Section 10 Answer Key:

0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

1) A simple random sample of 1000 New Yorkers finds that 87 are left-handed. (a) Find the 95% confidence interval for the proportion of New Yorkers who are left-handed.

$$\hat{p} = \frac{87}{1000} = 0.087$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.087 \pm 1.960 * \sqrt{\frac{0.087(1-0.087)}{1000}}$$
$$0.087 \pm 0.0175 = (0.0695, 0.1045)$$

We are 95% confident that the proportion of New Yorkers who are left-handed is 0.087 with a margin of error of 0.0175.

(b) Find the 99% confidence interval:

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.087 \pm 2.576 * \sqrt{\frac{0.087(1-0.087)}{1000}}$$
$$0.087 \pm 0.02296 = (0.06404, 0.10996)$$

We are 99% confident that the proportion of New Yorkers who are left-handed is 0.087 with a margin of error of 0.02296.

(c) We can be 99.9% confident that the proportion of New Yorkers who are left-handed is between what two numbers?

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.087 \pm 3.291 * \sqrt{\frac{0.087(1-0.087)}{1000}}$$
$$0.087 \pm 0.0293 \ = (0.0577, 0.1163)$$

We are 99.9% confident that the proportion of New Yorkers who are left-handed is 0.087 with a margin of error of 0.0293.

(d) Either our group of 1000 is among the 10% most unusual samples, or the proportion of New Yorkers who are left-handed is between what two numbers?

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.087 \pm 1.645 * \sqrt{\frac{0.087(1-0.087)}{1000}}$$
$$0.087 \pm 0.015 = (0.072{,}0.102)$$

We are 90% confident that the proportion of New Yorkers who are left-handed is 0.087 with a margin of error of 0.015.

2) We wish to know the probability that a suspect coin will land heads. We flip the coin 400 times and 190 times it lands heads. (a) Find the 80% confidence interval for the probability that the coin will land heads.

$$\hat{p} = \frac{190}{400} = 0.475$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.475 \pm 1.282 * \sqrt{\frac{0.475(1-0.475)}{400}}$$
$$0.475 \pm 0.032 = (0.443{,}0.507)$$

(b) We can be 95% confident that the probability of the coin landing heads is between what two numbers?

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.475 \pm 1.960 * \sqrt{\frac{0.475(1-0.475)}{400}}$$
$$0.475 \pm 0.049 = (0.426{,}0.524)$$

(c) Either our sample of 400 flips was among the 1% most unusual, or that the probability of the coin landing heads is between what two numbers?
This is a 99% confidence interval:

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.475 \pm 2.576 * \sqrt{\frac{0.475(1-0.475)}{400}}$$
$$0.475 \pm 0.064 = (0.411{,}0.539)$$

3) Picking 250 orders randomly from a mail-ordering company, we find that 210 arrived on time. Let p be the proportion of all orders that are on time. (a) Find the 98% confidence interval for p. (b)

$$\hat{p} = \frac{210}{250} = 0.84$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.84 \pm 2.326 * \sqrt{\frac{0.84(1-0.84)}{250}}$$
$$0.84 \pm 0.054 = (0.786{,}0.894)$$

We can be 99% confident that p is between what two numbers?

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.84 \pm 2.576 * \sqrt{\frac{0.84(1-0.84)}{400}}$$
$$0.84 \pm 0.060 = (0.780, 0.900)$$

(c) Either our sample of 250 orders was among the 5% most unusual, or p is between what two numbers?
This is a 95% confidence interval:

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.84 \pm 1.960 * \sqrt{\frac{0.84(1-0.84)}{400}}$$
$$0.84 \pm 0.045 = (0.795, 0.885)$$

4) In our effort to find out what percentage of all statistics are meaningless, we do a well-funded study and learn that of 420 examined statistics, 386 of them were meaningless. Find a 99.5% confidence interval for the true proportion of all statistics that are meaningless.

$$\hat{p} = \frac{386}{420} = 0.919$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.919 \pm 2.807 * \sqrt{\frac{0.919(1-0.919)}{420}}$$
$$0.919 \pm 0.037 = (0.882, 0.956)$$

5) The U.S.R.S. (Union of Starfleet Red Shirts) wants to know the probability of a Red Shirt dying when he beams down to a planet with Captain Kirk. Find a 99.8% confidence interval for this probability, after learning that 35 of the last 93 Red Shirts who beamed down with Kirk met an unfortunate end.

