Chapter 10

Price Competition

The access to and use of high-speed Internet connections is spreading rapidly. According to the Federal Communications Commission (FCC), such access grew in America at a rate of over 80 percent in 2001 and over 150 percent in 2000. So far, most of this growth has been service provided by cable companies such as Comcast. At the beginning of 2003, cable companies had more than twice as many American subscribers as did telephone companies, but this could be changing. In the early spring of 2003, two major phone companies, Verizon and SBC, reduced their prices for Internet connection from $50 per month to about $35 per month. This fee undercut the typical cable company's rate, which was then $45 per month. Not surprisingly, both phone companies added many new subscribers and closed in on the cable companies lead in this fast-growing market.

Consumers of high-speed Internet connection often buy their service from the lowest priced provider. The providers post their prices, consumers decide from whom to buy, and then the consumer's line is hooked up. In other words, the firm does not produce the service or output until the consumer makes the purchase at the firm's posted price. This is the way competition works in many markets, including restaurants, electricians, moving companies, consulting firms, and numerous financial services. Note, though, that this is quite different from the way competition works in the Cournot model. There each competing firm independently produces an amount of output so that production occurs before the consumer makes a purchase. It is only afterwards that the price adjusts so that consumers will buy the total output that firms produced. This is what is meant by the phrase “the price adjusts so that the market clears,” and it is perhaps an apt description of how automobile, aircraft, and other manufacturing firms compete.

In a monopolized market, it would make no difference whether the firm initially set a price and then produced whatever amount consumers demanded at that price or first chose its production and let the price settle at whatever level was necessary to sell that output. If the monopolist is truly profit maximizing, the optimal choice if it first selects a price will imply, via the demand curve, an output level which is precisely that amount of production that the monopolist would choose if it instead initially chose how much to produce. For a monopolized market, the equilibrium outcome is the same whether the firm regards price or output as its major decision variable.

However, once we leave the world of monopoly the equivalence of price and output strategies vanishes. As we shall see, once we move into a setting of oligopoly it matters very much whether firms compete in terms of quantities, as do perhaps aircraft manufacturers and other Cournot-type competitors, or in terms of price, as do the high-speed Internet providers. The nature of the competition is markedly different. To understand these differences we begin by turning the Cournot model on its head and looking at a market in which firms again produce identical products but now compete by first setting prices instead of production levels. This is known as the Bertrand model. Later, we retain the assumption of competition but allow the
products to be less than perfect substitutes, that is, to be differentiated. As in Chapter 9, we also focus on static or simultaneous models of price competition limited to a single market period.

10.1 THE BERTRAND DUOPOLY MODEL

The standard Cournot duopoly model, recast in terms of price strategies rather than quantity strategies, is typically referred to as the Bertrand model. Joseph Bertrand was a French mathematician who in 1883 reviewed and critiqued Cournot's work nearly fifty years after its publication in an article in the *Journal des Savants*. Bertrand saw the absence of price competition as a weakness in Cournot’s analysis and, indeed, in the general notion of mathematical modeling in economics altogether. The legacy of Bertrand is not, however, his criticism of what he termed “pseudo-mathematics” in economics. Instead, Bertrand’s contribution was the recognition that using price as a strategic variable is different from using quantity as the strategic variable, and that this difference is worth investigating.

Let us now rework the Cournot duopoly model with each firm choosing the price it will charge rather than the quantity it will produce. Otherwise, the model and the assumptions are exactly the same as before. There are two firms who choose their strategies simultaneously. Each produces the identical good at the same, constant marginal cost, \( c \). Each firm knows the structure of market demand. Before we described demand by a linear inverse demand function, \( P = A - BQ \). When firms choose prices rather than quantities it is more convenient to rewrite the demand function and have total output as the dependent variable.\(^1\) Therefore, we have

\[
Q = a - bP, \text{ where } a = \frac{A}{B} \text{ and } b = \frac{1}{B}. \tag{10.1}
\]

Consider the pricing problem first from firm 2’s perspective. In order to determine its best price response to its rival firm 1, firm 2 must first work out the demand for its product conditional on both its own price, denoted by \( p_2 \), and firm 1’s price, denoted by \( p_1 \). Rationally speaking, firm 2’s reasoning should be as follows. If \( p_2 > p_1 \), firm 2 will sell no output. The product is homogenous so that consumers always buy from the cheapest source. Setting a price above that of firm 1 therefore means that firm 2 will serve no customers. The opposite is true if \( p_2 < p_1 \). When firm 2 sets the lower price, it will supply the entire market, and firm 1 will sell nothing. Finally, we will assume that if \( p_2 = p_1 \), the two firms evenly split the market. When both firms charge identical prices the same number of customers patronize both producers.

The foregoing implies that the demand for firm 2’s output, \( q_2 \), may be described as follows:

\[
q_2 = 0 \quad \text{if } p_2 > p_1 \\
q_2 = \frac{a - bp_2}{2} \quad \text{if } p_2 = p_1 \\
q_2 = a - bp_2 \quad \text{if } p_2 < p_1 \tag{10.2}
\]

\(^1\) When firms choose quantities (as in Cournot’s model), it is best to work with the inverse demand curve and treat price as the dependent variable. When firms select prices, as in Bertrand’s analysis, it is usually best to let quantity be the dependent variable.
As Figure 10-1 shows, this demand structure is not continuous. For any \( p_2 \) greater than \( p_1 \), demand for \( q_2 \) is zero. But when \( p_2 \) falls and becomes equal to \( p_1 \), demand jumps from zero to \( \frac{a-bp_2}{2} \). When \( p_2 \) then falls still further so that it is below \( p_1 \), demand again jumps to \( a-bp_2 \).

**FIRM 2’S DEMAND CURVE IN THE BERTRAND MODEL**

Industry demand equal to \( a-bp_2 \) is the same as firm 2’s demand for all \( p_2 \) less than \( p_1 \). If \( p_2 = p_1 \), then the two firms share equally the total demand. For \( p_2 > p_1 \), firm 2’s demand falls to zero.

