Question 1. We are given that there are homotopies $F\colon X \times I \to Y$ from f_1 to f_2 and $G\colon Y \times I \to Z$ from g_1 and g_2. This gives the composition

$$\overline{H}\colon X \times (I \times I) = (X \times I) \times I \xrightarrow{F \times \mathrm{id}_I} Y \times I \xrightarrow{G} Z.$$

Take any path $\alpha\colon I \to I \times I$ so that $\alpha(0) = (0,0)$ and $\alpha(1) = (1,1)$. Now the composition

$$H\colon X \times I \xrightarrow{\mathrm{id}_X \times \alpha} X \times I \times I \xrightarrow{\overline{H}} Z$$

is exactly what we need:

$$H|X \times \{0\} = \overline{H}|X \times \{(0,0)\} = G \circ F|X \times \{0\} = g_1 \circ f_1$$

$$H|X \times \{1\} = \overline{H}|X \times \{(1,1)\} = G \circ F|X \times \{1\} = g_2 \circ f_2$$

I have included this solution because it generalizes the correct solutions I saw when grading papers. Some of your solutions which included formulas like

$$H(x,t) = G(F(x,t),t)$$

correspond to choosing the diagonal path $\alpha(t) = (t,t)$ in I^2. Other solutions have formulas such as

$$H(x,t) = \begin{cases} g_1(F(x,2t)), & \text{for } t \text{ between } 0 \text{ and } 1/2, \\ G(f_2(x), 2t-1), & \text{for } t \text{ between } 1/2 \text{ and } 1. \end{cases}$$

This corresponds to the choice of α which is the composition of the paths

$$H(x,t) = \begin{cases} (0,t), & \text{for } t \text{ between } 0 \text{ and } 1/2, \\ (2t-1,1), & \text{for } t \text{ between } 1/2 \text{ and } 1, \end{cases}$$

in I^2.

Of course, restricting to any choice of α from $(0,0)$ to $(1,1)$ in I^2 will give a desired homotopy.