CSI 445/660 – Network Science – Fall 2015

Handout 4.2 – Algorithm for Testing the Balance of a Signed Graph

Notes:

(a) A signed graph $G(V, E)$ is an undirected graph (not necessarily complete) where each edge has a label from {`'+'`, `'-'`}.

(b) A signed graph G is **balanced** if the vertices of G can be partitioned into two sets X and Y such that each internal edge (i.e., each edge between a pair of nodes in X or a pair of nodes in Y) has the label `'+'` and each external edge (i.e., an edge between a node in X and another in Y) has the label `'-'`).

(c) The following algorithm (taken from Chapter 5 of the text by Easley & Kleinberg) is based on Harary's characterization of balance: A signed graph is balanced if and only if it does not contain a cycle with an odd number of edges labeled `'-'`.

(d) Recall that an undirected graph H is **bipartite** if and only if H does not contain any cycle with an *odd* number of nodes.

Details of the Algorithm:

Input: A signed graph $G(V, E)$.

Output: "Yes" if G is balanced and "No" otherwise.

Steps:

1. Let $G^+(V, E^+)$ denote the subgraph of G containing only the `'+'` edges.

2. Find the connected components C_1, C_2, ..., C_k of G^+.

3. If for some C_i, there is a `'-'` edge joining two nodes in C_i, then output "No" and **stop**.

4. Collapse each C_i to a single node and add the `'-'` edges of G. Replace multi-edges with a single edge and let H denote the resulting graph.

5. If H is bipartite, then output "Yes"; else output "No".

Exercise: Assume that determining whether a given undirected graph $H(V_1, E_1)$ is bipartite can be done in $O(|V_1| + |E_1|)$ time. Find the running time of the above algorithm.