This discontinuity in firm 2’s demand curve was not present in the quantity version of the Cournot model. It turns out to make a crucial difference in terms of firms’ strategies. This is because the discontinuity in demand carries over into a discontinuity in profits. Firm 2’s profit, \( \Pi_2 \), as a function of \( p_1 \) and \( p_2 \) is

\[
\Pi_2(p_1, p_2) = \begin{cases} 
0 & \text{if } p_2 > p_1 \\
(p_2 - c) \frac{a - bp_2}{2} & \text{if } p_2 = p_1 \\
(p_2 - c)(a - bp_2) & \text{if } p_2 < p_1
\end{cases}
\] (10.3)

To find firm 2’s best response function, we need to find the price, \( p_2 \), that maximizes firm 2’s profits, \( \Pi_2(p_1, p_2) \), for any given choice of \( p_1 \). For example, suppose firm 1 chooses a very high price—higher even than the pure monopoly price, which is \( p^M = \frac{a + c}{2b} \). Since firm 2 can capture the entire market by selecting any price lower than \( p_1 \), its best response would be to choose the pure monopoly price, \( p^M \), and thereby earn the pure monopoly profits.

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2 This is, of course, the same monopoly price as we showed in Chapter 8 for the quantity version of the model with the notational change that \( a = A/B \), and \( b = 1/B \).
Conversely, what if firm 1 set a very low price, one below its unit cost, $c$? This would be an unusual choice, of course. However, if we wish to construct a complete best response function for firm 2, we must determine its value for all the possible values $p_1$ can take. Continuing then, if $p_1 < c$, firm 2 is best setting its price at some level above $p_1$. This will mean that firm 2 will sell nothing and earn zero profits. But any other choice will lead to negative profits. For if $p_2$ is less than or equal to $p_1$, firm 2 will sell a positive amount of output. Since such a price will be below its unit cost, firm 2 will lose money on each unit sold.

So much for firm 2's best response to the extreme choices of $p_1$. What about the more likely case in which firm 1 sets its price above marginal cost, $c$, but either equal to or below the pure monopoly price, $p_M$? How should firm 2 optimally respond in these circumstances? The simple answer is that it should set a price just a bit less than $p_1$. The intuition behind this strategy is illustrated in Figure 10-2, which shows firm 2's profit given a price, $p_1$, satisfying the relationship $p_1 > c$.

Firm 2's profits rise continuously as its price rises from the level of marginal cost, $c$, to just below firm 1's price. When $p_2$ equals $p_1$, firm 2's profits fall relative to those earned when $p_2$ is just below $p_1$. For $p_2$ greater than $p_1$, firm 2 earns zero profits.

Note that firm 2's profits rise continuously as $p_2$ rises from $c$ to just below $p_1$. Whenever $p_2$ is less than $p_1$, firm 2 is the only company that any consumer buys from. However, in the case where $p_1$ is less than or equal to $p_M$, the monopoly position that firm 2 obtains from undercutting $p_1$ is constrained. In particular, it cannot achieve the pure monopoly price, $p_M$, and associated profit because, at that price, firm 2 would lose all its customers. Still, the firm will wish to get as close to that result as possible. It could, of course, just match firm 1's price exactly. But whenever it does so it shares the market equally with its rival. If instead of setting $p_2 = p_1$, firm 2 just slightly reduces its price below the $p_1$ level, it will double its sales while incurring only an infinitesimal decline in its profit margin per unit sold. This is a trade well worth the making as Figure 10-2 makes clear. In turn, the implication is that for any $p_1$ such that
$p^M > p_1 > c$, firm 2's best response is to set $p_2^* = p_1 - \epsilon$, where $\epsilon$ is an arbitrarily small amount.

The last case to consider is the case in which firm 1 prices at cost so that $p_1 = c$. Clearly, firm 2 has no incentive to undercut this value of $p_1$. To do so would only lead to losses for firm 2. Instead, firm 2 will do best to set $p_2$ either equal to or above $p_1$. If it prices above $p_1$, firm 2 will sell nothing and earn zero profits. If it matches $p_1$, it will enjoy positive sales but break even on every unit sold. Accordingly, firm 2 will earn zero profits in this latter case, too. Thus, when $p_1 = c$, firm 2's best response is to set $p_2$ either greater than or equal to $p_1$.

Our preceding discussion may be summarized with the following description of firm 2's best price response:

$$
\begin{align*}
  p_2^* &= \frac{a + c}{2b} \quad \text{if } p_1 > \frac{a + c}{2b} \\
  p_2^* &= p_1 - \epsilon \quad \text{if } c < p_1 \leq \frac{a + c}{2b} \\
  p_2^* &\geq p_1 \quad \text{if } c = p_1 \\
  p_2^* &> p_1 \quad \text{if } c > p_1 \geq 0 \\
\end{align*}
$$

(10.4)

By similar reasoning, firm 1's best response, $p_1^*$, for any given value of $p_2$, would be given by

$$
\begin{align*}
  p_1^* &= \frac{a + c}{2b} \quad \text{if } p_2 > \frac{a + c}{2b} \\
  p_1^* &= p_2 - \epsilon \quad \text{if } c < p_2 \leq \frac{a + c}{2b} \\
  p_1^* &\geq p_2 \quad \text{if } c = p_2 \\
  p_1^* &> p_2 \quad \text{if } c > p_2 \geq 0 \\
\end{align*}
$$

(10.5)

We may now determine the Nash equilibrium for the duopoly game when played in prices. We know that a Nash equilibrium is one in which neither firm has an incentive to change its strategy. For example, the strategy combination $\left( p_1 = \frac{a + c}{2b}, \quad p_2 = \frac{a + c}{2b} - \epsilon \right)$ cannot be an equilibrium. This is because in this combination, firm 2 undercuts firm 1's price and sells at a price just below the monopoly level. However, in such a case, firm 1 would have no customers and earn zero profit. Since firm 1 could earn substantial profit by lowering its price to just below that set by firm 2, it would wish to do so. Accordingly, this strategy cannot be a Nash equilibrium. To put it another way, firm 2 could never expect firm 1 to set the monopoly price of $p_1 = \frac{(a + c)}{2b}$ precisely because firm 1 would know that so doing would lead to zero profit as firm 2 undercut that price by a small amount $\epsilon$ and stole all of firm 1's customers.

