COMPACT COMPOSITION OPERATORS ON BERGMAN SPACES OF THE UNIT BALL

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ABSTRACT. Under a mild condition we show that a composition operator C_φ is compact on the Bergman space A^p_α of the open unit ball in $\mathbb{C}^n$ if and only if $(1-|z|)/(1-|\varphi(z)|)\to 0$ as $|z|\to 1^-$.

1. INTRODUCTION

For any positive integer n we let

$$\mathbb{C}^n = \mathbb{C}\times\cdots\times\mathbb{C}$$

denote the n-dimensional complex Euclidean space. For any two points $z=(z_1,\cdots,z_n)$ and $w=(w_1,\cdots,w_n)$ in $\mathbb{C}^n$ we write

$$\langle z,w\rangle = z_1\overline{w}_1+\cdots+z_n\overline{w}_n,$$

and

$$|z| = \sqrt{|z_1|^2+\cdots+|z_n|^2}.$$

The open unit ball in $\mathbb{C}^n$ is the set

$$\mathbb{B}_n = \{z\in\mathbb{C}^n : |z|<1\}.$$

The space of holomorphic functions in $\mathbb{B}_n$ will be denoted by $H(\mathbb{B}_n)$.

Let dv be Lebesgue volume measure on $\mathbb{B}_n$, normalized so that $v(\mathbb{B}_n)=1$. For any $\alpha>-1$ we let

$$dv_\alpha(z) = c_\alpha(1-|z|^2)^\alpha\,dv(z),$$

where c_α is a positive constant chosen so that $v_\alpha(\mathbb{B}_n)=1$. The weighted Bergman space A^p_α, where $p>0$, consists of functions $f\in H(\mathbb{B}_n)$ such that

$$\int_{\mathbb{B}_n}|f(z)|^p\,dv_\alpha(z)<\infty.$$

The space A^2_α is a Hilbert space with inner product

$$\langle f,g\rangle = \int_{\mathbb{B}_n} f(z)\overline{g(z)}\,dv_\alpha(z).$$

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Every holomorphic $\varphi : \mathbb{B}_n \to \mathbb{B}_n$ induces a composition operator

$$C_\varphi : H(\mathbb{B}_n) \to H(\mathbb{B}_n),$$

namely, $C_\varphi f = f \circ \varphi$. When $n = 1$, it is well known that C_φ is always bounded on A^p_α; and C_φ is compact on A^p_α if and only if

$$\lim_{|z|\to 1^-} \frac{1-|z|^2}{1-|\varphi(z)|^2} = 0.$$

See [2] and [3].

When $n > 1$, not every composition operator is bounded on A^p_α. For example, it can easily be checked with Taylor coifficients that the composition operator C_φ is not bounded on A^2_α when

$$\varphi(z) = (\pi(z), 0, \cdots, 0),$$

where

$$\pi(z) = \sqrt{n^n}\, z_1 \cdots z_n.$$

See [2] for more examples and references.

The main result of the paper is the following.

Theorem. *Suppose $p > 0$ and $\alpha > -1$. If the composition operator C_φ is bounded on A^q_β for some $q > 0$ and $-1 < \beta < \alpha$, then C_φ is compact on A^p_α if and only if*

$$\lim_{|z|\to 1^-} \frac{1-|z|^2}{1-|\varphi(z)|^2} = 0. \tag{1}$$

Note that the compactness of C_φ on A^p_α always implies condition (1); we do not need any assumption on φ for this half of the theorem. The assumption that C_φ be bounded on A^q_β for some $\beta < \alpha$ is needed only for the other half the theorem. The exponents p and q are not important.

2. Preliminaries

We collect a few preliminary results in this section that will be needed later in the paper. We begin with the notion of compact composition operators on A^p_α.

When $p > 1$, the Bergman space A^p_α is a reflexive Banach space (see [6] for more information about Bergman spaces), and all reasonalble definitions of compactness of C_φ on A^p_α are equivalent. In general, for any $p > 0$, we say that the composition operator C_φ is compact on A^p_α if

$$\lim_{k\to\infty} \int_{\mathbb{B}_n} |C_\varphi f_k|^p \, dv_\alpha = 0$$

whenever $\{f_k\}$ is a bounded sequence in A^p_α that converges to 0 uniformly on compact subsets of $\mathbb{B}_n$.

