CSI 445/660 – Network Science – Fall 2015

Homework II

Date given: Sep. 22, 2015 **Due date:** Oct. 1, 2015

Instructions:

(a) All students must do Problems 1 and 2. Undergraduate and graduate students in Computer Science must also do Problem 3. Problem 4 is *optional*; however, Computer Science students are urged to give it a try.

(b) For all problems below, assume that the graphs are simple (i.e., they don't have multi-edges or self-loops).

(c) For Problems 3 and 4, bear in mind that when a node in an undirected graph has degree 0 or 1, the clustering coefficient of the node is not defined. (Thus, in those cases, the clustering coefficient is *not* zero.)

Problem 1: This problem has two parts.

(a) Consider a class of elementary school students consisting of 9 boys and 12 girls. Suppose a social network on this group exhibits extreme gender homophily; that is, it has no cross-gender edges. Compute the *maximum* number of possible edges in the social network.

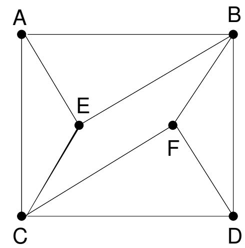
(b) Consider a set of high school students consisting of 120 girls and 80 boys. A social network on this set has a total of 1000 edges. Suppose the number of cross-gender edges in this network is exactly 40% of the value predicted by the random mixing model discussed in class. Find the number of cross-gender edges in the network.

Problem 2: Recall that an affiliation network is a bipartite graph with two sets of nodes: one set P represents people and the other set F represents focal points. Further, each edge is between a node in P and one in F. Also recall that given an affiliation network G_A, one can construct a **projected network** G_P of G_A as follows: the set of nodes for G_P is P itself and G_P has an edge between a pair of nodes x and y in P if and only if x and y have at least one common focal point in F. This problem has two parts.

(a) Show two different affiliation networks G_A^1 and G_A^2 such that the projected networks for the two are *identical* (i.e., the two projected networks have the same set of edges).

(over)

(b) Consider the following social network G. Construct an affiliation network G_A such that G is the projected network of G_A. The network G_A must use *at most* 4 focal points.



Problem 3: For any positive integer n, prove that there is an undirected graph with $N \geq n$ nodes and $\Omega(N^2)$ edges such that the clustering coefficient of each node is zero. (An undirected graph G with N nodes is said to be **dense** if the number of edges in G is $\Omega(N^2)$. This problem points out that dense graphs may have small clustering coefficients.)

Problem 4: Let n be an even positive integer. Suppose G is an undirected graph with n nodes such that each node of G has a clustering coefficient of zero. Prove that the number of edges in G is at most $n^2/4$.