions -> (weighted) edges
- Analysts use standard centrality measures (degree, closeness, and betweenness) to find key network members.
- Results are used in decision support systems.
- *These methods take only this network structure into account.*
Overview: Traditional Approach to Terrorist Network Analysis

- Intelligence is viewed and analyzed as network data.
  - Members of networks -> nodes
  - Their interactions -> (weighted) edges
- Analysts use standard centrality measures (degree, closeness, and betweenness) to find key network members.
- Results are used in decision support systems.
- These methods take only this network structure into account.

Intelligence gathered often includes information regarding the nature of relations between members and their individual skills/resources.
Overview: This paper’s contributions

- Introduces a new game theoretic centrality measure for ranking players in a terrorist network. (Elaborating Linderlauf’s earlier work.)
- This centrality measure incorporates individual and coalitional characteristics, such as special skills, and relational characteristics, like frequency of communication.
- This measure takes into account the operational strength of connected subnetworks, potentially providing a more suitable model of real-life networks.
- The robustness of the rankings is tested by performing a sensitivity analysis on the rankings of the terrorists in the 9/11 attack.
Overview: Roy Lindelauf’s earlier work

“Context specific cooperative coalitional games are defined that reflect the situation at hand taking all available information about the network structure and the individual members and their relations into account.

Next, the Shapley value is computed for the corresponding game to measure the importance of members of the network in order to construct a ranking of these members.

For each threat context a specific suitable game can be developed.”

(Bold face and italics added by presenter for emphasis.)
Cooperative/Coalitional Game Theory

- “An approach to modeling strategic situations that stands in contrast with Noncooperative Game Theory.”
- The essential difference is the *basic modeling unit*. 

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

- “An approach to modeling strategic situations that stands in contrast with Noncooperative Game Theory.”
- The essential difference is the basic modeling unit.

**Coalitional Game Theory**

The basic modeling unit is the group and what they can accomplish.

**Noncooperative Game Theory**

The basic modeling unit is the individual and what he/she can accomplish “playing a lone hand.”

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

- Given a set of agents, a coalitional game defines how well each group (or coalition) of agents can do for itself.
- NOT concerned with how agents make choices within a coalition, or how they coordinate.

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

Definition (Coalitional game with transferable utility)

A coalitional game with transferable utility is a pair \((N, v)\), where

- \(N\) is a finite set of players, indexed by \(i\); and
- \(v : 2^N \rightarrow \mathbb{R}\) associates with each coalition \(S \subseteq N\) a real-valued payoff \(v(S)\) that the coalition’s members can distribute among themselves. We assume that \(v(\emptyset) = 0\).
Cooperative/Coalitional Game Theory

Definition (Coalitional game with transferable utility)

A coalitional game with transferable utility is a pair \((N, v)\), where

- \(N\) is a finite set of players, indexed by \(i\); and
- \(v : 2^N \rightarrow \mathbb{R}\) associates with each coalition \(S \subseteq N\) a real-valued payoff \(v(S)\) that the coalition’s members can distribute among themselves. We assume that \(v(\emptyset) = 0\).

- \(v\), called the “characteristic function”, acts as utility function for a coalitional game.
Cooperative/Coalitional Game Theory

Definition (Coalitional game with transferable utility)

A coalitional game with transferable utility is a pair \((N, v)\), where

- \(N\) is a finite set of players, indexed by \(i\); and
- \(v : 2^N \to \mathbb{R}\) associates with each coalition \(S \subseteq N\) a real-valued payoff \(v(S)\) that the coalition’s members can distribute among themselves. We assume that \(v(\emptyset) = 0\).

- \(v\), called the “characteristic function”, acts as utility function for a coalitional game.
- Says: “For every subset of the players, \(S\), that could form in the game, what is the payoff \(v(S)\) that the coalition can achieve?”

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

The questions we use coalitional game theory to answer:

- Which coalitions will form?
- How should that coalition divide its payoff amongst its members?

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

The questions we use coalitional game theory to answer:

- Which coalitions will form?
- How should that coalition divide its payoff amongst its members?

Usually, all agents will agree to act together. However, sometimes this depends on how the coalition would divide its payoff amongst its members.

