This notebook presents the method used to derive the numerical solution for the break-even singularity asset level in the Markov Consumption Problem, as described in Section 3 in the paper, "The Markov Consumption Problem," by Michael Sattinger. The notebook then generates the figures used in the text. Derivations are provided in a separate notebook. The differential equations from 2.5 and 2.6 (with g for γ) are

```
dC1dA[C1_, C2_, A_] := (C1 * (r - b - p1 * (1 - (C2 / C1) ^ (g - 1))) / (1 - g)) / (y1 + r * A - C1)
dC2dA[C1_, C2_, A_] := (C2 * (r - b - p2 * (1 - (C1 / C2) ^ (g - 1))) / (1 - g)) / (y2 + r * A - C2)
```

Following are the two solutions to the slope of dC1 / dA at a singularity As.

x1[C1_, C2_, A_] :=

$$-\left(-C1\,C2\left(\frac{C2}{C1}\right)^{g} p1 + C1\,C2\left(\frac{C2}{C1}\right)^{g} g\,p1 + C2^{2}\,r - C2^{2}\,g\,r + A\,C1\left(\frac{C2}{C1}\right)^{g} p1\,r - A\,C1\left(\frac{C2}{C1}\right)^{g} g\,p1\,r - A\,C2\,r^{2} + A\,C2\,g\,r^{2} + C1\left(\frac{C2}{C1}\right)^{g} p1\,y2 - C1\left(\frac{C2}{C1}\right)^{g} g\,p1\,y2 - C2\,r\,y2 + C2\,g\,r\,y2 + \sqrt{\left((-1+g)\left(-4\,C1\,C2\left(\frac{C2}{C1}\right)^{g} p1\left(b\,C1 - \left(\frac{C1}{C2}\right)^{g} C2\,p2 + C1\,(p2 - r)\right)(C2 - A\,r - y2) + (-1+g)\left(C1\left(\frac{C2}{C1}\right)^{g} p1 - C2\,r\right)^{2}(-C2 + A\,r + y2)^{2}\right)\right)}\right) \Big/ \left(2\,C2\,(-1+g)\,(C2 - A\,r - y2)\right)$$

x2[C1_, C2_, A_] :=

$$\left(C1\,C2\left(\frac{C2}{C1}\right)^{g} p1 - C1\,C2\left(\frac{C2}{C1}\right)^{g} g\,p1 - C2^{2}\,r + C2^{2}\,g\,r - A\,C1\left(\frac{C2}{C1}\right)^{g} p1\,r + A\,C1\left(\frac{C2}{C1}\right)^{g} g\,p1\,r + A\,C2\,r^{2} - A\,C2\,g\,r^{2} - C1\left(\frac{C2}{C1}\right)^{g} p1\,y2 + C1\left(\frac{C2}{C1}\right)^{g} g\,p1\,y2 + C2\,r\,y2 - C2\,g\,r\,y2 + \sqrt{\left((-1+g)\left(-4\,C1\,C2\left(\frac{C2}{C1}\right)^{g} p1\left(b\,C1 - \left(\frac{C1}{C2}\right)^{g} C2\,p2 + C1\,(p2 - r)\right)(C2 - A\,r - y2) + (-1+g)\left(C1\left(\frac{C2}{C1}\right)^{g} p1 - C2\,r\right)^{2}(-C2 + A\,r + y2)^{2}\right)\right)}\right) \Big/ \left(2\,C2\,(-1+g)\,(C2 - A\,r - y2)\right)$$

The following are the parameter values used in the numerical solutions, as described in the paper.

```
b = 0.03
r = .0270352
p2 = .0736241 * 52
p1 = 0.24474850760576117`
y1 = 1
y2 = .5
g = .5
```

```
0.03

0.0270352

3.82845

0.244749

1

0.5

0.5
```

Following the procedure outlined in Section 3 for the natural debt limit $-y2/r$, the first step is to estimate the asset level where the singularity occurs, As, such that consumption approaches zero at $-y2/r$. Let Asing be the initial estimate and let C1sing and C2sing be the consumption levels at Asing. Suppose the initial estimate is Asing = −17. Then C1sing = $rAsing + y1$ and C2sing = $C1sing \left(\frac{b + p1 - r}{p1}\right)^{\frac{1}{-1+g}}$ from 2.8 in the text. Then using the positive slope x1[C1, C2, A] calculated from l' Hospital' s Rule and dC2dA[C1, C2, A], consumption in the two states is calculated at a small distance .0001 below Asing. Let Atmp (for A temporary) equal Asing − .0001 and let C1tmp and C2tmp be the corresponding consumption levels at Atmp. the differential equations are then solved backwards from Atmp down to a small increment (Ainc = .00000001) above the natural debt limit $-y2/r$. Then the value of C2 is calculated at Atmp and compared with the value it would have according to Result 3, given by $Ainc * (b + p2 - r * g) / (1 - g)$. The difference in consumption levels is given by C20 and used to determine the next value of Asing.

```
Asing = -17.5
C1sing = y1 + r * Asing
C2sing = C1sing ((b + p1 - r)/p1)^(1/(-1+g))
x1[C1sing, C2sing, Asing]
x2[C1sing, C2sing, Asing]
inc = .0001
Atmp = Asing - inc
C1tmp = C1sing - x1[C1sing, C2sing, Asing] * inc
C2tmp = C2sing - dC2dA[C1sing, C2sing, Asing] * inc
sola1 = NDSolve[{C1'[A] == dC1dA[C1[A], C2[A], A],
   C2'[A] == dC2dA[C1[A], C2[A], A], C1[Atmp] == C1tmp, C2[Atmp] == C2tmp},
  {C1, C2}, {A, -y2 / r + .00000000001, Atmp}, WorkingPrecision → 24]
Ainc = .00000001
C2eta = Ainc * (b + p2 - r * g) / (1 - g)
C2lim = C2[-y2 / r + Ainc] /. sola1
C20 = C2lim - C2eta
```

General::spell1 :
Possible spelling error: new symbol name "Asing" is similar to existing symbol "Using". More…