$$\hat{p} = \frac{35}{93} = 0.376$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.376 \pm 3.091 * \sqrt{\frac{0.376(1-0.376)}{93}}$$
$$0.376 \pm 0.155 = (0.221, 0.531)$$

6) A random sample of 1021 adults found that 38% said they believe in ghosts. Find a 90% confidence interval for the percentage of all adults who believe in ghosts. Find a 99% confidence interval.

$$\hat{p} = 0.38$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.38 \pm 1.645 * \sqrt{\frac{0.38(1-0.38)}{1021}}$$
$$0.38 \pm 0.0255 = (0.3545, 0.4055)$$
$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.38 \pm 2.576 * \sqrt{\frac{0.38(1-0.38)}{1021}}$$
$$0.38 \pm 0.039 = (0.341, 0.419)$$

7) An insurance company checks police records on 582 accidents selected at random and notes that teenagers were at the wheel in 91 of them. Find a 95% confidence interval for the true proportion of all auto accidents that involve teenage drivers.

$$\hat{p} = \frac{91}{582} = 0.156$$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.156 \pm 1.960 * \sqrt{\frac{0.156(1-0.156)}{582}}$$

$$0.156 \pm 0.030 = (0.126, 0.186)$$

8) A company wants to test the response to a new flier, and they send it to 1000 people randomly selected from their mailing list of over 200,000. They get orders from 123 of the recipients. Create a 90% confidence interval for the percentage of people the company contacts who will send in an order. The full mailing won't be cost effective unless it produces at least a 5% return. Should they do it or not?

$$\hat{p} = \frac{123}{1000} = 0.123$$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.123 \pm 1.645 * \sqrt{\frac{0.123(1-0.123)}{1000}}$$

$$0.123 \pm 0.017 = (0.106, 0.140)$$

One of three things is the case: either our sample is among the 90% most typical (in which case *p*, the proportion of the entire mailing list which will send in an order, is between 0.106 and 0.140), or it's among the 10% outlying tails. If it's among the 5% lowest samples (an unusually low sample proportion underestimating *p*) then *p* is more than 0.140. It is of course fine if more than 14% will send in orders. If it's among the 5% highest samples (an unusually high sample proportion overestimating *p*) then *p* is less than 0.106. But even so, it could still be much higher than the 0.05 which they require. So we can be very confident that *p* is more than the 0.05 which makes it worth their while.

9) It's believed that as many as 25% of adults over 50 never graduated from high school. We wish to see if this percentage is the same among the 25 to 30 age group. (a) How many of this younger age group must we survey in order to estimate the proportion of non-grads to within 6% with 90% confidence?
For 90% confidence, our *z* is 1.645, our margin of error is 0.06, and we require a *p* to use. Although the 0.25 refers to a different population, it at least is a wild guess than we can use, since we don't know our *p*, the proportion of young adults who never graduated, nor of course do we know $\hat{p}$, the outcome of the sample whose size we're still trying to determine:

$$n = \left(\frac{z}{m}\right)^2 p(1-p) = \left(\frac{1.645}{0.06}\right)^2 0.25(1-0.25) = 140.9, \text{so } n = 141.$$

Alternately, if we want to use *p*=0.5 (covering the worse-case scenario which overestimates) we get *n* = 188.
Which is better? In fact, these two types of people are probably not very similar regarding graduation rate, so our wild guess is pretty wild. But because it's more central (closer to 0.5) than the truth (likely less than 0.25 of young adults failed to graduate; i.e., *p* is more extreme), really the *p*=0.25 will overestimate as well, just not as badly as the *p*=0.5.

(b) Suppose we want to cut the margin of error down to 4%. How many?

$$n = \left(\frac{z}{m}\right)^2 p(1-p) = \left(\frac{1.645}{0.04}\right)^2 0.25(1-0.25) = 317.1, \text{so } n = 318.$$

(remembering that we have to round up to the smallest acceptable sample size).

(c) Down to 3% – how many?

$$n = \left(\frac{z}{m}\right)^2 p(1-p) = \left(\frac{1.645}{0.03}\right)^2 0.25(1-0.25) = 563.8, \text{so } n = 564.$$