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

The questions we use coalitional game theory to answer:

- How should that coalition divide its payoff amongst its members
  - in order to be fair?
  - in order to be stable?

http://www.game-theory-class.org/
Cooperative/Coalitional Game Theory

The questions we use coalitional game theory to answer:

- How should that coalition divide its payoff amongst its members
  - in order to be fair?
Shapley Value

One of the most prominent ways of dividing up the value of a set of individuals amongst its members.

http://www.game-theory-class.org/
Lloyd Shapley

Awards and honors [edit]

- Bronze Star, U.S. Army Air Corps, 1944
- Procter Fellow, Princeton University, 1951–52
- Fellow, Econometric Society, 1967
- Fellow, American Academy of Arts and Sciences, 1974
- Member, National Academy of Sciences, 1979
- John von Neumann Theory Prize, 1981
- Honorary Ph.D., Hebrew University of Jerusalem, 1986
- Fellow, INFORMS (Institute for Operations Research and the Management Sciences), 2002
- Distinguished Fellow, American Economic Association, 2007
- Fellow, American Mathematical Society, 2012[7]
- Sveriges Riksbank Nobel Memorial Prize in Economic Sciences, 2012
- Golden Goose Award, 2013[8]

(Screencapture from Wikipedia.)
Shapley Value

One of the most prominent ways of dividing up the value of a set of individuals amongst its members.

A solution to the question:

“What is a fair way to divide a coalition’s payout?”

http://www.game-theory-class.org/
Shapley Value

One of the most prominent ways of dividing up the value of a set of individuals amongst its members.

“What is a fair way to divide a coalition’s payout?”
- Depends on how we define fairness.

http://www.game-theory-class.org/
Shapley Value

One of the most prominent ways of dividing up the value of a set of individuals amongst its members.

“What is a fair way to divide a coalition’s payout?”
● Depends on how we define fairness.

Shapley’s idea:

Members should receive payments or shares proportional to their marginal contributions.

http://www.game-theory-class.org/
What does a person contribute when we add them to a group?

Their share of the group’s values should reflect what they contribute to the group’s value.

Members should receive payments or shares proportional to their marginal contributions.
Shapley Value

\[ \phi_i(N, v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N| - |S| - 1)! \left[ v(S \cup \{i\}) - v(S) \right] \]

Members should receive payments or shares proportional to their marginal contributions.

http://www.game-theory-class.org/
Members should receive payments or shares proportional to their marginal contributions.

http://www.game-theory-class.org/
Members should receive payments or shares proportional to their marginal contributions.
Members should receive payments or shares proportional to their marginal contributions.

\[ \phi_i(N, v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (|N| - |S| - 1)!}{|S|!(|N| - |S| - 1)!} [v(S \cup \{i\}) - v(S)] \]
Shapley Value

\[ \phi_i(N, v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \left[ v(S \cup \{i\}) - v(S) \right] \]

Summation over all possible coalitions that could be created from \( N \) without \( i \).

Marginal contribution

What does individual \( i \) add to coalitions that do not already include \( i \)?

The number of ways we could have build the coalition before adding \( i \).

Members should receive payments or shares proportional to their marginal contributions.

http://www.game-theory-class.org/
Members should receive payments or shares proportional to their marginal contributions.
Members should receive payments or shares proportional to their marginal contributions.
Overview: Roy Lindelauf’s earlier work

“Context specific cooperative coalitional games are defined that reflect the situation at hand taking all available information about the network structure and the individual members and their relations into account.

Next, the Shapley value is computed for the corresponding game to measure the importance of members of the network in order to construct a ranking of these members.

For each threat context a specific suitable game can be developed.”

(Bold face and italics added by presenter for emphasis.)
A coalition $S \subseteq N$ is called a connected coalition if the network $G[S]$ is connected, otherwise $S$ is called disconnected.

In monotonic weighted connectivity games the effectiveness of a disconnected coalition is determined by the most effective connected subcomponent.
A new game theoretic centrality measure

The idea is to create a game that takes into account the structure of the network

\[ G = (N, E) \]

individual strengths (e.g., special skills) of the members of the network

\[ \mathcal{I} = \left\{ w_i \right\}_{i \in N} \text{ with } w_i \geq 0 \]

as well as the relational strength (e.g., communication) between members of the network.

\[ \mathcal{R} = \left\{ k_{ij} \right\}_{ij \in E} \text{ with } k_{ij} \geq 0 \]
We define a *monotonic weighted connectivity game* $\left( N, \nu^{mwconn} \right)$ with respect to network $G = (N, E)$ based on $I$ and $R$ in the following way:

*For a connected coalition $S$ we define*

$$\nu^{mwconn}(S) = \begin{cases} f(S, I, R) & \text{if } |S| > 1, \\ 0 & \text{otherwise}, \end{cases}$$

where $f$ is a context specific and tailor-made non-negative function that measures the effectiveness of coalitions in the network.
A new game theoretic centrality measure

