

Homework I

Date given: Sep. 8, 2015

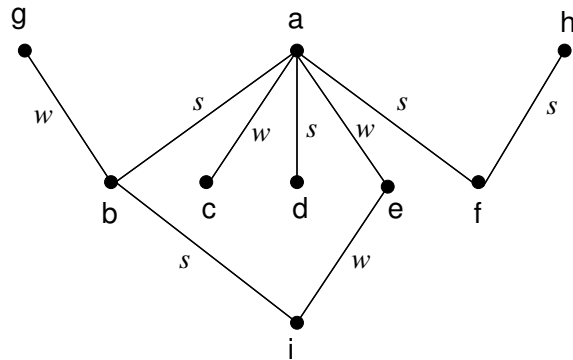
Due date: Sep. 17, 2015

Note: For all problems below, assume that the graphs are simple (i.e., they don't have multi-edges or self-loops).

Problem 1: Construct an undirected graph *without* edge weights that satisfies *all* of the following conditions: (i) the number of nodes in the graph is 16, (ii) each node has degree of at most 4 and (iii) the diameter of the graph is equal to 6.

Generalize the idea behind your construction for the above problem so that for any integer $n \geq 4$ such that n is a perfect square (i.e., $n = k^2$ for some integer k), your construction can be used to produce an undirected graph with n nodes so that the degree of each node is at most 4 and the diameter of the graph is close to $2\sqrt{n}$.

Problem 2: Consider the following graph in which edges are labeled s or w to indicate whether they represent strong or weak ties respectively.



Add a *minimum* number of undirected edges to the graph so that each node satisfies the *strong triadic closure* condition.

Your answer must show the graph that results after you add all the necessary edges. You must also indicate the reason for the addition of each new edge.

Problem 3: This problem has two parts. For each part, the graph example that you present must satisfy *both* of the following conditions: (i) it must be *connected* and (ii) it must have *at least* 5 nodes.

Recall that a **bridge** in a graph is an edge whose removal disconnects the graph. Further, an edge $\{x, y\}$ of a graph is a **local bridge** if x and y don't have any common neighbors.

- (a) Show an example of an undirected graph G_1 such that each edge of G_1 is a bridge.
- (b) Show an example of an undirected graph G_2 such that each edge of G_2 is a local bridge *but not* a bridge.