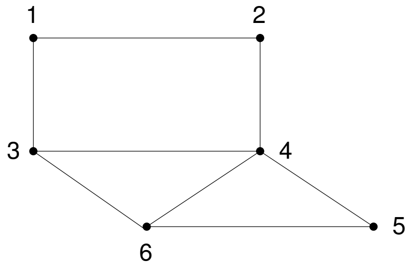
CSI 445/660 – Network Science – Fall 2015

Homework VI

Date given: Nov. 25, 2015 **Due date:** Dec. 8, 2015

Instructions: All students must do Problems 1 and 2. Undergraduate and graduate students in Computer Science must also do Problem 3.

Problem 1: The underlying graph of a deterministic synchronous dynamical system (SyDS), where each node has a state value from {`0`, `1`}, is shown below. Assume that this is also a *progressive* system; that is, once a node reaches the state 1, it stays in that state for ever.



It is known that the local function associated with each of the nodes 1 through 4 is the 1-threshold function. It is also known that the local functions associated with nodes 5 and 6 are threshold functions; however, we *don't know* the corresponding threshold values.

Recall that a configuration specifies the state value of each node. Since the graph has 6 nodes, we specify each configuration of the system as a vector with 6 components which represent the states of nodes 1 through 6 in that order. Observations of the system indicate the following.

(i) The configuration `(0, 0, 0, 0, 0, 0)` is a fixed point of the system.

(ii) When the system is started in the configuration `(0, 1, 0, 1, 0, 0)`, the configuration at the next time step is `(1, 1, 1, 1, 1, 0)`.

(iii) When the system is started in the configuration `(0, 1, 0, 1, 1, 0)`, the configuration at the next time step is `(1, 1, 1, 1, 1, 1)`.

Using these observations, find the threshold values of nodes 5 and 6 of the system. Be sure to indicate how you arrived at your solution.

(over)

Problem 2: Consider the 2-player game given by the following payoff matrix.

		P2	
		A	B
	A	(93, 70)	(85, 80)
P1	B	(89, 35)	(86, 45)
	C	(94, 32)	(87, 33)

Note that there are six combinations of the strategies by the two players. For each combination, indicate whether or not it is a pure Nash equilibrium. For each combination that is not a pure Nash Equilibrium, indicate which player has an incentive to switch and to which strategy.

Problem 3: Prove that there is **no** mixed Nash equilibrium for the following game when the probability values are required to be **strictly between** 0 and 1.

		B	
		P	E
A	P	(90,90)	(86,92)
	E	(92,86)	(88,88)