As it turns out, there is only one Nash equilibrium for the Bertrand duopoly game we have described. It is the price pair $(p_1^* = c, p_2^* = c)$.

If firm 1 sets this price in the

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\[3\] If prices cannot be set continuously but are restricted to whole dollar amounts, then there are two Nash equilibria. One is where both firms set price equal to marginal cost, $p_1 = p_2 = c$. The other is where each firm sets a price equal to $\$1$ above marginal cost, $p_1 = p_2 = c + 1$. [3]
Reality Checkpoint

Bertrand Competition—The Eyes Have It

Perhaps one of the most dramatic examples of Bertrand competition comes from the market for laser eye surgery. Such surgery uses a specialized laser machine to reshape the cornea, which acts like a second lens to focus light on the retina. Age and other factors can change the cornea's shape so that the focal point moves either to the front or the back of the retina wall. Laser machines originally designed for etching computer chips can correct this problem. Under the direction of a doctor, laser slices are made in the patient's eyeball, allowing the curvature to be reshaped so as to produce the proper focal point. The entire procedure takes about 15 minutes per eye. In 1997, the price for such surgery was close to $3,000 per eye. Currently, it is not uncommon to find prices as low as $499 per eye—a dramatic price reduction of 83 percent in just six years. It reflects two factors. First, the two initial makers of the laser machines, Visx and Summit Technologies (now owned by Nestle), were subsequently joined by other manufacturers, including Bausch & Lomb, Lasik, and Nidek. Each of these firms offers a virtually identical product to the eye surgery clinics. The second factor was the rapid discounting of eye surgery prices by the clinics themselves. Virtually every major city has at least two such clinics. Faced with a competitor offering a nearly identical service, each clinic can do little to attract customers except offer a lower price.


expectation that firm 2 will do so, and if firm 2 acts in precisely the same manner, neither will have an incentive to change. Hence, the outcome of the Bertrand duopoly game is that the market price equals marginal cost. This is, of course, exactly what occurs under perfect competition. The only difference is that here, instead of many small firms, we have just two large ones.

It is no wonder that Bertrand noted the different outcome obtained when price replaces quantity as the strategic variable. Far from being a cosmetic or minor change, this alternative specification has dramatic impact. It is useful, therefore, to explore the nature and the source of this powerful effect more closely.

Practice Problem 10.1

Let the market demand for carbonated water be given by \( Q^D = 100 - 5P \). Let there be two firms producing carbonated water, each with a constant marginal cost of 2.

a. What is the market equilibrium price and quantity when each firm behaves as a Cournot duopolist choosing quantities? What are the firms' profits?

b. What is the market equilibrium price and quantity when each firm behaves as a Bertrand duopolist choosing price? What are the firms' profits?
10.2 BERTRAND RECONSIDERED

Like its Cournot cousin, the Bertrand analysis of a duopoly market is not without its critics. The chief source of criticism with the Bertrand model is its assumption that any price deviation between the two firms leads to an immediate and complete loss of demand for the firm charging the higher price. It is this assumption that gives rise to the discontinuity in both firms' demand and profit functions. It is also this assumption that underlies our derivation of each firm's best response function.

Such a consumer response to minor price differences seems extreme. More importantly, there are two reasons why a firm's decision to charge a price higher than its rival would not result in the loss of all its customers. One is that because of a capacity constraint the rival firm will not be able to serve all of the customers demanding the product or service at the low price. The second is that the two products may not be perfect substitutes.

As an example of the importance of capacity constraints, consider a small New England area with two ski resorts, Pepall Ridge and Snow Richards, each located on different sides of Mount Norman. Skiers regard the services at these resorts to be the same and will choose whenever possible to ski at the resort that quotes the lowest lift ticket price. Pepall Ridge is a small resort that can accommodate 1,000 skiers per day. Snow Richards is slightly bigger and can handle 1,400 skiers a day. Skiing on Mount Norman has become extremely popular. The demand for skiing services on Mount Norman is estimated to be \( Q = 6,000 - 60P \), where \( P \) is the price of a daily lift ticket and \( Q \) is the number of skiers per day.

The two resorts compete in price. Suppose that the marginal cost of providing lift services is the same at each resort and is equal to $10 per skier. As a little thought will reveal, the outcome where each resort sets a price equal to marginal cost cannot be a Nash equilibrium. Demand when the price of a lift ticket is equal to $10 would be equal to 5,400, far exceeding the total capacity of the two resorts. If each resort had understood the extent of demand, each might have built additional lifts, ski runs, and parking facilities such that each would have had much greater capacity. Nevertheless, it is not likely that the Nash equilibrium will end up with each resort setting a price equal to the marginal cost of $10 per skier. Why? Think of it this way. If Pepall Ridge sets a price of $11, Snow Richards could set a price of $10.02, in which case it would steal all of Pepall Ridge's customers and, in fact, serve just about all 5,400 skiers. However, this is only a credible threat—one that will inhibit Pepall Ridge from pricing at $11 in the first place—if Snow Richards really can serve that many customers. However, to build that much capacity would be fairly short-sighted behavior for Snow Richards. For if Pepall Ridge is serving no skiers at a price of $11 while Snow Richards is serving all the skiers at a price of $10.02, Pepall Ridge will retaliate with a price of $10.01 and steal all the market demand for itself. Again, however, for this to be a credible threat Pepall Ridge must also have capacity of nearly 5,400.

