MAT 540A Midterm Fall 2014

The total number of points is 15.

Part 1. Here you need to give complete proofs.

Question 1 (5 points)**.** Prove that $f\colon X \to Y$ is continuous if and only if for all $A \subset Y$

$$\overline{f^{-1}(A)} \subset f^{-1}(\overline{A}).$$

Question 2 (6 points)**.** Let A be a subset of X given the subspace topology.
(a) If X is Hausdorff, does A have to be Hausdorff? Prove you are correct.
(b) If X is not Hausdorff, can A be Hausdorff? Prove you are correct.

Part 2 (4 points, each question is worth half point)**. True-False.** The questions in this section can be answered either "true" or "false". You do not need to give reasons for your answers, though a wrong answer with a largely correct explanation will receive partial credit.

1. Let $X = \{0, 1\}$ with the topology in which the open sets are $\emptyset$, $\{0\}$, and X.

- Is X Hausdorff?
- Is X metrizable?

2. Let $X = \mathbb{R}^2/\{y - \text{axis}\}$, i.e., the plane with the y-axis collapsed to a point, with the quotient topology.

- Is X Hausdorff?
- Is X metrizable?

3. Let X be the "line with two origins". As a set, this is the real line, except there are two points $0'$ and $0''$ in place of single "zero". There is a set map π to the usual real line mapping all nonzero numbers to themselves, and both $0'$ and $0''$ to 0. The topology on X is the coarsest topology which makes π continuous.

- Is X Hausdorff?
- Is X metrizable?

4. Let $X = \prod_{n=1}^{\infty}[0, n]$, an infinite product of the indicated closed intervals with the product topology.

- Is X Hausdorff?
- Is X metrizable?