-17.5

0.526884

0.514347

0.0853632

-0.306041

0.0001

-17.5001

0.526875

0.514337

NDSolve::precw :
The precision of the differential equation ({C1′[A] == (2. <<1>> (-0.0029648 - 0.244749 (1 - <<1>>)))/(1 + 0.0270352 A - C1[A]), <<1>> == <<1>>, <<1>>, C2[-17.5001] == 0.514337}) is less than WorkingPrecision (24.`). More…

NDSolve::ndsz :
At A == -18.2703, step size is effectively zero; singularity or stiff system suspected. More…

{{C1 → InterpolatingFunction[{{-18.2703, -17.500099999999997669420}}, <>],
  C2 → InterpolatingFunction[{{-18.2703, -17.500099999999997669420}}, <>]}}

General::spell1 : Possible spelling error: new symbol name "Ainc" is similar to existing symbol "inc". More…

$1. \times 10^{-8}$

7.68987×10^{-8}

```
InterpolatingFunction::dmval : Input value {-18.4944} lies outside
   the range of data in the interpolating function. Extrapolation will be used. More…
```

$\{-2.22938 \times 10^{28}\}$

$\{-2.22938 \times 10^{28}\}$

The solution above reaches a singularity ("step size is zero") at -18.2703, which is greater than the natural debt limit -y2/r = -18.4944. This occurs because consumption in state 2 reaches zero before assets fall to the natural debt limit. From step 5 in the list of steps for determining initial conditions, the next value to try for As should be lower (so that C2 will be higher). Next, try Asing = -18.

```
Asing = -18
C1sing = y1 + r * Asing
C2sing = C1sing ((b + p1 - r)/p1)^(1/(-1+g))
x1[C1sing, C2sing, Asing]
x2[C1sing, C2sing, Asing]
inc = .0001
Atmp = Asing - inc
C1tmp = C1sing - x1[C1sing, C2sing, Asing] * inc
C2tmp = C2sing - dC2dA[C1sing, C2sing, Asing] * inc
sola1 = NDSolve[{C1'[A] == dC1dA[C1[A], C2[A], A],
   C2'[A] == dC2dA[C1[A], C2[A], A], C1[Atmp] == C1tmp, C2[Atmp] == C2tmp},
  {C1, C2}, {A, -y2 / r + .00000000001, Atmp}, WorkingPrecision → 24]
Ainc = .00000001
C2eta = Ainc * (b + p2 - r * g) / (1 - g)
C2lim = C2[-y2 / r + Ainc] /. sola1
C20 = C2lim - C2eta
```

-18

0.513366

0.501151

0.0835999

-0.304278

0.0001

-18.0001

0.513358

0.501141

NDSolve::precw :
The precision of the differential equation (<<1>>) is less than WorkingPrecision (24.`). More…

{{C1 → InterpolatingFunction[{{-18.4944, -18.000099999999997669420}}, <>],
  C2 → InterpolatingFunction[{{-18.4944, -18.000099999999997669420}}, <>]}}

$1. \times 10^{-8}$

7.68987×10^{-8}

{0.409756}

{0.409756}

Using Asing = -18, consumption in state 2 at - y2 / r + Ainc is too high (0.4097). Following the rule in step 5 in the text, the next value of As to try is above - 18 and below - 17.5. Iterating in this way leads to the following solution.

```
Asing = -17.713509544035015
C1sing = y1 + r * Asing
C2sing = C1sing ((b + p1 - r)/p1)^(1/(-1+g))
x1[C1sing, C2sing, Asing]
x2[C1sing, C2sing, Asing]
inc = .0001
Atmp = Asing - inc
C1tmp = C1sing - x1[C1sing, C2sing, Asing] * inc
C2tmp = C2sing - dC2dA[C1sing, C2sing, Asing] * inc
sola1 = NDSolve[{C1'[A] == dC1dA[C1[A], C2[A], A],
   C2'[A] == dC2dA[C1[A], C2[A], A], C1[Atmp] == C1tmp, C2[Atmp] == C2tmp},
  {C1, C2}, {A, -y2 / r + .00000000001, Atmp}, WorkingPrecision → 24]
Ainc = .00000001
C2eta = Ainc * (b + p2 - r * g) / (1 - g)
C2lim = C2[-y2 / r + Ainc] /. sola1
C20 = C2lim - C2eta
```

General::spell1 :
Possible spelling error: new symbol name "Asing" is similar to existing symbol "Using". More…

-17.7135

0.521112

0.508712

0.0846119

-0.30529

0.0001

-17.7136

0.521103

0.508702

NDSolve::precw : The precision of the differential equation (
$\{C1'[A] == \frac{2.\,C1[A]\,(-0.0029648 - 0.244749\,(1 - \text{Power}[\ll 2 \gg]))}{1 + 0.0270352\,A - C1[A]},\ C2'[A] == \frac{2.\,(-0.0029648 - 3.82845\,(1 - \ll 1 \gg))\,C2[A]}{0.5 + 0.0270352\,A - C2[A]},$
$C1[-17.7136] == \ll 19 \gg,\ C2[-17.7136] == 0.508702\}$) is less than WorkingPrecision (24.`). More…

{{C1 → InterpolatingFunction[{{-18.4944, -17.71360954403501608567 24}}, <>],
C2 → InterpolatingFunction[{{-18.4944, -17.71360954403501608567 24}}, <>]}}

General::spell1 : Possible spelling error: new symbol name "Ainc" is similar to existing symbol "inc". More…

$1. \times 10^{-8}$

7.68987×10^{-8}

$\{7.68997 \times 10^{-8}\}$

$\{9.81818 \times 10^{-13}\}$

Following footnote 8 in the text,
the maximum error in C2 at Ainc is less than 7.6898712`*^-8 + 9.818176891567788`*^-13.

