CSI 445/660 – Network Science – Fall 2015

Handout 4.1 – Algorithms for Testing Structural Balance of a Labeled Clique

Algorithm I: This is a straightforward (but slow) algorithm based directly on the definition of structural balance for a labeled clique.

Input: A clique G with n nodes where each edge has a '+' or '-' label.

Output: "Yes" if G is balanced and "No" otherwise.

Outline of the Algorithm:

1. **for** each triple of nodes x, y and z **do**
   if (triangle $\{x, y, z\}$ is **not** balanced)
   Output "No" and **stop**.
2. Output "Yes".

Running time: $O(n^3)$ (since there are $\binom{n}{3} = O(n^3)$ triangles in a clique with n nodes).

Algorithm II: This algorithm is based on the Cartwright-Harary Theorem and is asymptotically faster. The description of the algorithm below ignores the trivial cases (where all edges have the '+' label or the '-' label).

Input: A clique G with n nodes where each edge has a '+' or '-' label.

Output: "Yes" if G is balanced and "No" otherwise.

Outline of the Algorithm:

1. Choose an arbitrary node a of G.
2. Construct set X consisting of a and all friends of a.
3. Let Y be the remaining set of nodes.
4. **if** (X has a pair of nodes p and q such that the label of edge $\{p, q\}$ is '-')
   Output "No" and **stop**.
5. **if** (Y has a pair of nodes p and q such that the label of edge $\{p, q\}$ is '-')
   Output "No" and **stop**.
6. **if** (X has a node p and Y has a node q such that the label of edge $\{p, q\}$ is '+')
   Output "No" and **stop**.
7. Output "Yes".

Running time: $O(n^2)$ (since each of the seven steps runs in $O(n^2)$ time). Since the graph has $\Omega(n^2)$ edges, the running time of this algorithm is linear in the size of the input.