Question 4. Connectedness is clear in all parts. For (1), we know from class that $S^2 -$ point is homeomorphic to $\mathbb{R}^2$. (2) can use part (1) because $S^2 -$ point $-$ point is homeomorphic to $\mathbb{R}^2 -$ point, and that deformation retracts to S^1. So the answer is $\mathbb{Z}$. In (3) $S^3 -$ point is homeomorphic to $\mathbb{R}^3$ just as in part (1), so $S^3 -$ point $-$ point is homeomorphic to $\mathbb{R}^3 -$ point, which deformation retracts onto the unit sphere S^2, so the answer is the trivial group. For (4), as in (1) we can deformation retract $S^2 -$ point to the disk D^2. So $S^2 -$ point $\times$ 3 deformation retracts to $D^2 -$ point $\times$ 2. This in tern clearly dformation retracts to the "figure 8" that can be traced around the two punctures. So the answer is the free group on two generators, or $\mathbb{Z} * \mathbb{Z}$. We need Baby Van Kampen again in (5), applied to the homeomorphic space $\mathbb{R}^3 -$ point $\times$ 2. This is the union of two $\mathbb{R}^3 -$ point which is simply connected. Now we can do induction until we get to 1001 punctures.