

1)

blood type	O	A	B	AB
probability	0.49	0.27	?	0.20

- (a) The probability of B is $1 - 0.49 - 0.27 - 0.20 = 0.04$. Alternately $0.49 + 0.27 + 0.20$ is the probability that someone does *not* have type B blood (O or A or AB), so subtract from 1.
- (b) Blood type must be “O or B” to be useable. $P(O \text{ or } B) = P(O) + P(B) = 0.49 + 0.04 = 0.53$, since there is no possibility of overlap (being O means it can’t be B and vice-versa).

2)

marble color	brown	red	yellow	green	orange	blue
probability	0.3	0.2	0.2	0.1	0.1	?

The probability of blue is $1 - 0.3 - 0.2 - 0.2 - 0.1 - 0.1 = 0.1$; that is, adding up what we have, 0.9 are *not blue*.

- (a) $P(\text{red or green}) = 0.2 + 0.1 = 0.3$ since a marble being red means it’s not green and vice-versa.
- (b) $P(\text{not yellow}) = 1 - P(\text{yellow}) = 1 - 0.2 = 0.8$. Alternately, this is “brown or red or green or orange or blue”, add up.
- (c) neither yellow nor blue: We can subtract from 1 what doesn’t count ($1 - 0.2 - 0.1$) or we can add up what does count (brown or red or green or orange is $0.3 + 0.2 + 0.1 + 0.1$), either way, **0.7**.

If we draw four marbles (for every draw, the above are independently the probabilities of what we get):

- (d) first yellow and second yellow and third yellow and fourth yellow: YYY Y is $0.2 * 0.2 * 0.2 * 0.2 = 0.0016$

For what remains, use B for “brown” (0.3) and N for “not brown” (0.7):

- (e) NNNN has probability $0.7 * 0.7 * 0.7 * 0.7 = 0.2401$

- (f) exactly one brown: The separate, non-overlapping ways of achieving this are:

BNNN	$0.3 * 0.7 * 0.7 * 0.7$	0.1029
NBNN	$0.7 * 0.3 * 0.7 * 0.7$	0.1029
NNBN	$0.7 * 0.7 * 0.3 * 0.7$	0.1029
NNNB	$0.7 * 0.7 * 0.7 * 0.3$	0.1029

“exactly one brown” means first row or second row or third row or fourth row in this chart, so add probabilities: **0.4116**

- (g) “at least one brown” is very complicated directly: tally up the probability of all sequences of four B’s and N’s that have at least one B in them. But the only possibility that *doesn’t count* is “no browns”, which we already know (e). So

“some browns” is “NOT no browns”, $1 - 0.2401 = 0.7599$

- (h) The “or” principle won’t work directly because “first brown” doesn’t preclude the possibility that the last is brown nor vice-versa; there is overlap. The non-overlapping ways of achieving the result are N**B or B**N or B**B. The middle slots are irrelevant; we’re not saying anything one way or the other. For instance, “first not-brown and last brown” is $0.7 * 0.3$. The next is $0.3 * 0.7$, and the third possibility is $0.3 * 0.3$. So the answer is $0.21 + 0.21 + 0.09 = 0.51$. Note that if you try to use the “or” principle directly ($0.3 + 0.3$), you get an answer that is 0.09 too big (0.6). This is because the third case has been double counted: “first brown” includes B**B, but so does “last brown”.

- 3) We want to know which of the three has the greatest probability (i.e., the least small). $P(G) = 4/6 = 2/3$ and $P(R) = 2/6 = 1/3$.

RGRRR	$(1/3) * (2/3) * (1/3) * (1/3) * (1/3)$	0.0082
RGRRRR	$(1/3) * (2/3) * (1/3) * (1/3) * (1/3) * (2/3)$	0.0055
GRRRRR	$(2/3) * (1/3) * (1/3) * (1/3) * (1/3) * (2/3)$	0.0027

In any case, they are the same number until the final multiplication. The top is biggest, the next is 2/3rds of it, and the last is 1/3 of the first.

- 4) $P(\text{bad chip}) = 0.05$, so $P(\text{good chip}) = 0.95$. We need GGGGGGGGGGGG, which is $(0.95) * (0.95) * \dots * (0.95) = (0.95)^{12} = 0.5404$. There is a better than even chance that the next car will work, but only 54% of cars made this way will work... bad design!

- 5) $P(\text{Albany rain and Schenectady rain}) = P(\text{Albany rain}) * P(\text{Schenectady rain})$ presupposes that they are independent; that knowledge of whether it’s raining in Albany doesn’t affect the chance that it’s raining in Schenectady and vice-versa. This is almost certainly **not true**. Although rain in one doesn’t mean it must be raining in the other, it does increase the chances, since they’re geographically close. Bear in mind that if things do affect each other, then we must study in detail *how* they affect each other; the point is that our shortcut formula doesn’t work. In this course, things are generally independent when we need them to be. We don’t have enough information about how rain in the two cities affects the other to answer this question.

- 6) (a) Bach or Beethoven or Brahms: add them, so $0.05 + 0.26 + 0.09 = 0.40$
- (b) Not Schubert or Schumann: We can subtract the cases we don't want from 1 ($1 - 0.12 - 0.07$) or we can add up all the others (Bach or Beethoven or...). Either way, this is 0.81 .
- (c) We can subtract Bach's and Wagner's cases from 1 (since we don't want them) or add up everyone else's: $1 - 0.05 - 0.01 = 0.94$.
- (d) First is Beethoven and Second is Beethoven (assuming people call in independently; i.e., don't know what each other have requested) is $0.26 * 0.26 = 0.0676$.
- (e) "exactly nine requests to get Bach": $B = \text{Bach}, 0.05$ and $N = \text{not Bach}, 0.95$. We want NNNNNNNNNB, which is $0.95 * 0.95 * \dots * 0.95 * 0.05 = (0.95)^8 * 0.05 = 0.03317$