

**Handout 3.1 – A Simple Model for Measuring Homophily**

**Notes:**

- (a) It is assumed that the characteristic for which homophily needs to be measured has only two distinct values. (The discussion below uses gender as the characteristic.)
- (b) The derivation below is based on a simple random mixing model and is discussed in Chapter 4 of the text by Easley & Kleinberg.

**Derivation of the Homophily Test:**

1. Let  $N_B$  and  $N_G$  denote respectively the numbers of boys girls in the network.
2. So, the total number of nodes  $n$  in the network is given by  $n = N_B + N_G$ . Let  $m$  denote the number of edges in the network.
3. A node chosen uniformly randomly will represent a boy with probability  $p = N_B/(N_B + N_G)$  and will represent a girl with probability  $q = 1 - p = N_G/(N_B + N_G)$ .
4. What is the probability of generating a cross-gender edge?



Each of these events happens with probability  $pq$ . So, the probability of generating a cross-gender edge in the model is  $2pq$ .

5. As a consequence, the average number of cross-gender edges under the model is  $2pq \times m$ .
6. Thus, under the simple random mixing model, on the average, the fraction  $2pq$  of the edges will be cross-gender edges.

So, the test for homophily can be summarized as follows.

**Homophily Test:** If the fraction of cross-gender edges in the network is significantly below  $2pq$ , then there is evidence for homophily.