MAT 540A | End-of-term exam | Fall 2014

The total number of points is 15.

Part 1. Here you need to give complete proofs.

Question 1 (4 points)**.** Let $f\colon X \to Y$ be a closed, continuous surjection. Assume that X is normal. Prove that Y is normal.

Question 2 (5 points)**.** Let X be a compact metric space with metric d and the property that for all $t < 1$, there are pairs of points x_t, y_t so that $d(x_t, y_t) = t$. Prove there are points x and y so that $d(x, y) = 1$.

Part 2 (6 points, each question is worth one point)**. True-False.** The questions in this section can be answered either "yes" or "no". You do not need to give reasons for your answers.

1-2. Let $X = \{0, 1\}$ with the topology in which the open sets are $\emptyset$, $\{0\}$, and X.

- Is X path-connected?
- Is X compact?

3-4. Let $X = \prod_{n=1}^{\infty}[0, n]$, an infinite product of the indicated closed intervals with the product topology.

- Is X connected?
- Is X compact?

5. Suppose X is a compact Hausdorff space. Is it true that X is metrizable if and only if X has a countable basis?

6. Is Urysohn lemma a "deep" theorem?