The logical extension of this analysis is that the pressure for each resort to cut price to marginal cost rests implicitly on each having sufficient capacity to serve the entire competitive market supply of 5,400. However, when each charges a price of $10, the market is split and each serves only 2,700. It seems unlikely that each will

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4 Edgeworth (1897) was one of the first economists to investigate the impact of capacity constraints on the Bertrand analysis.
build capacity of 5,400 if each will serve only 2,700 skiers in equilibrium. Yet unless each does so, there is little pressure on price to fall to the marginal cost of $10.

More generally, denote as $Q^C$ the competitive output or the total demand when price is equal to marginal cost, that is, $Q^C = a - bc$. If neither firm has the capacity to produce $Q^C$ (neither could individually meet the total market demand generated by competitive prices), but instead can each produce only a smaller amount, then the Bertrand outcome with $p_1 = p_2 = c$ will not be the Nash equilibrium. The reason for this should be clear from our previous work. In the Nash equilibrium, it must be the case that each firm’s choice is a best response to the strategy of the other. Consider the original Bertrand solution with prices chosen to be equal to marginal cost, $c$, and profit at each firm equal to zero. Because we have now imposed a capacity constraint, a firm such as firm 2 can contemplate raising its price. If firm 2 sets $P_2$ above marginal cost, and hence above $P_1$, it would surely lose some of its customers. But it would not lose all of them. Firm 1 does not have the capacity to serve them. Some customers would remain with firm 2. Yet firm 2 is now earning some profit from each such customer ($P_2 > c$), implying that its total profit is now positive whereas before it was zero. It is evident, therefore, that $P_2 = c$ is not a best response to $P_1 = c$. Accordingly, the strategy combination $(P_1 = c, P_2 = c)$ cannot be a Nash equilibrium if there are binding capacity constraints.

Once we bring a consideration of capacity constraints into the analysis, the game becomes a two-stage one. In the first stage, firms determine capacity. In the second, they then compete in price. Formal analysis of such games is tricky. However, as previously noted, neither firm is likely to acquire enough capacity to serve the entire market when pricing at marginal cost. Again, though, if neither acquires that large amount of capacity, then the Bertrand solution of each charging a price equal to marginal cost cannot be a Nash equilibrium. We will return to the issue of capacity choice in Chapter 12. It is worth noting, however, that the equilibrium in a model of price competition with capacity constraints takes us away from the efficient outcome, and closer towards the outcome in the Cournot model.5

Let us now return to the ski resort competition between Pepall Ridge and Snow Richards, and let’s also make a special additional assumption. Let’s assume that at any price at which a resort has demand beyond its maximum capacity, the skiers that it actually serves are those who are the most eager, that is, those who have the highest willingness to pay. For example, at a price of $50 at each resort, total market demand is 3,000. This is beyond the total market capacity of 2,400 and, therefore, each resort will need somehow to ration or choose which skiers will actually ski. Our assumption, sometimes called the efficient rationing assumption, is that they will do this by serving customers in order of their willingness to pay. For example, Pepall Ridge will choose those 1,000 potential skiers with the top willingness to pay. If we proceed in this way, then we can derive the residual demand curve facing Snow Richards at any price.

A price of particular interest is $60. Suppose that both resorts have set $P_1 = P_2 = 60$. At these prices, total demand is equal to 2,400, which is just equal to the total capacity of the two resorts. Is this a Nash equilibrium? We can answer this question by using the above logic to determine the demand function facing Snow Richards when Pepall Ridge sets a price equal to $60$. Under our assumption of efficient rationing, this is shown in Figure 10-3. It is the original demand curve shifted to the

5 This result is formally modeled in a two-stage game in Kreps and Scheinkman (1983).
left by 1,000 units, that is, it is \( Q = 5,000 - 60P \) (or, in inverse form, \( P = \frac{83.333}{60} - \frac{Q}{60} \)). The marginal revenue curve facing Snow Richards when Pepall Ridge charges a price of $60 is also shown.

Note that while changes in its price also change the demand facing Snow Richards, it is always constrained to serve no more than its capacity of 1,400. In this light, consider again the situation in which Snow Richards sets a price just equal to the $60 that Pepall Ridge is charging. Is this a best response? We check this by asking whether Snow Richards has an incentive to change its price. The answer is no. Lowering its price will not lead to any more customers since Snow Richards is at capacity. Yet raising its price is not an attractive option, either. This will lower its demand below capacity of 1,400. Since marginal revenue exceeds marginal cost, losing customers also loses profit. Accordingly, Snow Richards has no incentive to lower or to raise its price from $60, assuming that Pepall Ridge is also setting that price. By a similar logic, we can show that Pepall Ridge has no incentive to change its price from $60 given that Snow Richards is charging that amount. Therefore, \( P_1 = P_2 = $60 \) is the Nash equilibrium for this game.

As noted earlier, the logic of the above example is quite general. Firms competing in prices selling identical products will rarely choose the capacity necessary to serve the total market demand forthcoming at competitive prices. As a result, both output and capacity will be less than the competitive level. In turn, this implies that prices must rise to a level at which demand equals the total industry capacity—a level that is necessarily above marginal cost. Thus, the efficiency property of the Bertrand solution can break down when firms are capacity constrained.

**Practice Problem 10.2**

Suppose now market demand for skiing increases to \( Q^D = 9,000 - 60P \). However, because of environmental regulation the two resorts cannot increase their capacities and serve more skiers. What is the Nash equilibrium outcome for this case? That is, what are the profit-maximizing prices set by Pepall Ridge and Snow Richards?
10.3 BERTRAND IN A SPATIAL SETTING

Capacity constraints are one reason to question Bertrand’s view that price competition will automatically lead to marginal cost pricing. However, as previously noted, there is a second reason that the Bertrand efficient outcome may not be obtained. This is that the two firms typically do not produce identical products as Bertrand assumed. Think of hair salons, for example. No two hair stylists cut and style hair in exactly the same way. Nor will the salons have exactly the same sort of equipment or furnishings. Indeed, as long as the two firms are not side by side, they will differ in their location. This is often sufficient by itself to generate a preference by some consumers for one salon or the other, even when different prices are charged. In short, differences in locations, furnishings, or cutting styles can each be sufficient to permit one salon to price somewhat higher than its rival without immediately losing all of its customers.

