CSI 445/660 – Network Science – Fall 2015

Homework V

Date given: Nov. 17, 2015 **Due date:** Nov. 24, 2015

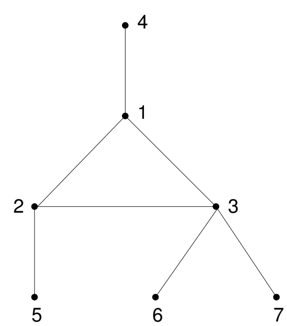
Instructions: All students must do Problems 1 and 2. Undergraduate and graduate students in Computer Science must also do Problem 3. Problem 1(b) is optional for all students.

Problem 1: Let G be a connected undirected graph with 100 nodes such that the degree of each node in G is at least 50. Find the *largest* possible value for the farness centrality of a node of G. Be sure to explain how you arrived at your answer.

Problem 1(b) (optional – for extra credit): Suppose the answer you arrived at for Problem 1 is α. Find a graph G which has 100 nodes and in which each node has a degree of at least 50 such that the farness centrality of *every* node in G is exactly α.

Your answer for Problem 1(b) must include a clear description of the graph (and *not* a drawing of the graph) along with an explanation of why the farness centrality of each node is α.

Problem 2: The underlying graph of a deterministic synchronous dynamical system (SyDS), where each node has a state value from {`0`, `1`}, is shown below. Assume that the system is *progressive*; that is, once a node reaches the state 1, it remains in that state forever.



The local function associated with each node is the 2-threshold function. Recall that a configuration specifies a state value for each node. This problem has two parts.

(a) Suppose the system starts at time 0 in the configuration where nodes 1, 6 and 7 are in state `1` while the other nodes are in state `0`. Show the successive configurations of the system until the system reaches a fixed point.

(b) Find an initial configuration with the *smallest number* of nodes in state `1` such that the system reaches the fixed point where every node is in state `1`. Be sure to indicate how you arrived at your solution.

Problem 3: Before trying to solve this problem, you may want to review the the definitions of an affiliation network and its projected network.

Let $G(V, E)$ be the projected network of an affiliation network G_A. Suppose G is connected and there is an independent set of size α in G. (In other words, G contains a set V' with α nodes such that there is no edge between any pair of nodes in V'.) **Prove or disprove:** The number of focal points in G_A is at least α.