

Handout 7.3 – Generating Random Graphs under Chung-Lu Model

Ref: F. Chung and L. Lu, *Complex Graphs and Networks*, CBMS Lecture Series, No. 107, AMS Publications, 2006.

Note: The following algorithm generates random undirected graphs. (The graph may have self loops). It is a generalization of the Erdős-Renyi (ER) model for generating random graphs. With appropriate choice of parameters, the algorithm can also generate undirected graphs whose degree distributions satisfy a power law.

Input: Integer n (number of nodes) and a sequence of n positive real numbers $\langle w_1, w_2, \dots, w_n \rangle$ such that $\max_{1 \leq i \leq n} \{w_i^2\} < \sum_{i=1}^n w_i$. (This condition is needed to ensure that values of certain expressions can represent probabilities.)

Output: An undirected graph with n nodes (numbered 1 through n) such that the expected degree of node i is w_i , $1 \leq i \leq n$.

Steps of the algorithm:

1. **for** $i = 1$ **to** n **do**

for $j = i$ **to** n **do**

Add the undirected edge $\{i, j\}$ with probability $= w_i w_j / \sum_{i=1}^n w_i$.

2. Output the resulting undirected graph.

Exercise: Show that for $1 \leq i \leq n$, the expected degree of node $i = w_i$.