For any holomorphic $\varphi : \mathbb{B}_n \to \mathbb{B}_n$ we can define a positive Borel measure $\mu_{\varphi,\alpha}$ on $\mathbb{B}_n$ as follows. Given a Borel set E in $\mathbb{B}_n$, we set

$$\mu_{\varphi,\alpha}(E) = v_\alpha(\varphi^{-1}(E)) = c_\alpha \int_{\varphi^{-1}(E)} (1 - |z|^2)^\alpha \, dv(z).$$

Obviously, $\mu_{\varphi,\alpha}$ is the pullback measure of dv_α under the map φ. Therefore, we have the following change of variables formula:

$$\int_{\mathbb{B}_n} f(\varphi) \, dv_\alpha = \int_{\mathbb{B}_n} f \, d\mu_{\varphi,\alpha}, \tag{2}$$

where f is either nonnegative or belongs to $L^1(\mathbb{B}_n, d\mu_{\varphi,\alpha})$. In particular, the composition operator C_φ is bounded on A^p_α if and only if there exists a constant $C > 0$ such that

$$\int_{\mathbb{B}_n} |f|^p \, d\mu_{\varphi,\alpha} \le C \int_{\mathbb{B}_n} |f|^p \, dv_\alpha \tag{3}$$

for all $f \in A^p_\alpha$. Measures satisfying this condition are called Carleson measures for the Bergman space A^p_α.

Similarly, a positive Borel measure μ on $\mathbb{B}_n$ is called a vanishing Carleson measure for the Bergman space A^p_α if

$$\lim_{k\to\infty} \int_{\mathbb{B}_n} |f_k|^p \, d\mu = 0 \tag{4}$$

whenever $\{f_k\}$ is a bounded sequence in A^p_α that converges to 0 uniformly on compact subsets of $\mathbb{B}_n$. In particular, a composition operator C_φ is compact on A^p_α if and only if the pullback measure $\mu_{\varphi,\alpha}$ is a vanishing Carleson measure for A^p_α.

It is well known that Carleson (and vanishing Carleson) measures for the Bergman space A^p_α is indenpendent of p. More precisely, the following result holds.

Lemma 1. *Suppose $p > 0$ and $\alpha > -1$. Then the following conditions are equivalent for any positive Borel measure μ on $\mathbb{B}_n$.*

(i) *μ is a Carleson measure for A^p_α, that is, there exists a constant $C > 0$ such that*

$$\int_{\mathbb{B}_n} |f|^p \, d\mu \le C \int_{\mathbb{B}_n} |f|^p \, dv_\alpha$$

for all $f \in A^p_\alpha$.

(ii) *For some (or each) $R > 0$ there exists a constant $C > 0$ (depending on R and α but independent of a) such that*

$$\mu(D(a,R)) \le C v_\alpha(D(a,R))$$

for all $a \in \mathbb{B}_n$, where $D(a,R)$ is the Bergman metric ball at a with radius R.

Proof. See [7] for example. □

A consequence of the above lemma is the following well-known result about composition operators; see [2].

Corollary 2. *Suppose $p > 0$, $q > 0$, and $\alpha > -1$. Then C_φ is bounded on A^p_α if and only if C_φ is bounded on A^q_α.*

A similar characterization of vanishing Carleson measures for A^p_α also holds.

Lemma 3. *Suppose $p > 0$ and $\alpha > -1$. The following two conditions are equivalent for a positive Borel measure on $\mathbb{B}_n$.*

(i) *μ is a vanishing Carleson measure for A^p_α.*

(ii) *For some (or any) $R > 0$ we have*

$$\lim_{|a|\to 1^-} \frac{\mu(D(a,R))}{v_\alpha(D(a,R))} = 0.$$

Proof. See [7] for example. □

As a result of the above lemma we see that the compactness of C_φ on A^p_α is indenpendent of p. We state this as the following corollary which can be found in [2] as well.