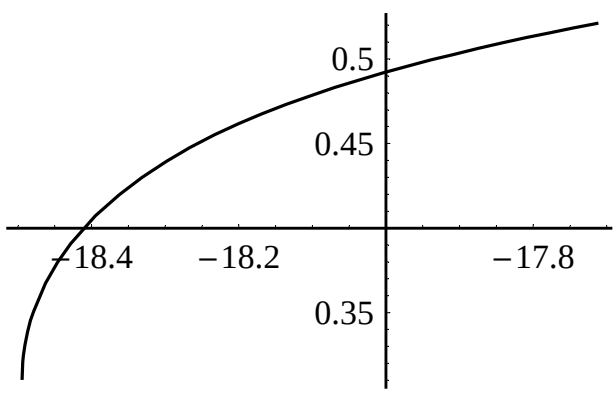
With As determined,
the next step is to construct Figure 1. The solution sola1 above yields the consumption functions between the natural debt limit and the break – even singularity asset level As. Below are the graphs for these consumption functions.

```
$TextStyle = {FontFamily -> "Times", FontSize -> 16}
<< Graphics`Arrow`
```

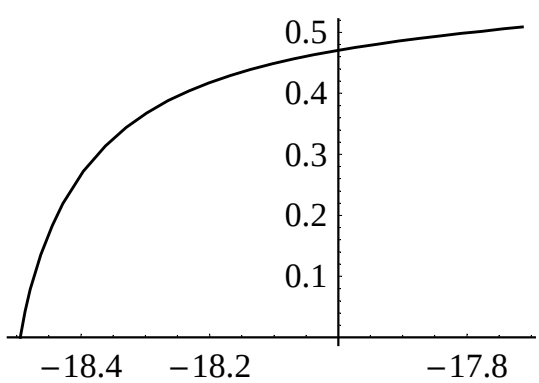
```
{FontFamily → Times, FontSize → 16}
```

```
Asing = -17.713509544035015
abeg = -y2 / r + .00000000001
aend = Asing - inc
g1 = Plot[C1[A] /. sola1, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
g2 = Plot[C2[A] /. sola1, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
glower = Show[g1, g2, AxesOrigin → {-y2 / r, 0}]
```

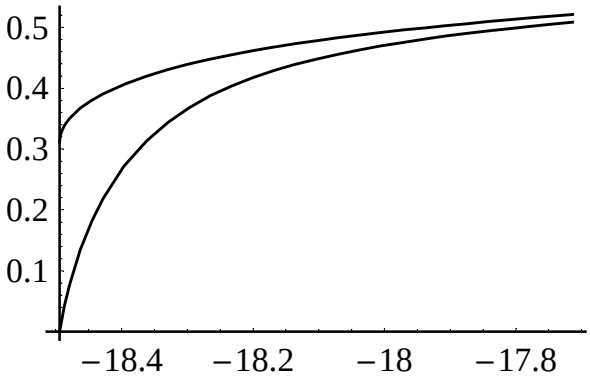
```
-17.7135

-18.4944

-17.7136
```




```
- Graphics -
```



- Graphics -



- Graphics -

Next is the solution for consumption above As.

```
Asing = -17.713509544035015
C1sing = y1 + r * Asing
C2sing = C1sing ((b + p1 - r)/p1)^(1/(-1+g))
x1[C1sing, C2sing, Asing]
x2[C1sing, C2sing, Asing]
inc = .0001
Atmp = Asing + inc
C1tmp = C1sing + x1[C1sing, C2sing, Asing] * inc
C2tmp = C2sing + dC2dA[C1sing, C2sing, Asing] * inc
sola2 = NDSolve[{C1'[A] == dC1dA[C1[A], C2[A], A], C2'[A] == dC2dA[C1[A], C2[A], A],
   C1[Atmp] == C1tmp, C2[Atmp] == C2tmp}, {C1, C2}, {A, Atmp, 10}, WorkingPrecision → 24]
```

-17.7135

0.521112

0.508712

0.0846119

-0.30529

0.0001

-17.7134

0.52112

0.508723

NDSolve::precw : The precision of the differential equation ($\{C1'[A] == \frac{2.\,C1[A]\,(-0.0029648 - 0.244749\,(1 - \text{Power}[\ll 2 \gg]))}{1 + 0.0270352\,A - C1[A]},\ C2'[A] == \frac{2.\,(-0.0029648 - 3.82845\,(1 - \ll 1 \gg))\,C2[A]}{0.5 + 0.0270352\,A - C2[A]},$ $C1[-17.7134] == \ll 19 \gg,\ C2[-17.7134] == 0.508723\}$) is less than WorkingPrecision (24.`). More…

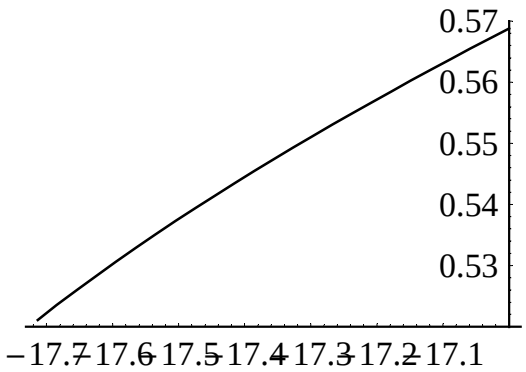
```
{{C1 → InterpolatingFunction[{{-17.713409544035016551788 4, 10.000000000000000000000}},
    <>], C2 → InterpolatingFunction[
    {{-17.713409544035016551788 4, 10.000000000000000000000}}, <>]}}
```

```
Asing = -17.713509544035015
abeg = Asing + .0001
aend = -17
g3 = Plot[C1[A] /. sola2, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
g4 = Plot[C2[A] /. sola2, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
gupper = Show[g3, g4]
```