We define a monotonic weighted connectivity game $(N, \nu_{mwconn})$ with respect to network $G = (N, E)$ based on $\mathcal{I}$ and $\mathcal{R}$ in the following way:

For a disconnected coalition $S$ we define

$$\nu_{mwconn}(S) = \max_{T \subset S, \ T \text{connected}} \nu_{mwconn}(T)$$
A new game theoretic centrality measure

The game theoretic centrality measure $C^m_i$ of member $i$ in network $G = (N, E)$ based on $I$ and $R$ is defined by

$$C^m(i) = \varphi_i(\nu^{mwconn})$$

where $\varphi_i(\nu^{mwconn})$ is the Shapley value of member $i$ in the game. The corresponding ranking of all members in $N$ is denoted by $R^m$. 
An example coalition effectiveness function

\[ f(S, \mathcal{I}, \mathcal{R}) = \left( \sum_{i \in S} w_i \right) \cdot \max_{ij \in E_S} k_{ij}. \]

This specific choice can be motivated for terrorist cells in which we need to focus on the total operational strength of the cell as well as the most prominent line of communication between members.

(This function is used throughout the paper in the analyses.)
Example

\[ f(S, I, R) = \left( \sum_{i \in S} w_i \right) \cdot \max_{ij \in E_S} k_{ij}. \]
**Example**

\[ f(S, T, R) = \left( \sum_{i \in S} w_i \right) \cdot \max_{ij \in E_S} k_{ij}. \]

---

**The value \( v_{\text{mwconn}}(S) \) for each coalition**

<table>
<thead>
<tr>
<th>Coalition S</th>
<th>( v_{\text{mwconn}}(S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {A, C} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {A, D} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {A, E} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {B, C} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {B, D} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {B, E} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {C, D} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( {A, B} )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

---

![Graph with nodes A, B, C, D, E and edges connecting them](image_url)
Example

The centrality measure $C^m$ for the example

<table>
<thead>
<tr>
<th>Member</th>
<th>Centrality measure $C^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>6.1667</td>
</tr>
<tr>
<td>$B$</td>
<td>6.4167</td>
</tr>
<tr>
<td>$C$</td>
<td>1.7500</td>
</tr>
<tr>
<td>$D$</td>
<td>5.5833</td>
</tr>
<tr>
<td>$E$</td>
<td>4.0833</td>
</tr>
</tbody>
</table>

The value $\nu^\text{wconn}(S)$ for each coalition

<table>
<thead>
<tr>
<th>Coalition $S$</th>
<th>$\nu^\text{wconn}(S)$</th>
<th>${A}$</th>
<th>${B}$</th>
<th>${C}$</th>
<th>${D}$</th>
<th>${E}$</th>
<th>${A, B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${A, C}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>${A, D}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>${A, E}$</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>${B, C}$</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>${B, D}$</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>${B, E}$</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>${C, D}$</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>${C, E}$</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>${A, B, C}$</td>
<td>24</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
A sensitivity analysis is run to investigate the robustness of the ranking obtained.

- To model individual strength, the data on individuals are expressed as weights on the nodes of the network.

- Not all interactional data between members may be completely known.

- Finally, some of the interactions between members may be considered more important than others.
A sensitivity analysis is run to investigate the robustness of the ranking obtained.

- To model individual strength, the data on Individuals are expressed as weights on the nodes of the network. Analysis is run to see how robust the derived ranking is with respect to small variations in the weights.

- Not all interactional data between members may be completely known.

- Finally, some of the interactions between members may be considered more important than others.
A sensitivity analysis is run to investigate the robustness of the ranking obtained.

- To model individual strength, the data on Individuals are expressed as weights on the nodes of the network. Analysis is run to see how robust the derived ranking is with respect to small variations in the weights.
- Not all interactional data between members may be completely known. Analysis is run to see how robust our ranking is with respect to the addition or deletion of a small percentage of the links in the network.
- Finally, some of the interactions between members may be considered more important than others.
Sensitivity analysis of Al Qaeda’s 9/11 attack

A sensitivity analysis is run to investigate the robustness of the ranking obtained.

- To model individual strength, the data on Individuals are expressed as weights on the nodes of the network. Analysis is run to see how robust the derived ranking is with respect to small variations in the weights.

- Not all interactional data between members may be completely known. Analysis is run to see how robust the ranking is with respect to the addition or deletion of a small percentage of the links in the network.

