MAT 540A Test No. 7 Fall 2014

Please turn in the solutions on Monday, December 8, in class. There is an automatic extension to Tuesday, December 9, in my office between 10 and 11am if you need it.

Question 1 (Qual Exam from January 2008)**.** Suppose that X is path-connected and let $h\colon X \to Y$ be a continuous map with $h(x_0) = y_0$ and $h(x_1) = y_1$. Recall that h induces a group homomorphism from $\pi_1(X, x_0)$ to $\pi_1(Y, y_0)$, which will be denote by $(h_{x_0})_*$, and also a homomorphism from $\pi_1(X, x_1)$ to $\pi_1(Y, y_1)$, which will be denote by $(h_{x_1})_*$. Prove that there are isomorphisms $\phi\colon \pi_1(X, x_0) \to \pi_1(X, x_1)$ and $\psi\colon \pi_1(Y, y_0) \to \pi_1(Y, y_1)$ so that

$$\psi \circ (h_{x_0})_* = (h_{x_1})_* \circ \phi.$$

Question 2 (Qual Exam from August 2005)**.** Let $p\colon E \to B$ be a covering map. Suppose that f and g are continuous functions from I, the closed unit interval, to E such that

1. $pf = pg$,
2. $f(0) \neq g(0)$.

Prove that for all t in I, $f(t) \neq g(t)$.

Question 3 (Qual Exam from January 2014)**.** Let X be the topological space consisting of the standard 2-sphere together with a line segment from the north pole to the south pole. Compute $\pi_1(X)$.

Question 4 (4 points)**.** Recall that a space is *simply connected* if it is connected and its fundamental group is trivial.

1. Prove that the punctured sphere S^2 is simply connected.
2. What is the fundamental group of S^2 without two points?
3. Prove that S^3 without two points is simply connected.
4. What is the fundamental group of S^2 without three points?
5. What is the fundamental group of S^3 without three points?
6. What is the fundamental group of S^3 without 1001 points?