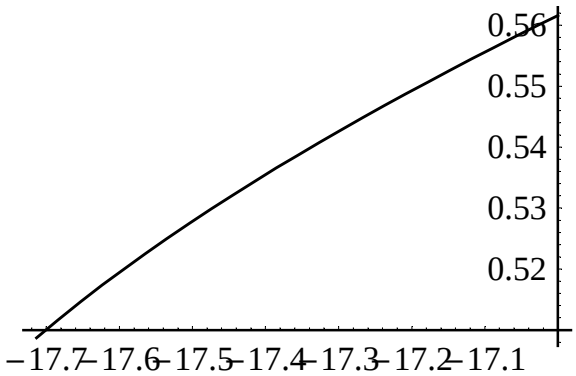
-17.7135

-17.7134

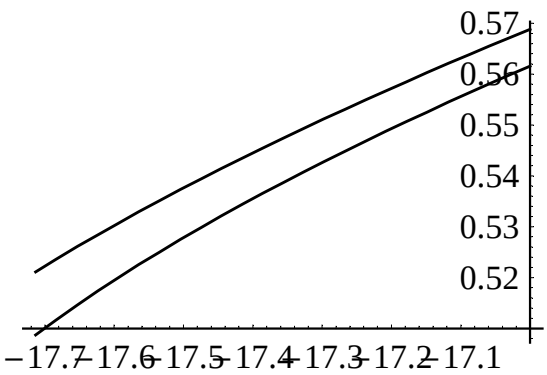
-17



- Graphics -

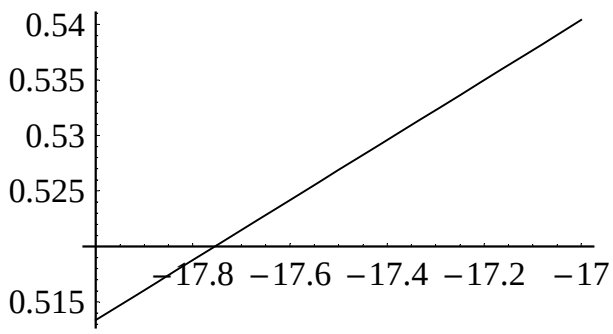


- Graphics -

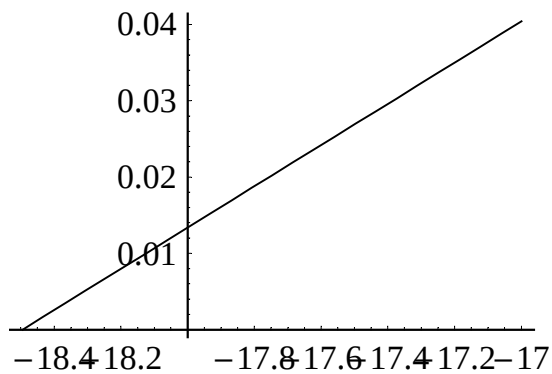


- Graphics -

```
ginc1nocc = Plot[r * a + y1, {a, -18, -17}, PlotStyle -> {Thickness[.004]},
    AxesStyle -> {Thickness[.005]}]
ginc2nocc = Plot[r * a + y2, {a, -y2 / r, -17}, PlotStyle -> {Thickness[.004]},
    AxesStyle -> {Thickness[.005]}]
```



- Graphics -



- Graphics -

```
ld1 = Graphics[
  {Dashing[{.02, .02}], Thickness[.005], Line[{{Asing, 0}, {Asing, r * Asing + y1}}]}]
gh1 = Graphics[{Text["rA+y1", {-17, .52}, {1, 0}], Text["rA+y2", {-17, .07}, {1, 0}]}]
gh2 = Graphics[Text["As", {Asing, -.01}, {0, 1}]]
gh3 = Graphics[{Text["C1", {-18.42, .34}, {1, 0}], Text["C2", {-18.4, .1}, {1, 0}]}]
ar1 = Graphics[{Thickness[.004], Arrow[{-18.1, .42}, {-18.3, .28}, HeadWidth → .3],
   Arrow[{-18.4, .445}, {-18.1, .515}, HeadWidth → .3],
   Arrow[{-17.2, .59}, {-17.5, .562}, HeadWidth → .3]}]
Show[glower, gupper, ginc1nocc, ginc2nocc, ar1, ld1, gh1, gh2, gh3,
 AxesLabel → {"Assets, A", "Consumption"}, PlotRange → {-.04, .6}]
```