- Finally, some of the interactions between members may be considered more important than others. Analysis is run to see how robust the ranking is with respect to changes in the weights assigned to interactions.
### Sensitivity analysis of Al Qaeda’s 9/11 attack

#### Weight assigned to each hijacker of Al Qaeda’s 9/11 attack.

<table>
<thead>
<tr>
<th>Hijacker</th>
<th>Weight</th>
<th>Hijacker</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahmed Alghamdi</td>
<td>1</td>
<td>Nawaf Alhazmi</td>
<td>2</td>
</tr>
<tr>
<td>Hamza Alghamdi</td>
<td>1</td>
<td>Khalid Al-Mihdhar</td>
<td>3</td>
</tr>
<tr>
<td>Mohand Alshehri</td>
<td>1</td>
<td>Hani Hanjour</td>
<td>1</td>
</tr>
<tr>
<td>Fayez Ahmed</td>
<td>1</td>
<td>Majed Moqed</td>
<td>1</td>
</tr>
<tr>
<td>Marwan Al-Shehhi</td>
<td>3</td>
<td>Mohamed Atta</td>
<td>4</td>
</tr>
<tr>
<td>Ahmed Alnami</td>
<td>1</td>
<td>Abdul Aziz Al-Omari</td>
<td>1</td>
</tr>
<tr>
<td>Saeed Alghamdi</td>
<td>1</td>
<td>Waleed Alshehri</td>
<td>1</td>
</tr>
<tr>
<td>Ahmed Al-Haznawi</td>
<td>1</td>
<td>Satam Suqami</td>
<td>1</td>
</tr>
<tr>
<td>Ziad Jarrah</td>
<td>4</td>
<td>Wail Alshehri</td>
<td>1</td>
</tr>
<tr>
<td>Salem Alhazmi</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Description indicator

<table>
<thead>
<tr>
<th>Description indicator</th>
<th>Example(s)</th>
<th>Hijacker(s)</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending meetings on terror attack planning</td>
<td>Kuala Lumpur meeting January 2000</td>
<td>Nawaf Al-Hazmi Khalid Al-Midhhar</td>
<td>+1</td>
</tr>
<tr>
<td>Signs of radicalization</td>
<td>Antisemitic and anti-American speech, talk about jihad and martyrdom, writing a will</td>
<td>Mohamed Atta Marwan Al-Shehhi</td>
<td>+1</td>
</tr>
<tr>
<td>Affiliations</td>
<td>Al-Quds mosque Hamburg</td>
<td>Mohamed Atta Ziad Jarrah</td>
<td>+1</td>
</tr>
<tr>
<td>Accomplice to previous attacks</td>
<td>Attack on USS Cole</td>
<td>Khalid Al-Midhhar</td>
<td>+1</td>
</tr>
<tr>
<td>Attending terrorist training camps</td>
<td>Traveling to training camps in Pakistan and Afghanistan</td>
<td>Mohamed Atta Marwan Al-Shehhi</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ziad Jarrah</td>
<td></td>
</tr>
</tbody>
</table>
### Sensitivity analysis of Al Qaeda’s 9/11 attack

Rankings for Al Qaeda's 9/11 network based on game theoretic and standard centrality.

