CSI 445/660 – Network Science – Fall 2015

Handout 7.4: An Algorithm for Generating Watts-Strogatz Graphs

Ref: R. Albert and A. Barabasi, "Statistical Mechanics of Complex Networks", *Reviews of Modern Physics*, Vol. 74, Jan. 2002, pp. 47–97. (See also Chapter 20 of the text by Easley & Kleinberg.)

Note: The following algorithm creates an undirected graph which has a number of properties including the "small world" property.

Input: Integers n (number of nodes) and K (assumed to be an even positive integer); a probability value β. The nodes are numbered 0 through $n-1$. (It is assumed that $n \gg K \gg \ln n \gg 1$.)

Output: An undirected graph with n nodes and $nK/2$ edges such that the average distance between any pair of nodes is $\ln(n)/\ln(K)$ with high probability.

Steps of the algorithm:

1. Start with the following graph: Arrange the node set $V = \{v_0, v_1, \ldots, v_{n-1}\}$ around a circle; each node v_i is adjacent to the $K/2$ nodes that precede or follow v_i in the circle. (Formally, edge $\{v_i, v_j\}$ is in the graph if and only if $|i - j| \equiv r \pmod{n}$ for some $r \in [1 \mathinner{..} K/2]$.)

   Note: An example of the starting graph with $n = 8$ and $K = 4$ is shown below.

2. **for** each edge $\{v_i, v_j\}$ in the graph created in Step 1 **do**

   Rewire edge $\{v_, v_j\}$ with probability β. (**Note:** The rewiring should not introduce self loops or multi-edges.)

3. Output the resulting undirected graph $G(V, E)$.

An example of a stating graph in Step 1: ($n = 8$ and $K = 4$)