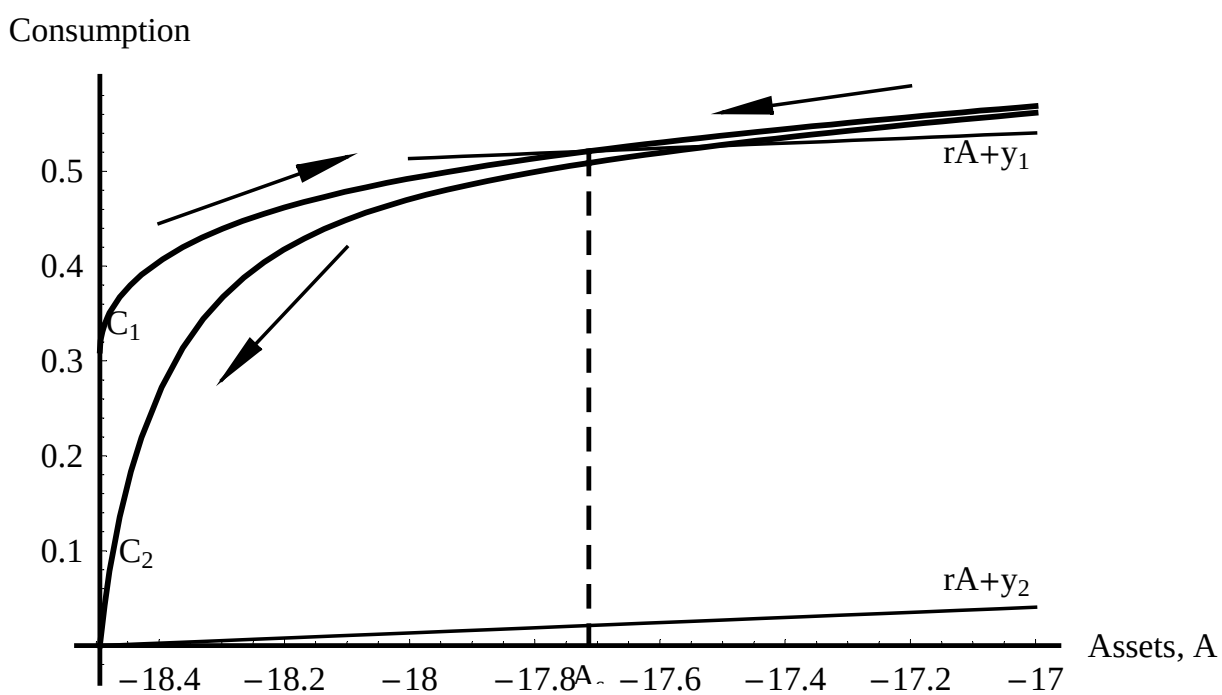
- Graphics -

- Graphics -

- Graphics -

- Graphics -

- Graphics -



- Graphics -

Next, consider the construction of Figure 2 in the text, which compares consumption while unemployed with an ad hoc debt limit versus a natural debt limit. The first step is to determine the break – even level of assets with an ad hoc debt limit along with the solution for consumption levels below the break – even level of assets. For Figure 2, the ad hoc debt limit is set at 0.

```
Asing = .439181803385
C1sing = y1 + r * Asing
C2sing = C1sing ((b + p1 - r)/p1)^(1/(-1+g))
x1[C1sing, C2sing, Asing]
x2[C1sing, C2sing, Asing]
inc = .0001
Atmp = Asing - inc
C1tmp = C1sing - x1[C1sing, C2sing, Asing] * inc
C2tmp = C2sing - dC2dA[C1sing, C2sing, Asing] * inc
sola3 = NDSolve[{C1'[A] == dC1dA[C1[A], C2[A], A],
   C2'[A] == dC2dA[C1[A], C2[A], A], C1[Atmp] == C1tmp, C2[Atmp] == C2tmp},
  {C1, C2}, {A, .0000000000001, Atmp}, WorkingPrecision → 24]
c2old = c20
c20 = C2[.0000000000001] /. sola3
c20 - c2old
```

0.439182

1.01187

0.987797

0.141779

-0.362457

0.0001

0.439082

1.01186

0.987776

NDSolve::precw :
The precision of the differential equation ({C1′[A] == (2. <<1>> (-0.0029648 - 0.244749 (1 - <<1>>)))/(1 + 0.0270352 A - C1[A]),
<<2>>, C2[0.439082] == 0.987776}) is less than WorkingPrecision (24.`). More…

{{C1 → InterpolatingFunction[{{1. × 10^{-13}, 0.439081803385000024597673}}, <>],
C2 → InterpolatingFunction[{{1. × 10^{-13}, 0.439081803385000024597673}}, <>]}}

General::spell1 : Possible spelling error: new symbol name "c20" is similar to existing symbol "C20". More…

c20

{0.500001}

{0.}

The following provides the solution for consumption above the break - even level of assets for the ad hoc debt limit.

```
Asing = .439181803385
C1sing = y1 + r * Asing
C2sing = C1sing ((b + p1 - r)/p1)^(1/(-1+g))
x1[C1sing, C2sing, Asing]
x2[C1sing, C2sing, Asing]
inc = .0001
Atmp = Asing + inc
C1tmp = C1sing + x1[C1sing, C2sing, Asing] * inc
C2tmp = C2sing + dC2dA[C1sing, C2sing, Asing] * inc
sola4 = NDSolve[{C1'[A] == dC1dA[C1[A], C2[A], A], C2'[A] == dC2dA[C1[A], C2[A], A],
   C1[Atmp] == C1tmp, C2[Atmp] == C2tmp}, {C1, C2}, {A, Atmp, 10}, WorkingPrecision → 24]
```

0.439182

1.01187

0.987797

0.141779

-0.362457

0.0001

0.439282

1.01189

0.987817

NDSolve::precw :
The precision of the differential equation ({C1′[A] == (2. <<1>> (-0.0029648 - 0.244749 (1 - <<1>>)))/(1 + 0.0270352 A - C1[A]), <<2>>, C2[0.439282] == 0.987817}) is less than WorkingPrecision (24.`). More…

```
{{C1 → InterpolatingFunction[{{0.439281803385000002570848, 10.000000000000000000000}},
    <>], C2 → InterpolatingFunction[
    {{0.439281803385000002570848, 10.000000000000000000000}}, <>]}}
```

The following constructs the consumption curves for the ad hoc debt limit below the singularity.

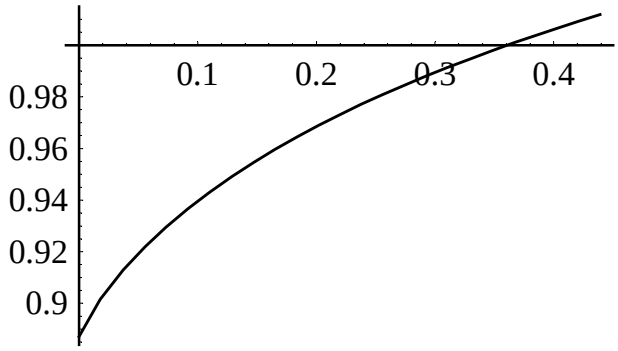
```
Asing = .439181803385
inc = .0001
abeg = .00000000001
aend = Asing - inc
g5 = Plot[C1[A] /. sola3, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
g6 = Plot[C2[A] /. sola3, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
gadlower = Show[g5, g6, AxesOrigin → {0, 0}]
```

0.439182

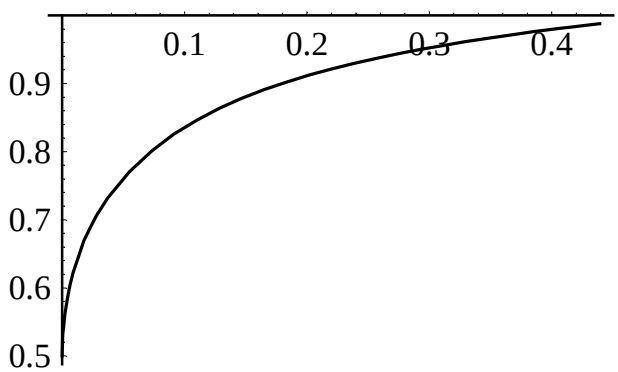
0.0001

$1. \times 10^{-11}$

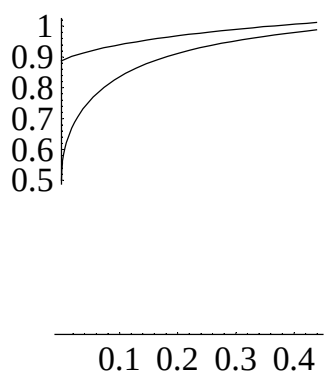
0.439082



- Graphics -



- Graphics -



- Graphics -

Next, the consumption curves for the ad hoc debt limit above the singularity.