We presented the basic spatial model of product differentiation based on the work of Hotelling (1929) in Chapter 7.6 There our aim was to understand the use of such differentiation by a monopoly firm to extract additional surplus. The same model of demand, however, may also be used to understand the nature of price competition when there is more than one firm marketing differentiated products. Let’s review the basic setup presented earlier. There is a line of unit (say, one mile) length along which consumers are uniformly distributed. This market is supplied by two stores. This time, however, the same company does not operate the two stores. Rival firms operate them. One firm—located at the west end of town—has the address \( x = 0 \). The other—located at the east end of town—has the location \( x = 1 \). Each of the firms has the same constant unit cost of production, \( c \).

Each point on the line is associated with a value of \( x \) measuring the location of that point relative to the west or left end of town. A consumer whose most preferred style or location is \( x' \) is called consumer \( x' \). While consumers differ about which variant or location of the good is best, they are the same insofar as each has the same reservation demand price, \( V \), for their most preferred good. Naturally we assume that \( V \) is substantially greater than the unit cost of production, \( c \). Each consumer will also buy at most one unit of the product. If consumers purchase a good located “far away” from their most preferred location, they incur a utility cost. In particular, consumer \( x' \) incurs the cost \( t x' \) if he or she consumes good 1 (located at \( x = 0 \)), and the cost \( t(1 - x') \) if he or she consumes good 2 (located at \( x = 1 \)). Figure 10-4 describes this market setting.

Again, it bears repeating that the location difference that we have introduced serves as a metaphor for other qualitative differences. Thus, instead of having two

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6 Friedman (1977), pp. 50–76, is a very readable discussion. See also our discussion in Chapter 4.
stores geographically separated we could think of two products marketed by two different firms that are differentiated by some characteristic, such as sugar content in the case of soft drinks, or an index of fuel efficiency and ride comfort in the case of automobiles. Our unit line in each case would represent the spectrum of products differentiated by this characteristic and each consumer would have a most preferred product specification on this line. For the case of soft drinks our two firms could be Pepsi and Coca-Cola. For the case of automobiles, our two firms could be Ford and GM.

As Bertrand assumed, the two firms compete for customers by setting prices, $p_1$ and $p_2$, respectively. These are chosen simultaneously. As always, we look for a Nash equilibrium as the solution to the game. One requirement for such an equilibrium is that both firms have a positive market share as long as both prices are greater than or equal to $c$. If this condition is not satisfied it would mean that at least one firm's price is set so high that it has zero market share and, therefore, zero profits. Since in our case the two firms have the same unit cost, $c < V$, a firm could always obtain positive profits by cutting its price, and hence a zero market share situation cannot be part of a Nash equilibrium.

We will make the further assumption that the Nash equilibrium outcome is one in which the entire market is served. That is, we will assume the outcome involves a market configuration in which every consumer buys the product from either firm 1 or firm 2. This assumption will be true so long as each consumer's reservation price, $V$, is sufficiently large. When $V$ is large, firms will have an incentive to sell to as many customers as possible because such a high willingness to pay will imply that each customer can be charged a price sufficiently high to make each such sale profitable.

As we saw in Chapter 7, an important implication of the assumption that the entire market is served is that there will be some consumer, whom we call the marginal consumer, $x^m$, who is indifferent between buying from either firm 1 or firm 2. That is, that consumer enjoys the same surplus either way. Algebraically, this means that for consumer $x^m$

$$V - p_1 - tx^m = V - p_2 - t(1 - x^m).$$

Equation (10.6) may be solved to find the address of the marginal consumer, $x^m$. This is

$$x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t}. \quad (10.7)$$

At any set of prices, $p_1$ and $p_2$, all consumers to the west or left of $x^m$ buy from firm 1. All those to the east or right of $x^m$ buy from firm 2. In other words, $x^m$ is the fraction of the market buying from firm 1 and $(1 - x^m)$ is the fraction buying from firm 2. If the total number of consumers is denoted by $N$, the demand function facing firm 1 at any price combination $(p_1, p_2)$ in which the entire market is served is

$$D^1(p_1, p_2) = x^m(p_1, p_2)N = \frac{(p_2 - p_1 + t)}{2t}N. \quad (10.8)$$

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7 Refer to Figure 7-3 in Chapter 7 for a discussion of this point.
8 We are using $N$ here to refer to the number of consumers in the market.
Similarly, firm 2’s demand function is
\[ D^2(p_1, p_2) = [1 - \alpha(p_1, p_2)]N = \frac{(p_1 - p_2 + t)}{2t}N. \] (10.9)

Notice that unlike the original Bertrand duopoly model shown earlier, the model presented here is one in which the demand function facing either firm is continuous in both \( p_1 \) and \( p_2 \). This is because when goods are differentiated, a decision by firm 1 to set \( p_1 \) a little higher than its rival’s price \( p_2 \) does not cause firm 1 to lose all of its customers. Some of its customers will still prefer to buy good 1 even at the higher price simply because they prefer that version of the good to the style (or location) marketed by firm 2.\(^9\)

The continuity in demand functions carries over into the profit functions. Firm 1’s profit function is
\[ \Pi^1(p_1, p_2) = (p_1 - c)(p_2 - p_1 + t)N. \] (10.10)

Similarly, firm 2’s profits are given by
\[ \Pi^2(p_1, p_2) = (p_2 - c)(p_1 - p_2 + t)N. \] (10.11)

In order to work out firm 1’s best response pricing strategy, we need to work out how firm 1’s profit changes as the firm varies price \( p_1 \) in response to a given price \( p_2 \) set by firm 2. The most straightforward way to do this is to take the derivative of the profit function in equation (10.10) with respect to \( p_1 \). We can then solve for the firm’s best response price \( p^*_1 \) to a given price \( p_2 \) where we set the derivative equal to zero.\(^{10}\) However, careful application of our standard alternative of converting firm 1’s demand curve into its inverse form and solving for the point at which marginal revenue equals marginal cost will also work.

