CSI 445/660 – Part 10
(Link Analysis and Web Search)

Ref: Chapter 14 of [EK] text.

Searching the Web – Ranking Web Pages

- Suppose you type "UAlbany" to Google.
- The web page for UAlbany is among the top few results displayed.
- Search engines use automated methods to **rank** pages.
  - These methods are generally based on link analysis.
  - Search engines also maintain and try to get clues from a user's search history.

Common difficulties:

- **Synonymy:** Multiple ways to describe the same thing (e.g. "scallions" vs "green onions").
- **Polysemy:** Multiple meanings for the same word (e.g. "mercury" may refer to the planet, a car model, the chemical element or newspaper).

Information Retrieval – Then and Now

Pre-web era: Problem of **scarcity**.

Example: A lawyer searching for certain types of cases could only locate a few documents.

Now: Problem of **abundance**.

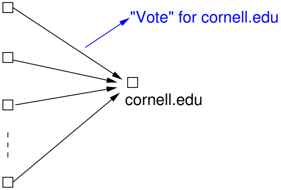
- The search engine should try to produce the most relevant information (from a whole lot of information).
- A popular area of research.
- **Focus:** Use of link analysis in ranking.

Some basic issues:

- Suppose a user types a one word query "Cornell" into a search engine.
- Are there clues within the web to suggest that `cornell.edu` is a good answer to the query?

Information Retrieval ... (continued)

Idea 1 – Voting by in-links:



- If many other pages link to `cornell.edu`, one can think of that page as receiving **collective endorsement**.
- Some of those pages may actually express negative opinions about `cornell.edu`.

Idea 2 – List finding:

- Consider the query "newspapers" to a search engine.
- There is no single "best" answer to this query.

Information Retrieval ... (continued)

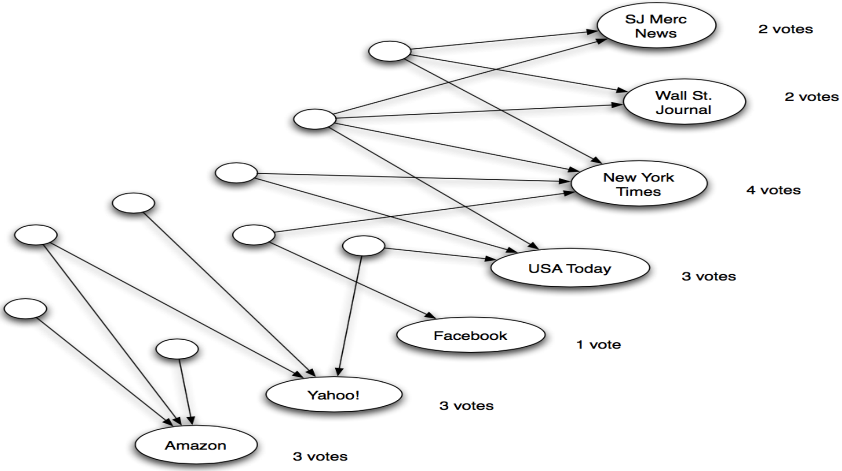
Idea 2: List finding (continued):

- Suppose we collect a set of web pages that have the word "newspapers" and then check which pages they "endorse" (i.e., to which pages they have in-links).
- The answers typically consist of the following:
  - High scores for web pages of prominent newspapers.
  - High scores for other web pages such as Google, Amazon, Facebook, etc.

Note: Web pages for Google, Amazon, Facebook, etc. generally receive a high score no matter what the query is.

Information Retrieval ... (continued)

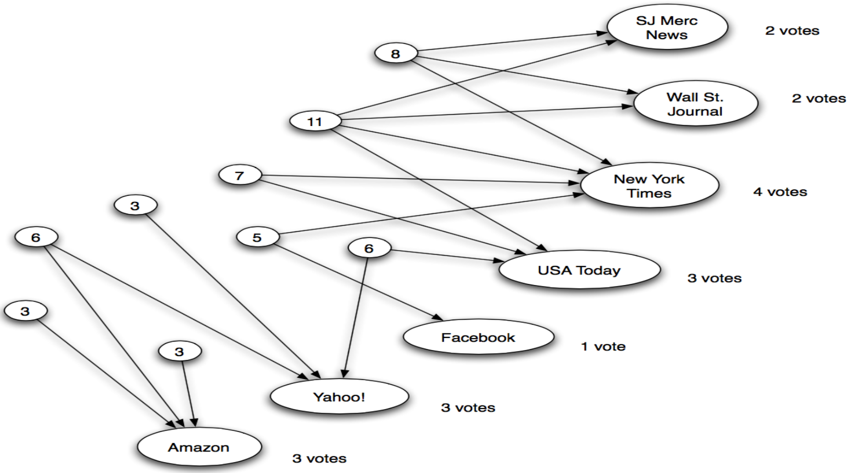
Example:



List Finding ... (continued)

- Pages that contain **lists** of resources relevant to a topic are also useful.
- For the query "newspapers", we may try to find pages that have lists of links to newspapers.
- We can try to compute a measure that represents the value of a page as a list.
- One possible measure: The list value of a page X is the **sum** of the votes received by the pages voted for by X.

