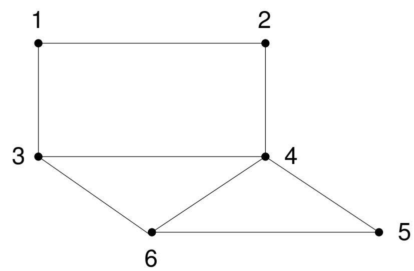
CSI 445/660 – Network Science – Fall 2015

Solutions to Homework VI

Problem 1: The underlying graph of a deterministic synchronous dynamical system (SyDS), where each node has a state value from {0, 1}, is shown below. Assume that this is also a *progressive* system; that is, once a node reaches the state 1, it stays in that state for ever.



It is known that the local function associated with each of the nodes 1 through 4 is the 1-threshold function. It is also known that the local functions associated with nodes 5 and 6 are threshold functions; however, we *don't know* the corresponding threshold values.

Recall that a configuration specifies the state value of each node. Since the graph has 6 nodes, we specify each configuration of the system as a vector with 6 components which represent the states of nodes 1 through 6 in that order. Observations of the system indicate the following.

(i) The configuration `(0, 0, 0, 0, 0, 0)` is a fixed point of the system.

(ii) When the system is started in the configuration `(0, 1, 0, 1, 0, 0)`, the configuration at the next time step is `(1, 1, 1, 1, 1, 0)`.

(iii) When the system is started in the configuration `(0, 1, 0, 1, 1, 0)`, the configuration at the next time step is `(1, 1, 1, 1, 1, 1)`.

Using these observations, find the threshold values of nodes 5 and 6 of the system. Be sure to indicate how you arrived at your solution.

Solution to Problem 1: Since the configuration `(0, 0, 0, 0, 0, 0)` is a fixed point, the threshold values for nodes 5 and 6 must be *at least* 1. (If their threshold values were 0, then the states of nodes 5 and 6 would have changed to 1 in the successor configuration of `(0, 0, 0, 0, 0, 0)`.)

In the configuration `(0, 1, 0, 1, 0, 0)`, exactly one of the inputs to nodes 5 and exactly one of the inputs to node 6 are 1. Since the next configuration is `(1, 1, 1, 1, 1, 0)`, it follows that the threshold for node 5 is 1 and that for node 6 is *greater than* 1.

In the configuration `(0, 1, 0, 1, 1, 0)`, exactly two of the inputs to node 6 are 1. Since the next configuration is `(1, 1, 1, 1, 1, 1)`, it follows that the threshold for node 6 is 2.

Thus, the threshold values of nodes 5 and 6 are 1 and 2 respectively.

Problem 2: Consider the 2-player game given by the following payoff matrix.

P2

P1	A	B
A	(93, 70)	(85, 80)
B	(89, 35)	(86, 45)
C	(94, 32)	(87, 33)

Note that there are six combinations of the strategies by the two players. For each combination, indicate whether or not it is a pure Nash equilibrium. For each combination that is not a pure Nash Equilibrium, indicate which player has an incentive to switch and to which strategy.

Solution:

Combination	**Pure NE?**	**Who has incentive to switch**
(A, A)	No	P1 has an incentive to switch to C (with payoff 94). Also, P2 has an incentive to switch to B (with payoff 80).
(A, B)	No	P1 has an incentive to switch to C (with payoff 87).
(B, A)	No	P1 has an incentive to switch to C (with payoff 94). Also, P2 has an incentive to switch to B (with payoff 45).
(B, B)	No	P1 has an incentive to switch to C (with payoff 87).
(C, A)	No	P2 has an incentive to switch to B (with payoff 33).
(C, B)	Yes	None

Problem 3: Prove that there is **no** mixed Nash equilibrium for the following game when the probability values are required to be **strictly between** 0 and 1.

B

A	P	E
P	(90,90)	(86,92)
E	(92,86)	(88,88)

Solution: The proof is by contradiction.

Suppose there is a mixed NE for this game where player B uses strategy P with probability q (and strategy E with probability $1-q$). We first compute the expected payoff to player A assuming that A uses a pure strategy.

Case 1: Player A uses pure strategy P.

In this case, the payoff to A is 90 with probability q and 86 with probability $1 - q$. So, A's expected payoff in this case $= 90q + 86(1 - q) = 4q + 86$.

Case 2: Player A uses pure strategy E.

In this case, the payoff to A is 92 with probability q and 88 with probability $1 - q$. So, A's expected payoff in this case $= 92q + 88(1 - q) = 4q + 88$.

To obtain a mixed Nash equilibrium, player B should choose q so that A's expected payoff is the same regardless of which pure strategy is used by that player. In other words, player B must choose q so that

$$4q + 86 \ = \ 4q + 88.$$

Obviously, there is no solution to the above equation under the requirement that $0 < q < 1$. Thus, there is no mixed NE for this game if all probability vales must be strictly between 0 and 1. ■

Note: The combination (E, E) is a pure NE for this game.