From equation (10.8), we can write firm 1’s inverse demand curve for a given value of firm 2’s price, \( p_2 \), as \( p_1 = p_2 + t - \frac{q_1^*}{N} \). Hence, firm 1’s marginal revenue is
\[ MR_1 = p_2 + t - \frac{q_1^*}{N}4t. \] This may be equated with firm 1’s marginal cost to yield the first-order condition for profit maximization, \( p_2 + t - \frac{q_1^*}{N}4t = c \). Solving for the optimal value of firm 1’s output, again given the price chosen by firm 2, we then obtain
\[ q_1^* = \frac{p_2 + t - c}{4t}N. \] (10.12)

When we substitute the value for \( q_1^* \) in equation (10.12) into firm 1’s inverse demand curve, we find the optimal price for firm 1 to set given the value of the price set by firm 2. This is by definition firm 1’s best response function. It is

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\(^9\) Our assumption that the equilibrium is one in which the entire market is served is critical to the continuity result.

\(^{10}\) Setting \( \Pi^1(p_1, p_2)/p_1 = 0 \) in equation (10.10) yields immediately \( p^*_1 = (p_2 + c + t)/2 \).
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\[ p_1^* = \frac{p_2 + c + t}{2}, \]  
\[ (10.13) \]

where \( t \) is the per-unit distance transportation or utility cost incurred by a consumer. Of course, we can replicate this procedure for firm 2. Because the firms are symmetric, the best response function of each firm is the mirror image of that of its rival. Hence, firm 2’s best price response function is

\[ p_2^* = \frac{p_1 + c + t}{2}. \]  
\[ (10.14) \]

The Nash equilibrium is a pair of best response prices, \( p_1^*, p_2^* \), such that \( p_1^* \) is firm 1’s best response to \( p_2^* \), and \( p_2^* \) is firm 2’s best response to \( p_1^* \). Thus, we may replace \( p_1 \) and \( p_2 \) on the right-hand side of equations (10.14) and (10.15) with \( p_1^* \) and \( p_2^* \), respectively. Then, solving jointly for the Nash equilibrium pair, \( p_1^*, p_2^* \), yields

\[ p_1^* = p_2^* = c + t. \]  
\[ (10.15) \]

The best response functions for the two firms are shown in Figure 10-5. They are upward sloping. The Nash equilibrium set of prices is shown as well. In equilibrium, each firm charges a price that is equal to the unit cost plus an amount, \( t \), the utility cost per unit of distance a consumer incurs in buying a good that is at some distance from the preferred good. At these prices, the firms split the market. The marginal consumer is located at the address \( x' = \frac{1}{2} \). The profit earned by each firm is the same and equal to \( \frac{Nt}{2} \).

Consider again, for example, the two hair salons located one mile apart on Main Street. All their potential customers live along this stretch of Main Street and they are

![Figure 10-5: Best Response Functions for Price Competition with Imperfect Substitutes](image)
Part Three · Game Theory and Oligopoly Markets

Reality Checkpoint

Unfriendly Skies: Price Wars in Airline Markets

Following general deregulation in 1977, the profitability of the airline industry has generally deteriorated and become much more volatile. An important source of these developments has been the continued outbreak of price wars. Morrison and Winston (1996) define such conflicts as any city-pair route market in which the average airfare declines by 20 percent or more within a single quarter. Based on this definition, they estimate that over 81 percent of airline city-pair routes experienced such wars in the 1979-95 time period. In the wars so identified, the average fare in fact typically falls by over 37 percent and sometimes by as much as 79 percent. These wars appear to be triggered by unexpected movements in demand and the entrance of new airlines on a route, especially low-cost airlines like Southwest. Morrison and Winston also find that the effect of such fare wars on industry profits is important. On average, they estimate that the intense price competition costs airlines $300 million in foregone profits in each of the first 16 years following deregulation. This amounts to over 20 percent of total net income over these same years. Of course, to the extent that this profit loss simply reflects movement toward the Bertrand outcome of marginal cost pricing it shows up as a gain to consumers and a net improvement in efficiency. Judging from the comments in the press, however—especially since September 11, 2001—airline executives take little comfort in such gains.


uniformly spread out. Each consumer is willing to pay at most $50 for a haircut done at the consumer’s home. However, if a consumer has to travel to get the haircut a travel cost of $5 per mile is incurred. Each of the hair salons can cut hair at a constant unit cost of $10 per cut, and each wants to set a price per haircut that maximizes the salon’s profit. Our model predicts that the equilibrium price of a haircut in this town will be $15, a price that is greater than the marginal cost of a haircut.

Two points are worth making in connection with the foregoing analysis. First, note the role that the parameter $t$ plays. It is a measure of the value each consumer places on obtaining the most preferred version of the product. The greater is $t$, the more the consumer is willing to pay a high price to avoid being “far away” from the favorite location. That is, a high $t$ value indicates that firms need not worry about charging a high price because consumers would prefer to pay that price rather than buy a low-price alternative that is “far away” from their preferred style. Thus, when $t$ is large, the price competition between the two firms is softened. A large value of $t$ implies that effective product differentiation makes price competition much less intense.

However, as $t$ falls, consumers place less value on obtaining a preferred style and focus more on simply obtaining the best price. This intensifies price competition. In
the limit, when \( t = 0 \), differentiation is of no value to consumers. They treat all goods as essentially identical. Price competition becomes fierce and, in the limit, forces prices to be set at marginal cost just as in the original Bertrand model.

