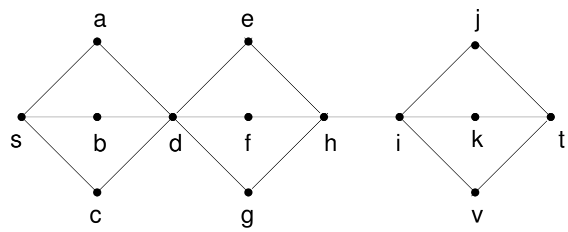
CSI 445/660 – Network Science – Fall 2015

Solutions to Homework IV

Problem 1: Consider the following graph.



The problem has three parts.

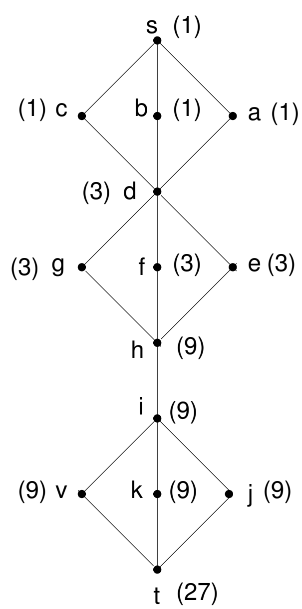
(a) Compute the total number of shortest paths between nodes s and t using the top-down algorithm discussed in class.

(b) Compute the total number of shortest paths between nodes s and t *that don't contain* node v, again using the top-down algorithm discussed in class.

(c) Using the answers from (a) and (b), compute the total number of shortest paths between nodes s and t *that contain* node v.

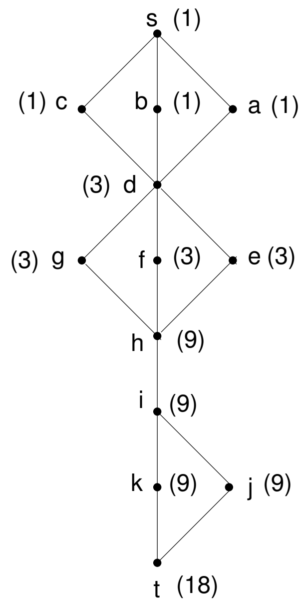
Solution:

Part (a): The following figure shows the graph above, with the integer appearing in parentheses next to the node's name giving the number of shortest paths from s to that node. These numbers have been computed using the top-down algorithm discussed in class.



Thus, the total number of shortest paths between s and t is 27.

Part (b): The following figure shows the graph above with node v deleted. Again, the integer appearing in parentheses next to the node's name is the number of shortest paths from s to that node.



Thus, the total number of shortest paths between s and t that don't contain node v is 18.

Part (c): From the results of Parts (a) and (b), it follows that the total number of shortest paths between s and t that contain node v is $27 - 18 = 9$.

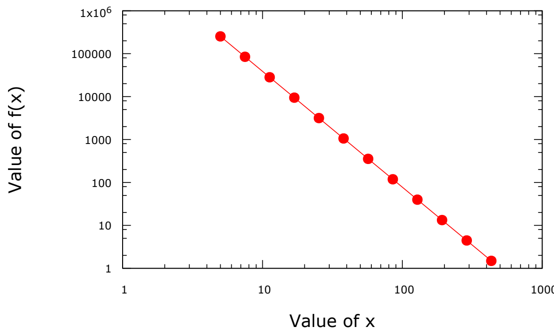
Problem 2: The following table shows the values of functions $f(x)$ and $g(x)$ for various values of the independent variable x.

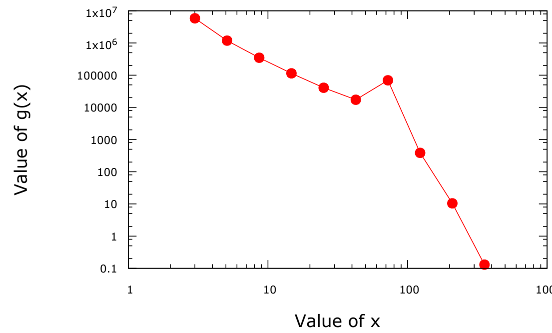
x	$f(x)$
5.00	252822.43
7.50	84599.77
11.25	28308.89
16.88	9472.76
25.31	3169.79
37.97	1060.68
56.95	354.93
85.43	118.77
128.14	39.74
192.22	13.30

x	$g(x)$
3.00	5824779.30
5.10	1185913.90
8.67	347172.77
14.74	114200.30
25.06	40746.59
42.60	17235.83
72.41	69248.85
123.10	384.66
209.27	10.41
355.76	0.13

By plotting these functions suitably, determine whether each of the above functions exhibits a power-law behavior. If yes, determine the power-law exponent.

Solution: Log-log plots of functions $f(x)$ and $g(x)$ are shown below.





Since the log-log plot for the function $f(x)$ is a straightline, it exhibits a power-law behavior. However, $g(x)$ does *not* exhibit a power-law behavior.

To compute the power-law exponent for $f(x)$, we will consider the values $(11.25, 28308.89)$ and $(16.88, 9472.76)$. The slope of the line joining these two points in the log-log plot is given by

$$\frac{\log_{10}(9472.76) - \log_{10}(28308.89)}{\log_{10}(16.88) - \log_{10}(11.25)} \quad = \quad -\,2.698028$$

So, the power law exponent for function $f(x)$ is 2.69808.

Note: The data for $f(x)$ was created using the exponent of 2.7.

Problem 3: Suppose G is a connected undirected graph. Let ρ and Δ denote respectively the radius and diameter of G. Prove that $\Delta \;\leq\; 2\rho$.

Solution: To establish the result, we will show that for any pair of nodes u and v, the shortest distance is at most 2ρ.

Consider any pair of nodes u and v. Let p be a center of the graph. From the definitions of center and radius, the distance between p and u is at most ρ and that between p and v is at most ρ. In other words, there is a path of length at most 2ρ between u and v (passing through p). Hence the shortest distance between u and v cannot be larger than 2ρ, and this completes the proof. ■

Note: It is easy to construct an example where the diameter of the graph is exactly twice the radius. (For example, consider a simple path with three nodes. The middle node of the path is the center and so the radius is 1. The diameter is 2, which is the shortest distance between the two end nodes.)