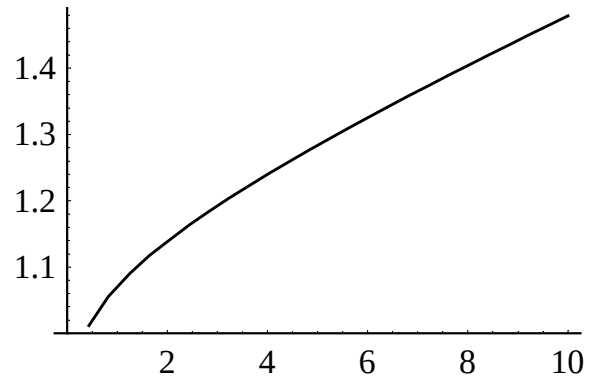
```
Asing = .439181803385
inc = .0001
abeg = Asing + inc
aend = 10
g7 = Plot[C1[A] /. sola4, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
g8 = Plot[C2[A] /. sola4, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
gadupper = Show[g7, g8, AxesOrigin → {0, 0}]
```

0.439182

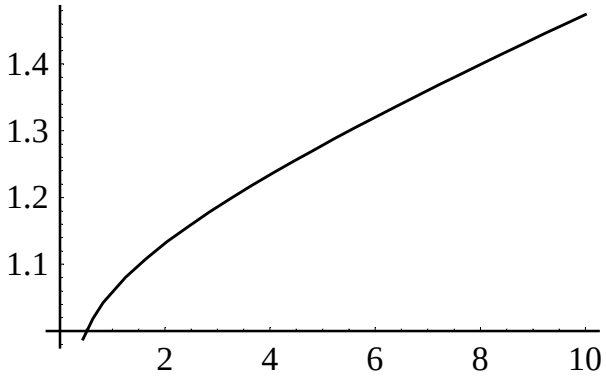
0.0001

0.439282

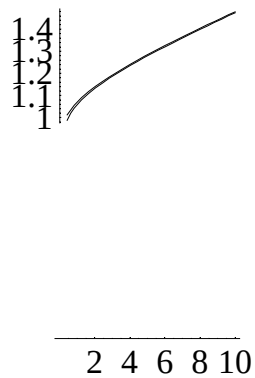
10



- Graphics -



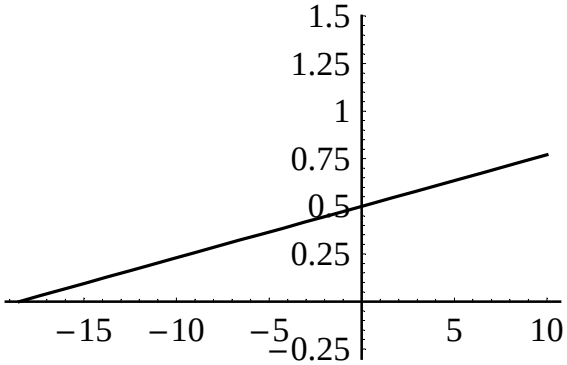
- Graphics -



- Graphics -

Graph for income in state 2.

```
ginc2nocc = Plot[r * A + y2, {A, -y2 / r, 10}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}, PlotRange → {-.3, 1.5}]
```



- Graphics -

Graphics for Figure 2.

```
gh1nocc = Graphics[Text["Income", {-10, .38}, {0, 1}]]
gh2nocc = Graphics[{Text["Ad Hoc Debt Limit", {2.4, 1.1}, {-1, 0}],
   Text["Natural Debt Limit", {-10, .88}, {1, 0}]}]
g11 = Graphics[Text["-y2/r", {-y2 / r, 0}, {0, 1}]]
```

- Graphics -

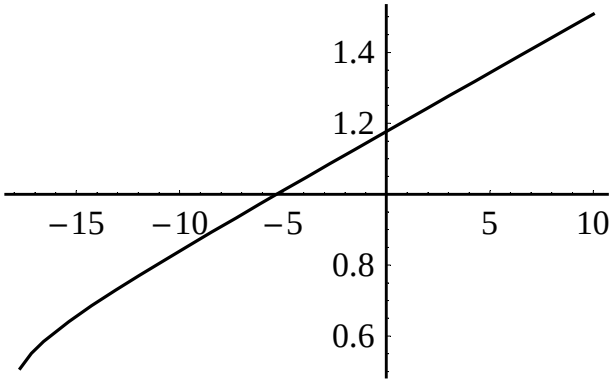
- Graphics -

- Graphics -

Graph for consumption in state 2 above break -
even asset level with natural debt limit (extending to A = 10)

```
Asing = -17.713509544035015
abeg = Asing + .0001
aend = 10
g9 = Plot[C2[A] /. sola2, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
```