Corollary 4. *Suppose $p > 0$, $q > 0$, and $\alpha > -1$. Then C_φ is compact on A^p_α if and only if C_φ is compact on A^q_α.*

We need two more technical lemmas. The first of which is called Schur's test and concerns the boundedness of integral operators on L^p spaces. Thus we consider a measure space (X, μ) and an integral operator

$$Tf(x) = \int_X H(x,y) f(y)\, d\mu(y), \tag{5}$$

where H is a nonnegative measurable function on $X \times X$.

Lemma 5. *Suppose that there exists a positive measurable function h on X such that*

$$\int_X H(x,y) h(y)\, d\mu(y) \le C h(x)$$

for almost all x and

$$\int_X H(x,y)h(x)\,d\mu(x) \le Ch(y)$$

for almost all y, where C is a positive constant. Then the integral operator T defined in (5) is bounded on $L^2(X, d\mu)$. Moreover, the norm of T on $L^2(X, d\mu)$ is less than or equal to the constant C.

Proof. See [5] or [6]. □

Lemma 6. *Suppose $\alpha > -1$ and $t > 0$. Then there exists a constant $C > 0$ such that*

$$\int_{\mathbb{B}_n} \frac{dv_\alpha(w)}{|1-\langle z,w\rangle|^{n+1+\alpha+t}} \le \frac{C}{(1-|z|^2)^t}$$

for all $z \in \mathbb{B}_n$.

Proof. See [4]. □

3. Characterizations in Terms of Kernel Functions

In order to understand the mild assumption made in the statement of the main theorem, we show in this section how the boundedness and compactness of composition operators on Bergman spaces can be described in terms of Bergman type kernel functions.

Theorem 7. *Suppose $p > 0$, $\alpha > -1$, and $t > 0$. Then the composition operator C_φ is bounded on A^p_α if and only if*

$$\sup_{a\in\mathbb{B}_n} (1-|a|^2)^t \int_{\mathbb{B}_n} \frac{dv_\alpha(z)}{|1-\langle a,\varphi(z)\rangle|^{n+1+\alpha+t}} < \infty. \tag{6}$$

Proof. It follows from Lemma 6 that the boundedness of C_φ on A^p_α implies condition (6).

Next we assume that condition (6) holds. Then by the change of variables formula (2) there exists a constant $C > 0$ such that

$$(1-|a|^2)^t \int_{\mathbb{B}_n} \frac{d\mu_{\varphi,\alpha}(z)}{|1-\langle a,z\rangle|^{n+1+\alpha+t}} \le C$$

for all $a \in \mathbb{B}_n$. For any fixed positive radius R we have

$$(1-|a|^2)^t \int_{D(a,R)} \frac{d\mu_{\varphi,\alpha}(z)}{|1-\langle a,z\rangle|^{n+1+\alpha+t}} \le C$$

for all $a \in \mathbb{B}_n$. It is well known that

$$|1-\langle a,z\rangle| \sim 1-|a|^2$$

for $z \in D(a, R)$, and it is also well known that

$$(1 - |a|^2)^{n+1+\alpha} \sim v_\alpha(D(a, R));$$

see [7]. It follows that there exists another positive constant C (independent of a) such that

$$\mu_{\varphi,\alpha}(D(a, R)) \le C v_\alpha(D(a, R))$$

for all $a \in \mathbb{B}_n$. By Lemma 1, the measure $\mu_{\varphi,\alpha}$ is Carleson for A^p_α, and so the composition operator C_φ is bounded on A^p_α. □

This result is probably well known to experts in the field. The main point here is that t can be an arbitrary positive constant. This also tells us roughly how far away the boundedness of C_φ on A^p_α is from that of C_φ on A^p_β.