Example (with list values):



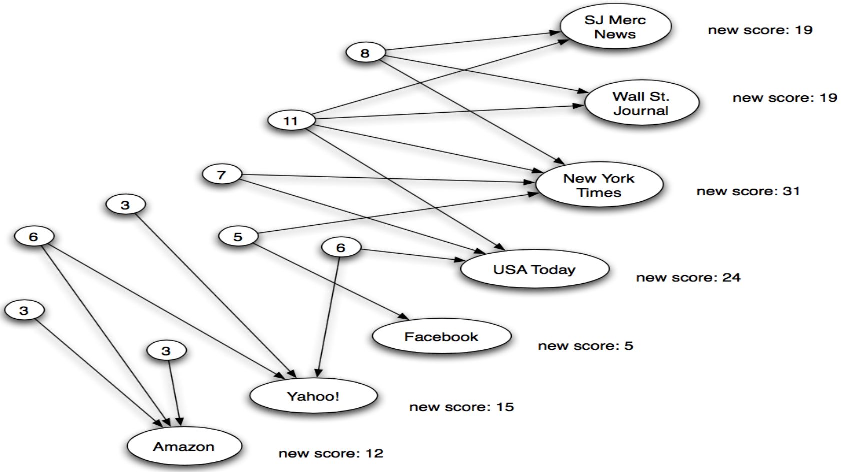
List Finding ... (continued)

Idea 3 – Principle of iterative improvement:

- Since pages with high list values are important, their votes should be weighted more heavily. (Endorsements from more important people should count more.)
- So, it is useful to tabulate the votes again, using the list values.
- After this, we can recompute the list values again; that is, repeat the vote count and list count steps.
- The resulting algorithm (due to Kleinberg) is called HITS (Hyperlink-Induced Topic Search).

Information Retrieval ... (continued)

Example (with list values and new vote counts):



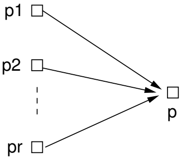
A Description of the HITS Algorithm

Definitions:

- **Authorities** for a query: Pages that are prominent and highly endorsed answers.
- **Hubs** for a query: Pages that have high list values.

Preliminary ideas:

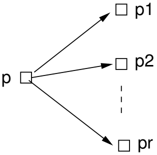
- For each page p, we maintain two numerical values denoted by $\text{auth}(p)$ and $\text{hub}(p)$. Initially, $\text{auth}(p) = \text{hub}(p) = 1$.
- Two update rules are used.



1. Authority update rule (or voting step): For each page p, update $\text{auth}(p)$ to be the sum of the hub scores for all the pages that point to p.

A Description of the HITS Algorithm (continued)

Update rules (continued):



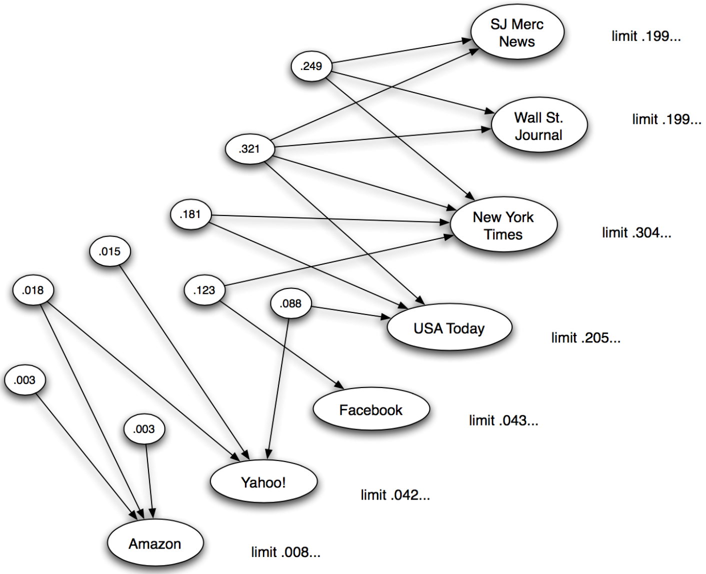
2. Hub update rule (or list finding step): For each page p, update $\text{hub}(p)$ to be the sum of the authority scores for all the pages to which p points.

Outline of the HITS Algorithm:

1. For each page p, set $\text{auth}(p) = \text{hub}(p) = 1$. Choose a value for the number of steps k.
2. **Repeat** the following steps k times:
   - Apply the Authority update rule.
   - Apply the Hub update rule.
3. Normalize the scores and output pages in non-increasing order of their authority scores.

