HOLIDAY PUZZLE SOLUTION: Groundhog Day, 2018

THIS YEAR: PEOPLE AND ORNAMENTS!

There is a set of 68 people, and a person in this set can pick anywhere from 0 to 8 different kinds of ornaments out of 8 different kinds total. (We assume that the kinds of ornaments never run out.) Must there be 6 people in this set and 3 kinds of ornaments such that each of these 6 people pick all of these 3 kinds of ornaments or each of of these 6 people pick none of these 3 kinds of ornaments? Justify your answer!

SOLUTION: This solution is based on one submitted by Richard Goldstein. There does not have to be 6 people in the set and 3 kinds of ornaments such that each of these 6 people pick all of these 3 kinds of ornaments or each of these 6 people pick none of these 3 kinds of ornaments. To see this, consider a 68×8 matrix consisting of 0's and 1's. If the element in the i-th row and the j-th column is 0, that means the i-th person did not choose the j-th ornament, and if that element is 1, that means the i-th person did choose the j-th ornament. Since $\binom{8}{4} = 70$, there is a 68×8 matrix comnsisting of 0's and 1's in which no 2 rows are equal and each row consists of 4 0's and 4 1's. Now suppose there are 6 rows and 3 columns so that any element which is in one of these rows and one of these columns is 1. By interchanging rows and columns, we may assume that we have a 6×3 matrix in the upper left corner which consists solely of 1's. The new matrix still has the property that none of the rows are equal. The first 3 elements in each of the 6 rows are 1; the remaining entries in each of these rows consists of 1 1 and 4 0's. There are only 5 permutations of 1 1 and 4 0's. Hence 2 of the rows are equal, but that gives a contradiction. So here there are not 6 people and 3 kinds of elements such that each of the 6 people pick all of these 3 kinds of ornaments. To show that there are not 6 people and 3 kinds of ornaments such that each of the 6 people pick none of these 3 kinds of ornaments, use a similar proof where the values 0 and 1 are interchanged.

This problem was inspired by problem A3 of the 1996 Putnam Exam. That problem involved 20 students picking from 0 to 6 courses out of 6 courses and asked if there are 5 students and 2 courses such that all 5 students have chosen both courses or all 5 students have chosen neither course.

Next year I hope to have another holiday puzzle. Look for it at
`http://www.albany.edu/~martinhi/puzzle.html`

M. Hildebrand, Department of Mathematics and Statistics, University at Albany, Albany, NY 12222 e-mail: `mhildebrand@albany.edu`