Corollary 8. *Suppose $p > 0$, $q > 0$, and $-1 < \beta < \alpha$. If C_φ is bounded on A^q_β, then C_φ is bounded on A^p_α.*

Proof. Write $\alpha = \beta + \epsilon$ with $\epsilon > 0$. Since

$$\frac{(1 - |z|^2)^\epsilon}{|1 - \langle a, \varphi(z)\rangle|^\epsilon} \le C_1 \left(\frac{1 - |z|^2}{1 - |\varphi(z)|^2}\right)^\epsilon \le C_2,$$

where the last inequality is an easy consequence of Schwarz lemma for the unit ball, we have

$$\int_{\mathbb{B}_n} \frac{(1 - |w|^2)^\alpha \, dv(w)}{|1 - \langle a, \varphi(w)\rangle|^{n+1+\alpha+t}} \le C_2 \int_{\mathbb{B}_n} \frac{(1 - |w|^2)^\beta \, dv(w)}{|1 - \langle a, \varphi(w)\rangle|^{n+1+\beta+t}}.$$

This shows that

$$\sup_{a \in \mathbb{B}_n} (1 - |a|^2)^t \int_{\mathbb{B}_n} \frac{dv_\beta(z)}{|1 - \langle a, \varphi(z)\rangle|^{n+1+\beta+t}} < \infty$$

implies

$$\sup_{a \in \mathbb{B}_n} (1 - |a|^2)^t \int_{\mathbb{B}_n} \frac{dv_\alpha(z)}{|1 - \langle a, \varphi(z)\rangle|^{n+1+\alpha+t}} < \infty.$$

The desired result then follows from Theorem 7. □

A similar argument gives the following characterization of the compactness of C_φ on A^p_α.

Theorem 9. *Suppose $p > 0$, $\alpha > -1$, and $t > 0$. Then C_φ is compact on A^p_α if and only if*

$$\lim_{|a| \to 1^-} (1 - |a|^2)^t \int_{\mathbb{B}_n} \frac{dv_\alpha(z)}{|1 - \langle a, \varphi(z)\rangle|^{n+1+\alpha+t}} = 0. \tag{7}$$

Corollary 10. *Suppose $p > 0$, $q > 0$, and $-1 < \beta < \alpha$. Then the compactness of C_φ on A^q_β implies the compactness of C_φ on A^p_α.*

Although we do not need Hardy spaces in this paper, we mention here that if C_φ is bounded (or compact) on a Hardy space H^q of the unit ball, then C_φ is bounded (or compact) on very Bergman space A^p_α. This result, along with Corollaries 8 and 10 above, can be found in [1].

4. Compactness on Bergman Spaces

We now prove the main result of the paper.

Theorem 11. *Suppose $p > 0$ and $\alpha > -1$. If C_φ is bounded on A^q_β for some $q > 0$ and $-1 < \beta < \alpha$, then C_φ is compact on A^p_α if and only if*

$$\lim_{|z|\to 1^-} \frac{1-|z|^2}{1-|\varphi(z)|^2} = 0. \tag{8}$$

Proof. According to Corollary 4, we may assume that $p = 2$.

The normalized reproducing kernels of A^2_α are given by

$$k_z(w) = \frac{(1-|z|^2)^{(n+1+\alpha)/2}}{(1-\langle w, z\rangle)^{n+1+\alpha}}.$$

Each k_z is a unit vector in A^2_α and it is clear that

$$\lim_{|z|\to 1^-} k_z(w) = 0, \qquad w \in \mathbb{B}_n.$$

Furthermore, the convergence is uniform when w is restricted to any compact subset of $\mathbb{B}_n$. A standard computation shows that

$$\int_{\mathbb{B}_n} |C^*_\varphi k_z|^2 \, dv_\alpha = \left(\frac{1-|z|^2}{1-|\varphi(z)|^2}\right)^{n+1+\alpha},$$

so the compactness of C_φ on A^2_α (which is the same as the compactness of C^*_φ on A^2_α) implies condition (8).