HITS Algorithm ... (continued)

Result produced by the HITS Algorithm:



HITS Algorithm ... (continued)

Final remarks:

- Kleinberg [1999] shows that the scores converge to appropriate limits as $k \to \infty$ (except in some degenerate cases).
- It is possible to express the HITS Algorithm as an iterative algorithm on matrices MM^T and M^TM, where M is the **adjacency matrix** formed by the initial pages.
- The authority scores and hub scores of pages converge to specific eigenvectors of MM^T and M^TM respectively.
- The resulting authority and hub scores represent a form of equilibrium (under the authority update and hub update rules).

Page Rank

- HITS Algorithm works well in commercial contexts where competing firms don't (generally) link to each other.
- In other contexts (e.g. academic pages, scientific literature), page rank algorithm generally outperforms the HITS Algorithm.
- Page rank computation uses ideas similar to those of HITS:
    - A page rank update rule.
    - Idea of iterative improvement.

A physical model for page rank:

- Think of page rank as a **fluid** that circulates through the links of the web network.
- The fluid accumulates at nodes that are "most important".

Page Rank Computation (continued)

Notation: For any page u,

- $\mathrm{PR}(u)$ denotes its page rank.
- $\mathrm{OD}(u)$ denotes its outdegree.

Outline of the algorithm:

1. Let n denote the number of pages. For each node u, let $\mathrm{PR}(u) = 1/n$.
2. Choose a value for k (the number of iterations).
3. **Repeat** the following step k times:
   - Apply the **Basic Page Rank Update Rule** to all the nodes **in parallel**.

Page Rank Computation (continued)

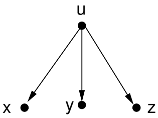
Basic Page Rank Update Rule: For any node u,

1. (**Flow generation step**)
   - If $\mathsf{OD}(u) = 0$ then u sends $\mathsf{PR}(u)$ to itself.
   - If $\mathsf{OD}(u) \geq 1$, then u sends $\mathsf{PR}(u)/\mathsf{OD}(u)$ along each of its outgoing edges.
2. (**Flow accumulation step**)
   - Suppose node u has r incoming edges and the flow along the i^{th} edge is α_i.
   - If $\mathsf{OD}(u) = 0$, then
     $$\mathsf{PR}(u) = \mathsf{PR}(u) + \alpha_1 + \alpha_2 + \cdots + \alpha_r.$$
   - If $\mathsf{OD}(u) \geq 1$, then
     $$\mathsf{PR}(u) = \alpha_1 + \alpha_2 + \cdots + \alpha_r.$$

Page Rank Computation (continued)

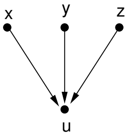
Examples for the flow generation step:

Example 1:



- Suppose $\text{PR}(u) = 1/2$.
- Since $\text{OD}(u) = 3$, u sends $1/6$ along each of the three outgoing edges.

Example 2:

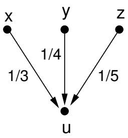


- Suppose $\text{PR}(u) = 1/10$.
- Since $\text{OD}(u) = 0$, u sends $1/10$ to itself.

Page Rank Computation (continued)

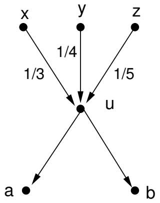
Examples for the flow accumulation step:

Example 3:



- Here, $\mathrm{OD}(u) = 0$.
- Let the current value of $\mathrm{PR}(u)$ be $1/10$.
- New value of $\mathrm{PR}(u) =$ $1/10 + 1/3 + 1/4 + 1/5 = 53/60$.

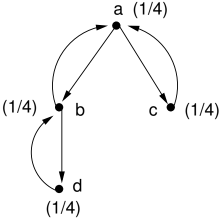
Example 4:



- Let the current value of $\mathrm{PR}(u)$ be $1/10$.
- Since $\mathrm{OD}(u) = 2$, u has already sent $1/20$ to each of a and b.
- New value of $\mathrm{PR}(u) = 1/3 + 1/4 + 1/5 =$ $47/60$.

Page Rank Computation (continued)

A more detailed example:



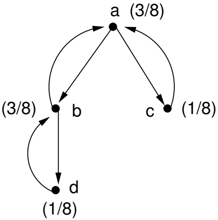
- Initially, $\text{PR}(a) = \text{PR}(b) = \text{PR}(c) = \text{PR}(d) = 1/4$.
- Each node has outdegree > 0. So, in every step, each node sends out its page rank along the outgoing edges.

Step 1:

- Node a receives $1/8$ from b and $1/4$ from c. So, $\text{PR}(a) = 1/8 + 1/4 = 3/8$.
- Node b receives $1/8$ each from a and d. So, $\text{PR}(b) = 1/8 + 1/4 = 3/8$.
- Node c receives $1/8$ from a. So, $\text{PR}(c) = 1/8$.
- Node d receives $1/8$ from b. So, $\text{PR}(d) = 1/8$.