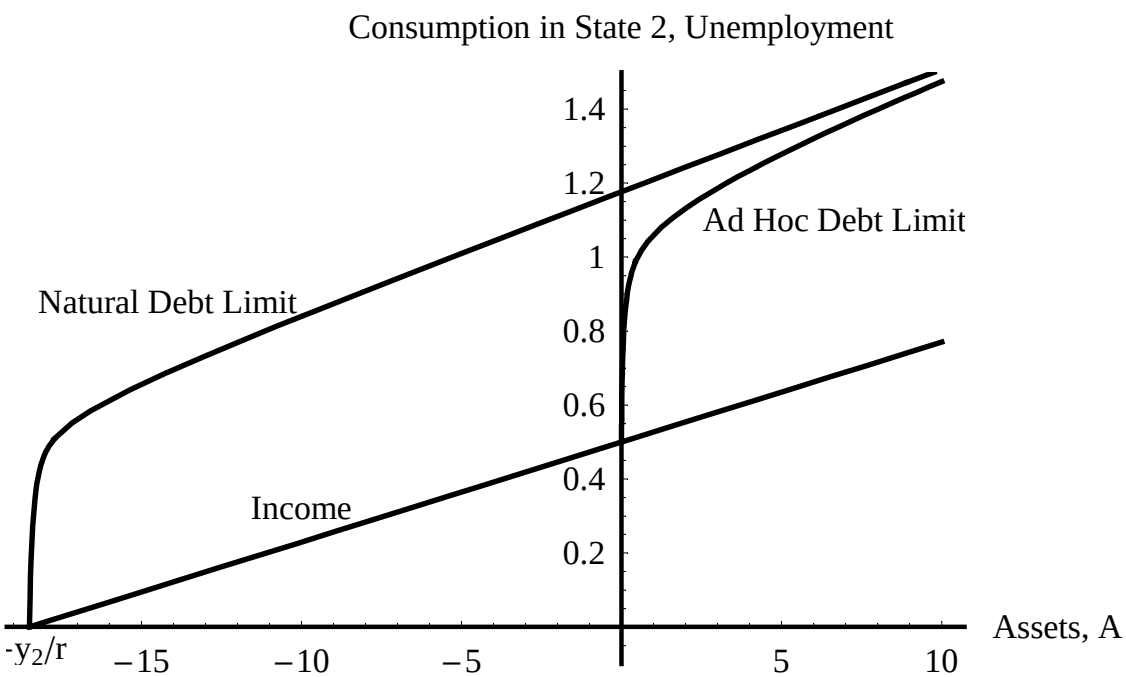
```
-17.7135

-17.7134

10
```



- Graphics -

Figure 4.1

```
Show[g6, g2, g9, g8, g11, ginc2nocc, gh1nocc, gh2nocc, AxesOrigin → {0, 0},
 AxesLabel → {"Assets, A", "Consumption in State 2, Unemployment"},
 PlotRange → {-.1, 1.5}]
```



- Graphics -

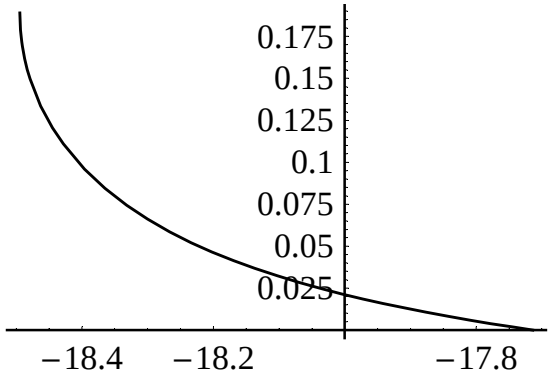
As an alternative to Figure 1,
it is possible to graph savings as a function of assets in the two states. The following generates a graph of savings in the case with the natural debt limit.

```
Inc1[A_] := r * A + y1
Inc2[A_] := r * A + y2

Asing = -17.713509544035015
abeg = -y2 / r + .00000000001
aend = Asing - inc
g10 = Plot[Inc1[A] - C1[A] /. sola1, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
g11 = Plot[Inc2[A] - C2[A] /. sola1, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
gslower = Show[g10, g11, AxesOrigin → {-y2 / r, 0}]
```

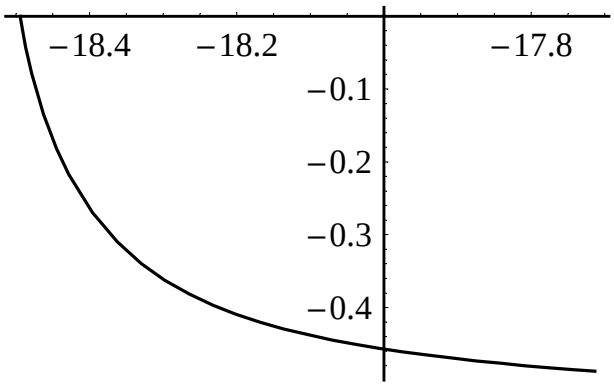
-17.7135

-18.4944

-17.7136

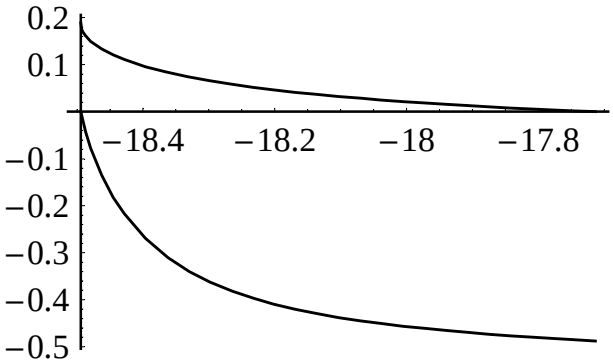


- Graphics -



- Graphics -

```
General::spell1 :
 Possible spelling error: new symbol name "gslower" is similar to existing symbol "glower". More…
```



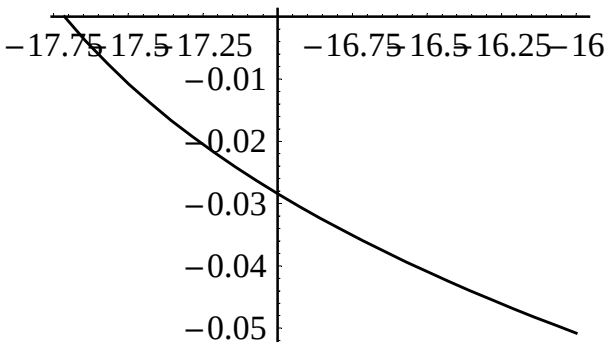
- Graphics -

```
Asing = -17.713509544035015
abeg = Asing + .0001
aend = -16
g12 = Plot[Inc1[A] - C1[A] /. sola2, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
g13 = Plot[Inc2[A] - C2[A] /. sola2, {A, abeg, aend}, PlotStyle -> {Thickness[.006]},
    AxesStyle -> {Thickness[.005]}]
gsupper = Show[g12, g13, PlotRange → {-.6, .2}]
```