We proceed to show that condition (8) implies the compactness of C_φ on A^2_α, provided that C_φ is bounded on A^q_β for some $\beta \in (-1, \alpha)$. An easy computation shows that the operator

$$C_\varphi C^*_\varphi : A^2_\alpha \to A^2_\alpha$$

admits the following integral representation:

$$C_\varphi C^*_\varphi f(z) = \int_{\mathbb{B}_n} \frac{f(w)\, dv_\alpha(w)}{(1-\langle \varphi(z), \varphi(w)\rangle)^{n+1+\alpha}}, \quad f \in A^2_\alpha. \tag{9}$$

We will actually prove the compactness of $C_\varphi C^*_\varphi$ on A^2_α, which is equivalent to the compactness of C_φ on A^2_α. In fact, our arguments will prove the compactness of the following integral operator on $L^2(\mathbb{B}_n, dv_\alpha)$:

$$Tf(z) = \int_{\mathbb{B}_n} \frac{f(w)\, dv_\alpha(w)}{|1 - \langle \varphi(z), \varphi(w)\rangle|^{n+1+\alpha}}. \tag{10}$$

For any $r \in (0,1)$ let χ_r denote the characteristic function of the set $\{z \in \mathbb{C}^n : r < |z| < 1\}$. Consider the following integral operator on $L^2(\mathbb{B}_n, dv_\alpha)$:

$$T_r f(z) = \int_{\mathbb{B}_n} H_r(z,w) f(w)\, dv_\alpha(w), \tag{11}$$

where

$$H_r(z,w) = \frac{\chi_r(z)\chi_r(w)}{|1 - \langle \varphi(z), \varphi(w)\rangle|^{n+1+\alpha}}$$

is a nonnegative integral kernel. We are going to estimate the norm of T_r on $L^2(\mathbb{B}_n, dv_\alpha)$ in terms of the quantity

$$M_r = \sup_{r<|z|<1} \frac{1-|z|^2}{1-|\varphi(z)|^2}.$$

We do this with the help of Schur's test.

Let $\alpha = \beta + \sigma$, where $\sigma > 0$, and consider the function

$$h(z) = (1-|z|^2)^{-\sigma}, \qquad z \in \mathbb{B}_n.$$

We have

$$\begin{aligned}
\int_{\mathbb{B}_n} H_r(z,w) h(w)\, dv_\alpha(w) &= \frac{c_\alpha}{c_\beta} \int_{\mathbb{B}_n} \frac{\chi_r(z)\chi_r(w)\, dv_\beta(w)}{|1 - \langle \varphi(z), \varphi(w)\rangle|^{n+1+\beta+\sigma}} \\
&\le \frac{c_\alpha}{c_\beta} \int_{\mathbb{B}_n} \frac{\chi_r(z)\, dv_\beta(w)}{|1 - \langle \varphi(z), \varphi(w)\rangle|^{n+1+\beta+\sigma}}.
\end{aligned}$$

By the boundedness of C_φ on A^q_β, there exists a constant $C_1 > 0$, independent of r and z, such that

$$\int_{\mathbb{B}_n} H_r(z,w) h(w)\, dv_\alpha(w) \le C_1 \chi_r(z) \int_{\mathbb{B}_n} \frac{dv_\beta(w)}{|1 - \langle \varphi(z), w\rangle|^{n+1+\beta+\sigma}}.$$

We apply Lemma 6 to find another positive constant C_2, independent of r and z, such that

$$\begin{aligned}
\int_{\mathbb{B}_n} H_r(z,w) h(w)\, dv_\alpha(w) &\le \frac{C_2 \chi_r(z)}{(1-|\varphi(z)|^2)^\sigma} \\
&= C_2 \chi_r(z) \left(\frac{1-|z|^2}{1-|\varphi(z)|^2} \right)^\sigma h(z) \\
&\le C_2 M_r^\sigma h(z)
\end{aligned}$$

for all $z \in \mathbb{B}_n$. By the symmetry of $H_r(z,w)$, we also have

$$\int_{\mathbb{B}_n} H_r(z,w)h(z)\,dv_\alpha(z) \le C_2 M_r^\sigma h(w)$$

for all $w \in \mathbb{B}_n$. It follows from Lemma 5 that the operator T_r is bounded on $L^2(\mathbb{B}_n, dv_\alpha)$ and the norm of T_r on $L^2(\mathbb{B}_n, dv_\alpha)$ does not exceed the constant $C_2 M_r^\sigma$.