Page Rank Computation (continued)

A more detailed example (continued):



- At the end of Step 1, $\text{PR}(a) = \text{PR}(b) = 3/8$ and $\text{PR}(c) = \text{PR}(d) = 1/8$.

Step 2:

- Node a receives $3/16$ from b and $1/8$ from c. So, $\text{PR}(a) = 3/16 + 1/8 = 5/16$.
- Node b receives $3/16$ from a and $1/8$ from d. So, $\text{PR}(b) = 3/16 + 1/8 = 5/16$.
- Node c receives $3/16$ from a. So, $\text{PR}(c) = 3/16$.
- Node d receives $3/16$ from b. So, $\text{PR}(d) = 3/16$.

Page Rank Computation (continued)

Table showing successive page rank values:

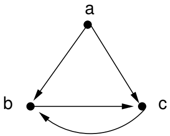
Step	PR(a)	PR(a)	PR(a)	PR(a)
0	1/4	1/4	1/4	1/4
1	3/8	3/8	1/8	1/8
2	5/16	5/16	3/16	3/16

Remarks:

- There is **no** normalization here; the total page rank is always 1.
- It can be shows that (except for degenerate cases), the page rank values converge to a limit as $k \to \infty$.

Page Rank Computation (continued)

Example for equilibrium state (or fixed point):



- Suppose $\mathrm{PR}(a) = 0$, $\mathrm{PR}(b) = 1/2$ and $\mathrm{PR}(c) = 1/2$.
- These values won't change; that is, this is an equilibrium state.

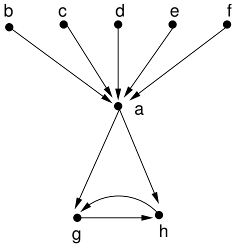
Another form of equilibrium:

- Initially, let $\mathrm{PR}(a) = 0$, $\mathrm{PR}(b) = 3/4$ and $\mathrm{PR}(c) = 1/4$.
- At the end of Step 1: $\mathrm{PR}(a) = 0$, $\mathrm{PR}(b) = 1/4$ and $\mathrm{PR}(c) = 3/4$.
- At the end of Step 2: $\mathrm{PR}(a) = 0$, $\mathrm{PR}(b) = 3/4$ and $\mathrm{PR}(c) = 1/4$ (which is the initial state).

Remark: If the network is strongly connected, it can be shown that there is a **unique** equilibrium state.

Page Rank Computation (continued)

A drawback: In some networks, the page rank update rule allows "wrong" nodes to end up with all the page rank.



- One would expect node *a* to have a high page rank.
- However, the current page rank update rule cause all the page rank to flow out of *a*.
- All the page rank accumulates at *g* and *h*; it doesn't flow back to the other nodes.

Remedy: Modify the page rank update rule.

Scaled Page Rank Update Rule

Steps:

1. Pick a **scaling factor** s, where $0 < s < 1$.
2. Apply the basic page rank update rule.
3. Scale down **all** the page rank values by the factor s. (This step reduces the total page rank from 1 to s.)
4. Divide the residual $1 - s$ units of page rank equally among the n nodes; that is, add $(1 - s)/n$ units of page rank to each node. (This step restores the total page rank value to 1.)

Remarks:

- It can be shows that (except for degenerate cases), the scaled page rank values converge to a limit as $k \to \infty$.
- It is believed that the value of s used by Google is in the range 0.8 to 0.9.

A Random Walk Interpretation of Page Rank

Basic page rank update rule and random walks:

1. Suppose we have n web pages p_1, p_2, ..., p_n.
2. Choose an initial page: each page is chosen with probability $= 1/n$. Let p_i be the chosen page.
3. **Repeat** k times:
   - Suppose the $\mathrm{OD}(p_i) = r$.
     - If $r = 0$, stay at p_i itself.
     - If $r \geq 1$, choose one of the outgoing edges of p_i with probability $= 1/r$.
     - Update p_i to the other end point of that edge.

Theorem: For each i, $1 \leq i \leq n$, the probability that the above random walk is at node p_i is **equal to the page rank of** p_i after k applications of the **basic page rank update rule**.

A Random Walk Interpretation of Page Rank (continued)

Notes:

- The random walk approach provides another way to estimate page ranks.
- The approach can also be extended to the **scaled page rank update rule**. In the body of the loop for Step 3, do the following:
  - With probability s (the chosen scale factor) continue the random walk as before.
  - With probability $1 - s$ choose another node, say p_j, with all nodes being equally likely and continue the random walk from p_j.
- Search engine companies are generally very secretive about the exact methods they use for computing page ranks.