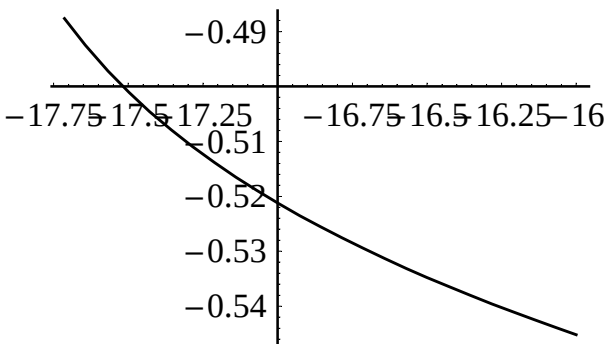
-17.7135

-17.7134

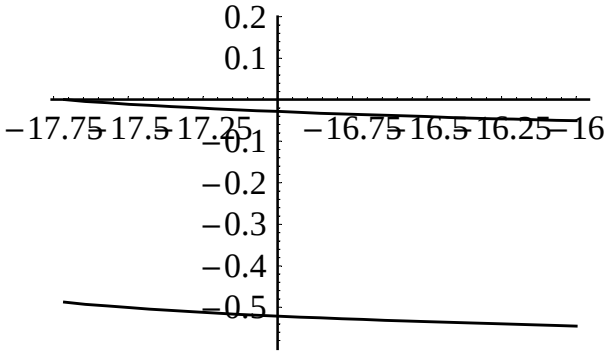
-16



- Graphics -

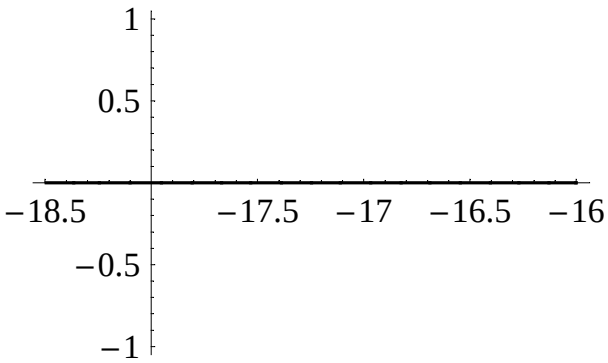


- Graphics -



- Graphics -

```
g0 = Plot[0, {A, -y2 / r, -16}, PlotStyle -> Thickness[.006]]
```



- Graphics -

```
ld1 = Graphics[{Dashing[{.02, .02}], Thickness[.005], Line[{{Asing, -.6}, {Asing, 0}}]}]
gh1 = Graphics[{Text["rA+y1", {-17, .52}, {1, 0}], Text["rA+y2", {-17, .07}, {1, 0}]}]
gh2 = Graphics[Text["As", {Asing, -.6}, {0, 1}]]
gh3 = Graphics[{Text["C1", {-18.42, .34}, {1, 0}], Text["C2", {-18.4, .1}, {1, 0}]}]
ar1 = Graphics[{Thickness[.004], Arrow[{-18.1, .42}, {-18.3, .28}, HeadWidth → .3],
   Arrow[{-18.4, .445}, {-18.1, .515}, HeadWidth → .3],
   Arrow[{-17.2, .59}, {-17.5, .562}, HeadWidth → .3]}]
```

- Graphics -

- Graphics -

- Graphics -

- Graphics -

- Graphics -

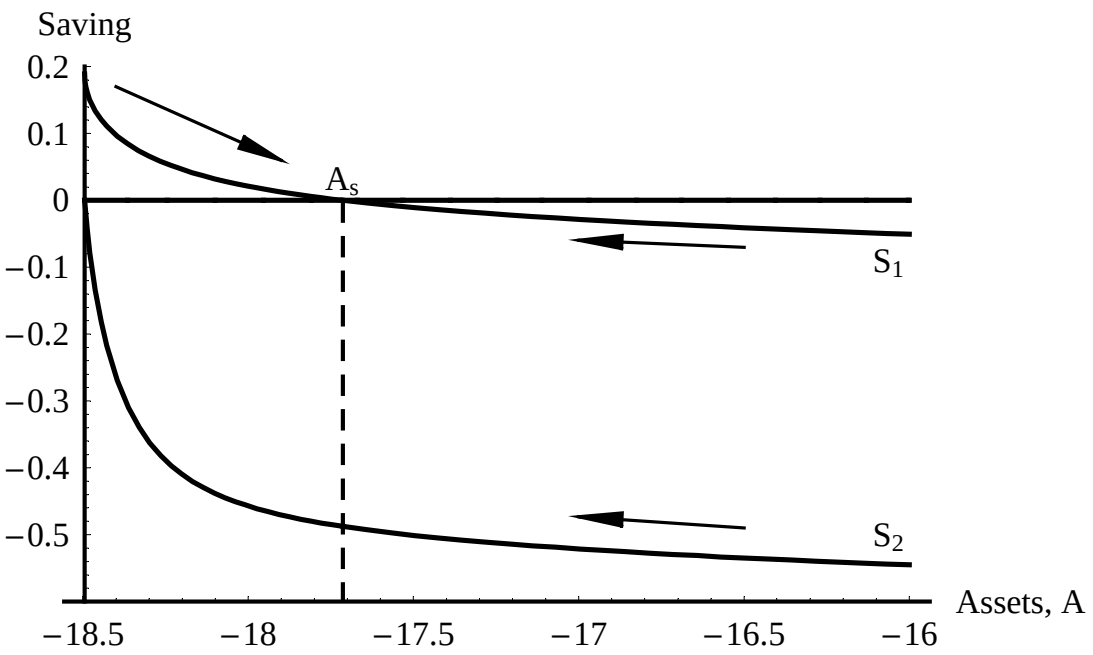
```
ar1 = Graphics[{Thickness[.004], Arrow[{-16.5, -.49}, {-17, -.473}, HeadWidth → .3],
   Arrow[{-16.5, -.07}, {-17, -.06}, HeadWidth → .3],
   Arrow[{-18.4, .17}, {-17.9, .06}, HeadWidth → .3]}]
ld1 = Graphics[{Dashing[{.02, .02}], Thickness[.005], Line[{{Asing, -.6}, {Asing, 0}}]}]
gh1 = Graphics[Text["As", {Asing, 0}, {0, -1}]]
gh3 = Graphics[{Text["S1", {-16, -.09}, {1, 0}], Text["S2", {-16, -.5}, {1, 0}]}]
Show[gsupper, gslower, g0, ld1, gh1, gh3, ar1, AxesOrigin → {-y2 / r, -.6},
 AxesLabel → {"Assets, A", "Saving"}, PlotRange → {-.6, .2}]
```

- Graphics -

- Graphics -

- Graphics -

- Graphics -



- Graphics -