Question 1. This was a homework problem.

Question 2. Let $\{a_n\}$ be an increasing sequence pof numbers converging to 1, say $1/2, 2/3, 3/4, ..., n/(n+1), ...$ For each a_n choose a pair of points x_n and y_n in X such that $d(x_n, y_n) = a_n$. Since X is a compact space, the sequence $\{x_n\}$ has finitely many cluster points. Select any one of them, say x. Let x_i be a subsequence converging to x. Similarly, $\{y_n\}$ has finitely many cluster points, choose one y and a subsequence y_i converging to y. Given $\epsilon > 0$ thete is $N > 0$ so that $d(x, x_i) < \epsilon$, $d(y, y_i) < \epsilon$ foa ll $i > N$. Since $d(x_i, y_i) = a_i$, we have two triangle inequalities: $d(x, y) \leq a_i + 2\epsilon$ and $a_i \leq d(x, y) + 2\epsilon$ OR $a_i - 2\epsilon \leq d(x, y) \leq a_i + 2\epsilon$. Taking $\epsilon > 0$ increasingly small, get $d(x, y) = \lim a_N = 1$.

Part 2. (1) Yes: connect 0 to 1 by the path that sends $[0, 1)$ to 0 and 1 to 1. (2) Yes: all finite spaces are compact. (3) Yes: products of connected spaces are conected. (4) Yes, by Tychonoff's theorem. (5) Yes, by Urysohn's lemma. (6) Yes, see the beginning of section 33.