The second point to be made in connection with this analysis concerns the location of the firms. We simply assumed that the two firms were located at either end of town. However, as we discussed in Chapter 7, the location or product design of the firm is also an object of choice. Unfortunately, allowing the firms in the model to choose both their price and their location strategies makes the problem too complicated to work out here. Still, the intuition behind this indeterminacy is instructive. Two opposing forces make the combined choice of price and location difficult. On the one hand, the two firms will wish to avoid locating at the same point because to do so eliminates all differences between the two products. Price competition in this case will be fierce as in the original Bertrand model. On the other hand, each firm also has some incentive to locate near the center of town. This enables a firm to reach as large a market as possible. Evaluating the balance of these two forces is what makes determination of the ultimate equilibrium so difficult.\(^{11}\)

**Practice Problem 10.3**

Imagine that the two hair salons located on Main Street no longer have the same unit cost. In particular, one salon has a constant unit cost of $10, whereas the other salon has a constant unit cost of $20. The low-cost salon, Cheap-Cuts, is located at the east end of town, \( x = 0 \). The high-cost salon, The Ritz, is located at the west end of town, \( x = 1 \). There are 100 potential customers who live along the one-mile stretch, and they are uniformly spread out along the mile. Consumers are willing to pay $50 for a haircut done at their home. If a consumer has to travel to get a haircut then a travel cost of $5 per mile is incurred. Each salon wants to set a price for a haircut that maximizes the salon’s profit.

a. The demand functions facing the two salons are not affected by the fact that now one salon is high-cost and the other is low-cost. However, the salons’ best response functions are affected. Compute the best response function for each salon. How does an increase in the unit cost of one salon affect the other salon’s best response?

b. Work out the Nash equilibrium in prices for this model. Compare these prices to the ones derived in the text for the case when the two salons had the same unit cost equal to $10. Explain why prices changed in the way they did. It may be helpful in your explanation to draw the best response functions when the salons are identical and compare them to those when the salons have different costs.

### 10.4 STRATEGIC COMPLEMENTS AND SUBSTITUTES

Best response functions in simultaneous-move games are extremely useful tools for understanding what we mean by a Nash equilibrium outcome. But an analysis of such functions also serves other useful purposes. In particular, examining the properties of

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\(^{11}\) There is a wealth of literature on this topic, with the outcome often depending on the precise functional forms assumed. See, for example, Eaton (1976); D’Aspremont, Gabszewicz, and Thisse (1979); Novshek (1980); and Economides (1989).
best response functions can aid our understanding of how strategic interaction works and how that interaction can be made “more” or “less” competitive.

Figure 10-6 shows both the best response functions for the standard Cournot duopoly model and the best response functions for the Bertrand duopoly model with differentiated products. One feature in the diagram is immediately apparent. The best response functions for the Cournot quantity model are negatively sloped—firm 1’s best response to an increase in $q_2$ is to decrease $q_1$. But the best response functions in the Bertrand price model are positively sloped. Firm 1’s best response to an increase in $p_2$ is to increase $p_1$ as well.

**FIGURE 10-6**

**BEST RESPONSE FUNCTIONS FOR THE COURNOT (QUANTITY) CASE AND THE BERTRAND (PRICE) CASE**

A rise in firm 2’s cost shifts its response function inwards in the Cournot model but outwards in the Bertrand model. Firm 1 reacts aggressively to increase its market share in the Cournot case. It reacts mildly in the Bertrand price by raising its price.

Whether the best response functions are negatively or positively sloped is quite important. The slope reveals much about the nature of competition in the product market. To see this, consider the impact of an increase in firm 2’s unit cost, $c_2$. Our analysis of the Cournot model indicated that the effect of a rise in $c_2$ would be to shift inward firm 2’s best response curve. As Figure 10-6 indicates, this leads to a new Nash equilibrium in which firm 2 produces less and firm 1 produces more than each did before $c_2$ rose. That is, in the Cournot quantity model, firm 1’s response to firm 2’s bad luck is a rather aggressive one in which it seizes the opportunity to expand its market share at the expense of firm 2.

Consider now the impact of a rise in $c_2$ in the context of the differentiated goods Bertrand model. The rise in this case shifts out firm 2’s best response function. Given the rise in its cost, firm 2 now finds it better to set a higher $p_2$ than it did previously in response to any given value of $p_1$. How does firm 1 respond? Unlike the Cournot
case, firm 1's reaction is not aggressive. Quite to the contrary, firm 1—seeing that firm 2 is now less able to set a low price—realizes that the price competition from firm 2 is now less intense. Hence, firm 1 now reacts by raising $p_1$.

When the best response functions are upward sloping, we say that the strategies (prices in the Bertrand case) are strategic complements. When we have the alternative case of downward-sloping response functions, we say that the strategies (quantities in the Cournot case) are strategic substitutes. This terminology comes from Bulow, Geanakopolos, and Klemperer (1985) and reflects similar terminology in consumer demand theory. When a consumer reacts to a change in the price of one good by buying either more or less of both that good and another product, we say that the two goods are complements. When a consumer reacts to a change in the price of one product by buying more (less) of it and less (more) of another, we say that the two goods are substitutes. This is the source of the similarity. Prices in the spatial Bertrand model are called strategic complements because a change (the rise in $c_2$) inducing an increase in $p_2$ also induces an increase in $p_1$. Similarly, quantities in Cournot analysis are strategic substitutes because such a change in $c_2$ induces a fall in $q_2$, but a rise in $q_1$.

Clearly, the choice of whether to use price or quantity as the strategic variable to model competition in a market is an important one. What factors influence the choice? In those industries in which firms set their production schedules far in advance of putting the goods on the market for sale, there is a good case to assume that firms compete in quantities. Examples include the world energy market, coffee growers, automobile producers, and the cement industry. In many service industries, such as banking, insurance, and air travel, it is more natural to think in terms of price competition. In certain manufacturing industries, such as cereal and detergents, the price competition for customers is a stronger factor than the setting of production schedules, and so Bertrand price competition may be the more appropriate model.