<table>
<thead>
<tr>
<th>Mwconn</th>
<th>Degree</th>
<th>Betweenness</th>
<th>Closeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>M. Atta</td>
<td>N. Alhazmi*</td>
<td>N. Alhazmi</td>
<td>N. Alhazmi*</td>
</tr>
<tr>
<td>Z. Jarrah</td>
<td>M. Al-Shehhi*</td>
<td>A. Aziz Al-Omari*</td>
<td>M. Atta*</td>
</tr>
<tr>
<td>M. Al-Shehhi*</td>
<td>H. Alghamdi*</td>
<td>M. Atta</td>
<td>M. Al-Shehhi*</td>
</tr>
<tr>
<td>N. Alhazmi</td>
<td>H. Hanjour*</td>
<td>M. Al-Shehhi*</td>
<td>H. Hanjour*</td>
</tr>
<tr>
<td>H. Hanjour</td>
<td>M. Atta*</td>
<td>Wd. Alshehri</td>
<td>Z. Jarrah</td>
</tr>
<tr>
<td>K. Al-Midhar*</td>
<td>Z. Jarrah*</td>
<td>H. Alghamdi*</td>
<td>H. Alghamdi°</td>
</tr>
<tr>
<td>A. Aziz Al-Omari</td>
<td>S. Alghamdi</td>
<td>H. Hanjour</td>
<td>S. Alhazmi°</td>
</tr>
<tr>
<td>H. Alghamdi*</td>
<td>A. Aziz Al-Omari°</td>
<td>Z. Jarrah</td>
<td>A. Aziz Al-Omari*</td>
</tr>
<tr>
<td>Wd. Alshehri*</td>
<td>Wd. Alshehri°</td>
<td>F. Ahmed</td>
<td>S. Alghamdi*</td>
</tr>
<tr>
<td>A. Al-Haznawi*</td>
<td>A. Al-Haznawi°</td>
<td>M. Alshehri</td>
<td>A. Al-Haznawi°</td>
</tr>
<tr>
<td>S. Alhazmi*</td>
<td>S. Alhazmi°</td>
<td>A. Al-Haznawi*</td>
<td>F. Ahmed*</td>
</tr>
<tr>
<td>F. Ahmed*</td>
<td>A. Alnami*</td>
<td>S. Alhazmi</td>
<td>A. Alnami*</td>
</tr>
<tr>
<td>S. Alghamdi*</td>
<td>F. Ahmed*</td>
<td>S. Alghamdi*</td>
<td>K. Al-Midhar*</td>
</tr>
<tr>
<td>M. Alshehri*</td>
<td>M. Alshehri*</td>
<td>A. Alnami*</td>
<td>M. Alshehri</td>
</tr>
<tr>
<td>A. Alnami*</td>
<td>K. Al-Midhar*</td>
<td>K. Al-Midhar*</td>
<td>M. Moqed*</td>
</tr>
<tr>
<td>M. Moqed</td>
<td>S. Suqami*</td>
<td>S. Suqami*</td>
<td>Wd. Alshehri</td>
</tr>
<tr>
<td>A. Alghamdi*</td>
<td>W. Alshehri*</td>
<td>W. Alshehri*</td>
<td>A. Alghamdi*</td>
</tr>
<tr>
<td>S. Suqami*</td>
<td>A. Alghamdi°</td>
<td>A. Alghamdi*</td>
<td>W. Alshehri°</td>
</tr>
<tr>
<td>W. Alshehri*</td>
<td>M. Moqed°</td>
<td>M. Moqed*</td>
<td>S. Suqami°</td>
</tr>
</tbody>
</table>
Performing a sensitivity analysis on the ranking $R^m$ (the original ranking):

The difference between the rankings $R^m$ and $R_1$ is represented by the number $\rho(R^m, R_1) \geq 0$ where $R_1$ is a ranking obtained by slightly perturbing the data.

Highly ranked hijackers that leave the top-5, and lowly ranked hijackers that enter the top-5, result in a large value of $\rho$. 
Sensitivity analysis of Al Qaeda’s 9/11 attack

**Network structure**: 1000 simulations run in which up to 4 links were randomly added or deleted.

**Individual strength**: 1000 simulations where the weight for each of 4 randomly selected individuals was set randomly equal to 1, 2, 3 or 4.

**Relational strength**: The weight of a single link was set to 4, with the rest of the weights kept to 1.
Sensitivity analysis of Al Qaeda’s 9/11 attack

**Network structure:** 1000 simulations run in which up to 4 links were randomly added or deleted.

**Individual strength:** 1000 simulations where the weight for each of 4 randomly selected individuals was set randomly equal to 1, 2, 3 or 4.

**Relational strength:** The weight of a single link was set to 4, with the rest of the weights kept to 1.
Conclusions

The new game theoretic centrality measure takes all available information about the members of the network and their relations into account, incorporates the strength of connected subnetworks, and is robust to small changes in the available data, which makes it a promising measure to construct rankings of terrorists in real-life networks.
Extra Resources

Tomasz P. Michalak:
- http://users.ecs.soton.ac.uk/tr/pub/DefeatingTerroristNetworks.pdf

The Value of an n-person Game by Lloyd Shapley

Theory Of Games And Economic Behavior by von Neumann & Mortgenstern
- https://archive.org/details/theoryofgamesand030098mbp

Game Theory In Economics by Shapley and Shubik
- https://www.rand.org/content/dam/rand/pubs/reports/2006/R904.6.pdf

Game Theory Online Courses
- http://www.game-theory-class.org/
Photo Credits:

9/11 Suspected Terrorists:
http://cdn.history.com/sites/2/2014/01/picture-of-suspected-hijackers.jpg