CSI 445/660 – Network Science – Fall 2015

Handout 7.2 – Algorithm for Generating Undirected Power Law Graphs

Ref: R. Albert & A. Barabasi, "Statistical Mechanics of Complex Networks", *Reviews of Modern Physics*, Vol. 74, Jan. 2002, pp. 47–97.

Note: The following algorithm creates an undirected graph where node degrees have a power law distribution (with power law exponent 3).

Input: Integers m_0 (the initial number of nodes), $m \leq m_0$ (the number of edges to be added at each step) and n (the total number of nodes in the graph).

It is assumed that the nodes of the graph are numbered $1, 2, \ldots n$. Initially, the graph has nodes $1, \ldots m_0$, with a self-loop around each of the m_0 nodes. (See Item 1 in "Additional Notes" below.)

Output: A undirected graph where node degrees have a power law distribution.

Steps of the algorithm:

1. **for** $j = m_0 + 1$ **to** n **do**

   for $i = 1$ **to** m **do**

   (i) Choose a node from $\{1, \ldots, j-1\}$ using the following probability distribution: node y is chosen with probability p_y given by

   $$p_y \;=\; \frac{\text{degree}(y)}{\sum_{k=1}^{j-1} \text{degree}(k)} \;.$$

   (ii) Let x be the node chosen in Step (i). Add an edge between nodes j and x.

2. Output the resulting undirected graph $G(V, E)$.

Additional Notes:

1. Self loops are used on the initial set of m_0 nodes so that the denominator of the expression in Step 1(i) doesn't become zero.

2. Step 1(i) directly uses the "preferential attachment" idea; a node with a higher degree has a better chance of getting an edge to node j.

3. During the inner **for** loop of Step 1, node degrees change. So, Step 1(i) must take the new degrees into account.

4. An analysis is presented in the Albert/Barabasi paper to argue that (for large enough n), the fraction of nodes with indegree k follows the (approximate) power law k^{-c}, where $c = 3$. (Thus, the power law exponent is independent of m and m_0.)