**SUMMARY**

The Bertrand model makes clear that price competition is quite different from quantity competition. Under quantity or Cournot competition, prices remain substantially above marginal cost so long as the number of firms is not large and high-cost firms can survive in equilibrium. Under Bertrand competition, prices are pushed to marginal cost even if there are just two firms. Moreover, high-cost firms cannot survive Bertrand competition against a firm with lower costs. In short, the simplest Bertrand model predicts competitive and efficient market outcomes even when the number of firms is quite small.

However, the efficient outcomes predicted by the pure Bertrand model are predicated on two key assumptions. One of these is that firms have extensive capacity so that it is possible to serve all a rival's customers after undercutting the rival's price. The other key assumption is that the firms in question produce identical products so that relative price is all that consumers use in choosing between brands. If either of these assumptions is relaxed, the efficiency outcomes of the simplest Bertrand model are no longer obtained. If firms must choose production capacities in advance, the outcome with Bertrand price competition approaches that of the Cournot model. If products are differentiated, prices are again likely to remain above marginal cost. Indeed, given the fierceness of price competition, firms have a real incentive to differentiate their products.
A useful model of product differentiation is the Hotelling spatial model explored in Chapter 7. This model uses geographic location as a metaphor for more general distinctions between different versions of the same product. It thereby makes it possible to consider price competition between firms selling differentiated products. As noted, the model demonstrates that Bertrand competition with differentiated products does not result in efficient marginal cost pricing. It also makes clear that the deviation from such pricing depends on how much consumers value variety. The greater value that the typical consumer places on getting the most preferred brand or version of the product, the more prices will rise above marginal cost even with Bertrand competition.

Ultimately, the differences between Cournot and Bertrand competition reflect an underlying difference between quantities and prices as strategic variables. The quantities chosen by Cournot firms are strategic substitutes—increases in one firm’s production lead to decreases in the rival’s output. In contrast, the prices chosen by Bertrand competitors are strategic complements. A rise in one firm’s price permits its rival to raise price, too.

**PROBLEMS**

1. Suppose firm 1 and firm 2 each produce the same product and face a market demand curve described by \( Q = 5,000 - 200P \). Firm 1 has a unit cost of production, \( c_1 \), equal to 6, whereas firm 2 has a higher unit cost of production, \( c_2 \), equal to 10.
   a. What is the Bertrand Nash equilibrium outcome?
   b. What are the profits of each firm?
   c. Is this outcome efficient?

2. Suppose that market demand for golf balls is described by \( Q = 90 - 3P \), where \( Q \) is measured in kilos of balls. There are two firms that supply the market. Firm 1 can produce a kilo of balls at a constant unit cost of $15 whereas firm 2 has a constant unit cost equal to $10.
   a. Suppose firms compete in quantities. How much does each firm sell in a Cournot equilibrium? What is the market price and what are the firms’ profits?
   b. Suppose firms compete in price. How much does each firm sell in a Bertrand equilibrium? What is market price and what are the firms’ profits?
   c. Would your answer in (b) change if there were three firms, one with unit cost = $20 and two with unit cost = $10? Explain why or why not.
   d. Would your answer in (b) change if firm 1’s golf balls were green and endorsed by Tiger Woods, whereas firm 2’s are plain and white? Explain why or why not.

3. In Tuftsville everyone lives along Main Street, which is 10 miles long. There are 1,000 people uniformly spread up and down Main Street, and each day they each buy one fruit smoothie from one of the two stores located at either end of Main Street. Customers ride their motor scooters to and from the store and the motor scooters use $0.50 worth of gas per mile. Customers buy their smoothies from the store offering the lowest price, which is the store’s price plus the customer’s travel expenses getting to and from the store. Ben owns the store at the west end of Main Street and Will owns the store at the east end of Main Street.
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1. If both Ben and Will charge $1 per smoothie, how many will each of them sell in a day? If Ben charges $1 per smoothie and Will charges $1.40, how many smoothies will each sell in a day?

2. If Ben charges $3 per smoothie what price would enable Will to sell 250 smoothies per day? 500 smoothies per day? 750 smoothies per day? 1,000 smoothies per day?

3. If Ben charges \( p_1 \) and Will charges \( p_2 \), what is the location of the customer who is indifferent between going to Ben’s and going to Will’s? How many customers go to Will’s store and how many go to Ben’s store? What are the demand functions that face Ben and Will?

4. Rewrite Ben’s demand function with \( p_1 \) on the left-hand side. What is Ben’s marginal revenue function?

5. Assume that the marginal cost of a smoothie is constant and equal to $1 for both Ben and Will. In addition, each of them pays Tuftsville $250 per day for the right to sell smoothies. Find the equilibrium prices, quantities sold, and profits.

4. Return to Main Street in Tuftsville. Now suppose that George would like to open another store at the midpoint of Main Street. He, too, is willing to pay Tuftsville $250 a day for the right to sell smoothies.

a. If Ben and Will do not change their prices, what is the best price for George to charge? How much profit would he earn?

b. What do you think will happen if George opens another store in the middle of Main Street? Will Ben and Will have an incentive to change their prices? Their locations? Would one or both leave the market?

5. Suppose that two firms, firm B and firm N, produce complementary goods, say bolts and nuts. The demand curve for each firm is described as follows:

\[
Q_B = Z - P_B - P_N \quad \text{and} \quad Q_N = Z - P_N - P_B
\]

For simplicity, assume further that each firm faces a constant unit cost of production, \( c = 0 \).

a. Show that the profits of each firm may be expressed as \( \Pi_B = (P_B)(Z - P_B - P_N) \) and \( \Pi_N = P_N(Z - P_B - P_N) \).

b. Show that each firm’s optimal price depends on the price chosen by the other as given by the optimal response functions \( P_B^* = (Z - P_N)/2 \) and \( P_N^* = (Z - P_B)/2 \).

c. Graph these functions. Show that the Nash equilibrium prices are \( P_B = P_N = Z/3 \).

d. Describe the interaction between two monopolists selling separate but complementary goods.

References