Now fix some $r \in (0,1)$ and fix a bounded sequence $\{f_k\}$ in A^2_α that converges to 0 uniformly on every compact subset of $\mathbb{B}_n$. In particular, $\{f_k\}$ converges to 0 uniformly on $|z| \le r$. We use (9) to write

$$C_\varphi C_\varphi^* f_k(z) = F_k(z) + G_k(z), \qquad z \in \mathbb{B}_n,$$

where

$$F_k(z) = \int_{|w|\le r} \frac{f_k(w)\,dv_\alpha(w)}{(1-\langle \varphi(z), \varphi(w)\rangle)^{n+1+\alpha}},$$

and

$$G_k(z) = \int_{\mathbb{B}_n} \frac{\chi_r(w) f_k(w)\,dv_\alpha(w)}{(1-\langle \varphi(z), \varphi(w)\rangle)^{n+1+\alpha}}.$$

Since $\{f_k(w)\}$ converges to 0 uniformly on for $|w| \le r$, we have

$$\lim_{k\to\infty} \int_{\mathbb{B}_n} |F_k(z)|^2\,dv_\alpha(z) = 0.$$

For any fixed $z \in \mathbb{B}_n$, the weak convergence of $\{f_k\}$ to 0 in $L^2(\mathbb{B}_n, dv_\alpha)$ implies that $G_k(z) \to 0$ as $k \to \infty$. In fact, by splitting the ball into $|z| \le \delta$ and $\delta < |z| < 1$, it is easy to show that

$$\lim_{k\to\infty} G_k(z) = 0$$

uniformly for z in any compact subset of $\mathbb{B}_n$.

It follows from the definition of T_r that

$$\int_{\mathbb{B}_n} |G_k|^2\,dv_\alpha \le \int_{|z|\le r} |G_k|^2\,dv_\alpha + \int_{\mathbb{B}_n} |T_r(|f_k|)|^2\,dv_\alpha.$$

Since $\{f_k\}$ is bounded in $L^2(\mathbb{B}_n, dv_\alpha)$, and since the norm of the operator T_r on $L^2(\mathbb{B}_n, dv_\alpha)$ does not exceed $C_2 M_r^\sigma$, we can find a constant $C_3 > 0$, independent of r and k, such that

$$\int_{\mathbb{B}_n} |T_r(|f_k|)|^2\,dv_\alpha \le C_3 M_r^{2\sigma}$$

for all k. Combining this with

$$\lim_{k\to\infty} \int_{|z|\le r} |G_k|^2\,dv_\alpha = 0,$$

we obtain
$$\limsup_{k\to\infty}\int_{\mathbb{B}_n}|G_k|^2\,dv_\alpha\le C_3M_r^{2\sigma}.$$
This along with the estimates for F_k in the previous paragraph gives
$$\limsup_{k\to\infty}\int_{\mathbb{B}_n}|C_\varphi C_\varphi^* f_k|^2\,dv_\alpha\le C_3M_r^{2\sigma}.$$
Since r is arbitrary and $M_r\to 0$ as $r\to 1^-$ (which is equivalent to the condition in (8)), we conclude that
$$\lim_{k\to\infty}\int_{\mathbb{B}_n}|C_\varphi C_\varphi^* f_k|^2\,dv_\alpha=0.$$
So C_φ is compact on A^2_α, and the proof of the theorem is complete. □

Note that when $n=1$, C_φ is bounded on every Bergman space A^q_β, so the characterization of compact composition operators on A^p_α does not need any extra assumption. However, our proof here still works. The idea of using Schur's test to prove the compactness of composition operators seems to be new even in the case $n=1$.

A similar compactness result was proven in [1] for composition operators on A^p_α. But the condition in [1] involves the derivatives of φ and is much stronger than our condition here.

Finally we mention that our results and proofs generalize to strongly pseudo-convex domains. The key preliminary results we need are Lemmas 1, 3, 5, and 6, which are all known to be true for strongly pseudo-convex domains. For the same reasons, our results and proofs also work for products of balls in $\mathbb{C}^n$. In particular, they remain valid for the polydisk.

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